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A Mathematical Model that Estimates Input Demand in Respect to the Costs for Cotton Production

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Abstract

The agricultural sector is the backbone of economic activities to the farmers in Tanzania. Tanzania produces cotton as a commercial crop that is also produced by more than 80 countries in the world particularly those located in the tropics and temperate climate regions. In Tanzania, cotton is mostly grown in the western cotton growing area (WCGA) and eastern cotton growing area (ECGA). The serious problem facing farmers for cotton production is the continuous increasing costs of inputs in Tanzania. The aim of this paper was to build a mathematical model that estimates input demand in respect to the costs for cotton production. The primary data from 2003 to 2014 were collected from cotton farmers at Bariadi in Shinyanga. Moreover, secondary data were collected from the Tanzania Cotton Board, Ukiliguru Agriculture Training Institute, and Shinyanga Regional Commissioners. A mathematical model was estimated after taking out the reasonable statistical tests. The fixed effect and random effect were compared in the Hausman's specification test. The coefficients (elasticities) in respect to the inputs and other elasticities were estimated by applying Ordinary Least Squares techniques facilitated by STATA 11 and EXCEL.

Keywords: Mathematical model; Cotton production; Cost of inputs; Demand of input.

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1. Introduction

Tanzania is a sub-Saharan African country which has a total area of 945090km2 and a population of about 44928923. The area which can be used for agricultural purpose is approximately 40million hectares or 42% of the total land area. Moreover, Tanzania's economy continues to be led by the agricultural sector which is at least 50% of the GDP and offers employment opportunities for more than 79% of the total economical active population [1]. About 40% of the country's population has been employed with the cotton sector. The people who have been employed with the cotton sector include farmers, cotton ginners and merchants, inputs suppliers, researchers, and other cotton processors and service providers, clothes wholesalers, retailers, and their dependents [2].

In this case, cotton is a crop that requires an extensive investment in land, seeds, pesticides and fertilizer as well as modern technology in order to ensure effective production. Generally, cotton is mostly important in agriculture as it provides raw materials for industries and it contributes to foreign currencies for the benefits of the country and providing employment opportunities in various sectors. However, it is very significant for international traded agricultural commodities in terms of quantity and quality as well. Also, a larger increase in incomes to the cotton farmers and processors depends on balancing of input use in the reasonable costs [3]. It insures sustainability of commercial commodities especially in manufacture of textile and clothes, cooked oil, soap and livestock feeds as well as farm yield manures. The profit model formulation proposed by Lau and Yotopoulos [4] help in making derivation of the input demand as a function of normalized input rates and the quantities of fixed inputs. They developed a mathematical model for estimation of coefficients that was based on input demand function in relation to the input prices for cotton production.

The author in [5] used the Cobb-Douglas production model to estimate the relationship between total aggregated output and inputs such as fertilizer, labour, tractor and machinery services, animal manure, irrigated area, seeds, pesticides, consumed water, and input prices in different crops. The crops involved in his study were wheat, pea, onion, tomato, potato, watermelon, cotton, sugar beet. He found that the estimated coefficient for quantity of output with respect to crops was positively significant.

Also, Muriith [6] used the Cobb Douglas production model to determine the small scale impacts and indicated the expansion of total labour force per capital increase rate for a decentralized economy. The results in his study revealed that there is a positive effect of labour on marginal product of capital. The study was so much concentrated to observe the allocation of inputs and required resources towards production in respect to each crop at River Njoro watershed in Kenya.

The Cobb-Douglas production model was used by [7] to estimate the relationship between total cumulated farm output and its inputs required for production such as fertilizer use, labor supply, and the like. The author noted that irrigation water demand is price inelastic and that estimated water usage goes beyond the actual use across the sample. This gives the same idea to the cotton production that; profitable farm output per hectare depends on the usage of inputs in respect to elasticity of input demand in a particular given price.

The authors in [8] conducted a research about the factors affecting cotton production at Multan district in Pakistan. They used the Cobb-Douglas production model in order to evaluate the impact of different inputs such as cultivation, seed and sowing, irrigation, fertilizers, pesticides, hoeing and labour costs for cotton output. Moreover, the results analyzed from the model of the study have shown that the coefficients for inputs like cultivation and seed have noted to be statistically significant at 1 percent level. This means the variables of cultivation and seed are mostly important factors for determining rate of output with respect to cotton crop.

The concepts of Cobb-Douglas production model were applied to derive the log-linear model in order to estimate irrigation water demand for tomato in Iran. In this study found that water has a low price elasticity of demand for tomato. Also, the concepts of Cobb-Douglas production model have used in this study to develop a mathematical model that estimates input demand in respect to costs in cotton production [9]. In addition to that, the same model was used to evaluate the structure of barley water demand and found that water has a small price elasticity of demand for barley. This means barley farmers are not sensitive enough towards price of water in production [10].

Moreover, the authors in [11] used the dynamic optimization model to derive optimal decision rules of input use for cotton experiment. This model was used for the purpose of determining the technological efficiency gains for irrigated cotton production in USA. The study has identified that precision farming overall would be more profitable than whole field farming.

The Auto Regressive Integrated Moving Average (ARIMA) models have been used for the purpose of predicting the cotton production in India by using time series data from 1951 to 2013. In this study they found that in year 2021 the production of cotton in India will increase to 47.5 million bales. Usually rate of increase in cotton production depends on efficient use of inputs in the reasonable costs to the cotton farmers. Thus, the prediction of cotton production was determined in respect to reasonable costs to the farmers and efficient use of inputs for the given cultivated area [12].

The effective mechanism to construct the price strategy, one requires reliable knowledge that can be used in real circumstances concerning the extent of reaction in respect to requirements of individuals, consistent prices, and technological advances. The study concluded that any change in input and output prices would affect the input demand and output supply at the same time [13].

Cotton farmers are faced with problem of increasing costs in production as the result return is not comparable to the costs used in production. Most studies have used Cobb-Douglas production function to determine the relationship between output and inputs in production. Some have used the function to evaluate the impacts of inputs in output production and others used the model for estimation of irrigation water demand in different crops. All these studies did not give sufficient information for estimation of outputs, total costs, net profit, and input demand in respect to costs in cotton production. Therefore, this study aims to use the concepts of Cobb-Douglas production function to build a mathematical model for estimation of outputs, total costs, net profit, input demand in respect to costs, and analyze the sensitivity of inputs to the input demand function in cotton production in Tanzania.

2. Materials and Methods

The primary data from 2003 to 2014 were collected from the cotton farmers by using questionnaires at Bariadi in Shinyanga; these questionnaires were distributed to 35 people who have enough knowledge about cotton production process. Moreover, secondary data were collected from the Tanzania Cotton Board, Ukiliguru Agriculture Training Institute and Shinyanga Regional Commissioners. A mathematical model was estimated after taking out the reasonable statistical test. The fixed effect and random effect were compared in the Hausman's specification test. The coefficients (elasticities) in respect to inputs and other elasticities were estimated by applying Ordinary Least Squares (OLS) that have been facilitated by STATA 11 and EXCEL.

2.1 Model development

2.1.1 Economic model

Economic model refers to the production function for agricultural purposes because frequently is used to determine the correlation between different inputs and output. The production inputs have been defined to mean land, labour, and capital as the fundamental economic factors in agriculture [1].

The model can be written in the form Q= (LD, C, L_l), where Q is the output and LD, C, L_l represent land, capital and labour respectively. This is the mathematical model for production that creates output from combination of inputs.

The Cobb-Douglas production model has been used in the ground of theoretical and practical point view based on different research purposes. It specifies clearly the relationship of output and inputs in production of any crop. The Cobb-Douglas model is used due to its simplicity to solve and help to make interpretation of the elasticity of production with respect to inputs.

The Cobb-Douglas production model is formulated as follows [5]:

$$Q = A\prod_{i=1}^{n} X_i^{\beta_i} \tag{1}$$

where $Q =$ output, $X_i =$ each bundle of inputs respectively,

A= constant earnings to scale Cobb-Douglas function and

$\beta_i =$ Statistical parameters for elasticity of Q and its sum is written symbolically as $\sum_{i=1}^{n} \beta_i = 1$.

Suppose that C and l respectively represent capital and labour force required to run a particular project. Then the

Cobb-Douglas model can be written in the simpler way as

$$Q = C^{\alpha} l^{\beta} \tag{2}$$

where α and β are parameters. Thus, the total costs (Tc) is formulated as

$$Tc = \sigma C + \lambda l \tag{3}$$

where σ and λ are the parameters associated with labour and capital as well.

From equation (2) and equation (3), a mathematical model can be formulated in order to minimize the costs on producing Q as

$$\begin{aligned} &Min \quad Tc = \sigma C + \lambda l \\ &Subject\ to\ Q = C^{\alpha} l^{\beta} \end{aligned}$$

The Lagrangian multiplier form for the cost minimization problem is:

$$L(C, l, \mu) = \sigma C + \lambda l + \mu\left(Q^{o} - C^{\alpha} l^{\beta}\right) \tag{4}$$

Differentiating the Lagrangian multiplier equation (4) with respect to C, l, μ and equating it to zero will satisfy the first order condition for cost minimization.

$$\frac{dL}{dC} = \sigma - \alpha\mu C^{\alpha-1} l^{\beta} = 0$$

Hence

$$\sigma = \alpha\mu C^{\alpha-1} l^{\beta} \tag{5}$$

$$\frac{dL}{dl} = \lambda - \beta\mu C^{\alpha} l^{\beta-1} = 0$$

Hence

$$\lambda = \beta\mu C^{\alpha} l^{\beta-1} \tag{6}$$

$$\frac{dL}{d\mu} = Q^{0} - C^{\alpha} l^{\beta} \;\Rightarrow\; Q^{0} = C^{\alpha} l^{\beta} \tag{7}$$

However, ratio technical substitution (RTS) can be applied to measure the given inputs in the mathematical model. Thus, dividing equation (6) by equation (5) and on simplifying it, we get

$$C=\left(\frac{\lambda}{\sigma}\right)\left(\frac{\alpha}{\beta}\right).l \tag{8}$$

The input value (C) that is obtained should be substituted into the output equation (2) and making *l* subject, we obtain

$$l=Q^{\frac{1}{\alpha+\beta}}\left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}\sigma^{\frac{\alpha}{\alpha+\beta}}\lambda^{\frac{-\alpha}{\alpha+\beta}} \tag{9}$$

Similarly, the input value of C is obtained by using the same procedures.

Hence

$$C=Q^{\frac{1}{\alpha+\beta}}\left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}}\sigma^{\frac{-\beta}{\alpha+\beta}}\lambda^{\frac{\beta}{\alpha+\beta}} \tag{10}$$

From equation (3) the total cost is written in form

$$Tc=\sigma\left[Q^{\frac{1}{\alpha+\beta}}\left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}}\lambda^{\frac{\beta}{\alpha+\beta}}\sigma^{\frac{-\beta}{\alpha+\beta}}\right]+\lambda\left[Q^{\frac{1}{\alpha+\beta}}\left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}\lambda^{\frac{-\alpha}{\alpha+\beta}}\sigma^{\frac{\alpha}{\alpha+\beta}}\right]$$

$$Tc=Q^{\frac{1}{\alpha+\beta}}\lambda^{\frac{\beta}{\alpha+\beta}}\sigma^{\frac{\alpha}{\alpha+\beta}}\left[\left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}}+\left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}\right] \tag{11}$$

Equation (11) can be written in form

$$Tc=Q^{\frac{1}{\alpha+\beta}}\lambda^{\frac{\beta}{\alpha+\beta}}\sigma^{\frac{\alpha}{\alpha+\beta}}J \tag{12}$$

where $J=\left[\left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}}+\left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}\right]$ which stands for a constant that include parameters of α and β

Hence

$$Tc=Q^{\frac{1}{\alpha+\beta}}\lambda^{\frac{\beta}{\alpha+\beta}}J\sigma^{\frac{\alpha}{\alpha+\beta}} \tag{13}$$

Moreover, on utilizing Shephard's lemma condition by taking partial derivatives can give the functions that are equivalent to input functions in form

$$C(\lambda, \sigma, Q) = \frac{\alpha}{\alpha+\beta} Q^{\frac{1}{\alpha+\beta}} J\left(\frac{\lambda}{\sigma}\right)^{\frac{\alpha}{\alpha+\beta}} \tag{14}$$

Therefore by introducing natural logarithm in equation (14) gives the general expression of log-linear in the form

$$InC(\lambda,\sigma,Q) = In\frac{\alpha}{\alpha+\beta} + \frac{1}{\alpha+\beta} InQ - \frac{\alpha}{\alpha+\beta} In\sigma + InJ + \frac{\beta}{\alpha+\beta} In\lambda\,. \tag{15}$$

2.1.2 Empirical model

A mathematical model was generated through total demanded inputs function in respect to the costs, which considers the average input price, labour force costs, transport costs and cultivation costs. The average input price based on fertilizers, seeds, pesticides, sprayers and animal fertilizers. In mathematical form it can be written as

$$W = f(Q, P_i, P_l, P_c, P_t)$$

The following is the development of a mathematical model that could be used to estimate and minimize the costs of inputs for the cotton production.

Suppose that *Tc* is total costs for cotton production and given in the form

$$Tc = WP_i + LP_l + CP_c + TP_t \tag{16}$$

where *Tc* =total costs and *L*=labour required per hectare,

Labour force has been considered in terms of harvesting by hand picking, weeding in three times per hectare, preparation of land, and planting as well.

$\sum_{j=1}^{5} W_j = W$ is the sum of inputs demanded by a cotton farmer per hectare.

$$\sum_{j=1}^{5} W_j = W = F + Fa + S + P + R \text{ for } j = F, Fa, S, P, R \tag{17}$$

where F = chemical fertilizer; Fa = animal fertilizer; S= seed; P = pesticide; R = sprayers and

W = total demanded inputs in respect to costs, P_i is average input price in respective year, and where P_l, P_c, P_t represents the average of prices with respect to the variables for cotton production.

$$Q = AW^{\alpha} L^{\beta} C^{\gamma} T^{\phi} \quad (18)$$

Thus, from equation (16) and equation (18), the minimization problem can be formulated as follows:

Minimize $Tc = WP_i + LP_l + CP_c + P_t T.$

Subject to $Q = AW^{\alpha} L^{\beta} C^{\gamma} T^{\phi}.$

The Lagrangian function for cost minimization of producing Q is

$$l(\mu, P_i, P_l, P_c, P_t) = WP_i + LP_l + CP_c + TP_t + \mu\left(Q^0 - AW^{\alpha} L^{\beta} C^{\gamma} T^{\phi}\right) \quad (19)$$

where μ is the Lagrangian multiplier. By using the first order conditions for cost minimization and rearranging yields

$$\frac{dl}{dW} = P_i - A\mu\alpha W^{\alpha-1} L^{\beta} C^{\gamma} T^{\phi} = 0$$

Hence

$$P_i = A\mu\alpha W^{\alpha-1} L^{\beta} C^{\gamma} T^{\phi} \quad (20)$$

$$\frac{dl}{dL} = P_l - A\mu\beta W^{\alpha} L^{\beta-1} C^{\gamma} T^{\phi} = 0$$

Hence

$$P_l = A\mu\beta W^{\alpha} L^{\beta-1} C^{\gamma} T^{\phi} \quad (21)$$

$$\frac{dl}{dC} = P_c - A\mu\gamma W^{\alpha} L^{\beta} C^{\gamma-1} T^{\phi} = 0$$

Hence

$$P_c = A\mu\gamma W^{\alpha} L^{\beta} C^{\gamma-1} T^{\phi} \quad (22)$$

$$\frac{dl}{dT} = P_t - A\mu\phi W^{\alpha} L^{\beta} C^{\gamma} T^{\phi-1} = 0$$

Hence

$$P_t = A\mu\phi W^{\alpha} L^{\beta} C^{\gamma} T^{\phi-1} \quad (23)$$

$$\frac{dl}{d\mu} = Q^0 - AW^{\alpha}L^{\beta}C^{\gamma}T^{\phi} = 0$$

Hence

$$Q^0 = AW^{\alpha}L^{\beta}C^{\gamma}T^{\phi} \qquad (24)$$

Take equation (21) to equation (23), and then divide each by equation (20) by using ratio technical substitution (RTS), gives

$$\frac{P_l}{P_i} = \frac{\beta}{\alpha} \times \frac{W}{L} \Rightarrow L = \frac{P_i}{P_l} . \frac{\beta}{\alpha} .W$$

$$\frac{P_c}{P_i} = \frac{\gamma}{\alpha} \times \frac{W}{C} \Rightarrow C = \frac{P_i}{P_c} . \frac{\gamma}{\alpha} .W$$

$$\frac{P_t}{P_i} = \frac{\phi}{\alpha} \times \frac{W}{T} \Rightarrow T = \frac{P_i}{P_t} . \frac{\phi}{\alpha} .W$$

Thus, on substituting in the output formula for Q, we obtain

$$Q = AW^{\alpha} . \left[\frac{P_i}{P_l} . \frac{\beta}{\alpha} .W \right]^{\beta} . \left[\frac{P_i}{P_c} . \frac{\gamma}{\alpha} .W \right]^{\gamma} . \left[\frac{P_i}{P_t} . \frac{\phi}{\alpha} .W \right]^{\phi} .$$

$$W^{\sum \alpha_i} = \frac{Q}{A\left[\frac{\beta}{\alpha} \times \frac{P_i}{P_l} \right]^{\beta} \left[\frac{\gamma}{\alpha} \times \frac{P_i}{P_c} \right]^{\gamma} \left[\frac{\phi}{\alpha} \times \frac{P_i}{P_t} \right]^{\phi}}$$

Required to find *W*

$$W = \frac{Q^{\frac{1}{\sum \alpha_i}} \alpha^{\frac{\beta+\phi+\gamma}{\sum \alpha_i}} .P_l^{\frac{\beta}{\sum \alpha_i}} P_c^{\frac{\gamma}{\sum \alpha_i}} P_t^{\frac{\phi}{\sum \alpha_i}}}{A^{\frac{1}{\sum \alpha_i}} \beta^{\frac{\beta}{\sum \alpha_i}} .\gamma^{\frac{\gamma}{\sum \alpha_i}} .\phi^{\frac{\phi}{\sum \alpha_i}} .P_i^{\frac{\beta+\phi+\gamma}{\sum \alpha_i}}}$$

where $\alpha q = \frac{1}{\sum \alpha_i}$, $z_1 = \frac{\beta}{\sum \alpha_i}, z_3 = \frac{\phi}{\sum \alpha_i}$, $z_0 = \frac{\alpha + \gamma + \phi}{\sum \alpha_i}$, $z_2 = \frac{\gamma}{\sum \alpha_i}$ and

$$B = \frac{\alpha^{\frac{\beta+\phi+\gamma}{\sum \alpha_i}}}{A^{\frac{1}{\sum \alpha_i}} \beta^{\frac{\beta}{\sum \alpha_I}} \gamma^{\frac{\gamma}{\sum \alpha_I}} \phi^{\frac{\phi}{\sum \alpha_I}}}$$

Table 1: Variables and Parameters of Mathematical Model Descriptions

Variables	Descriptions for variables
Q	Output of production obtained from the use of combination of inputs per hectare in Tshs.
W	Quantity of inputs demanded per hectare
P_i	Price of inputs required per hectare in Tshs.
P_l	Price of labour force used per hectare in Tshs.
P_c	Price for cultivation per hectare in Tshs.
P_t	Price of transport per cart in Tshs
ε	Omitted variable
Parameters	Descriptions for Parameters
B	Constant production that obtained from the combination of inputs.
αq	Coefficient of output variable indicates elasticity for cotton production
z_0, z_1, z_2, z_3	Coefficients of the variables that indicates elasticity of cotton production.

$$W = \frac{Q^{\alpha q} B.P_l^{z_1} P_c^{z_2} P_t^{z_3}}{P_i^{z_0}} \tag{25}$$

Similarly, the other factors are solved in the same way to obtain

$$L = \frac{Q^{\alpha q} B.P_i^{z_0} P_c^{z_2} P_t^{z_3}}{P_l^{z_1}} \tag{26}$$

$$C = \frac{Q^{\alpha q} B.P_i^{z_0} P_l^{z_1} P_t^{z_3}}{P_c^{z_2}} \tag{27}$$

$$T = \frac{Q^{\alpha q} B.P_i^{z_0} P_l^{z_1} P_c^{z_2}}{P_t^{z_3}} \tag{28}$$

Then substituting equations (25) to (28) into the total cost function (T_C), yields

$$Tc = WP_i + LP_l + CP_c + TP_t.$$

$$Tc(P_i, P_l, P_c, P_t) = P_i[W] + P_l[L] + P_c[C] + P_t[T]$$

$$= P_i\left[\frac{Q^{\alpha q} B.P_l^{z_1} P_c^{z_2} P_t^{z_3}}{P_i^{z_0}}\right] + P_l\left[\frac{Q^{\alpha q} B.P_i^{z_0} P_c^{z_2} P_t^{z_3}}{P_l^{z_1}}\right] + P_c\left[\frac{Q^{\alpha Q} B.P_i^{z_0} P_l^{z_1} P_t^{z_3}}{P_c^{z_2}}\right] +$$

$$P_t\left[\frac{Q^{\alpha q} B.P_i^{z_0} P_l^{z_1} P_c^{z_2}}{P_t^{z_3}}\right].$$

On simplifying gives

$$Tc(P_i, P_l, P_c, P_t) = Q^{\alpha q} B\left(P_i^{z_0} P_l^{z_1} P_c^{z_2} P_t^{z_3}\right) \times \left\lfloor P_i^{1-2z_0} + P_l^{1-2z_1} + P_c^{1-2z_2} + P_t^{1-2z_3} \right\rfloor \tag{29}$$

The production's system of cost minimizing input demand functions would be calculated by differentiating partially the cost function (T_C) with respect to input prices under Shephard's Lemma condition. Thus, Shephard's Lemma is stated when $W(P_i, P_l, P_c, P_t)$ is assumed to be cotton's conditional input demand function in production and $Tc(P_i, P_l, P_c, P_t)$ is differentiable at P_i for $P_i > 0$ and therefore

$$\frac{\partial Tc(P_i, P_l, P_c, P_t)}{\partial P_i} = W(P_i, P_l, P_c, P_t)$$

$$\frac{\partial Tc}{\partial P_i} = Q^{\alpha q} B\left(P_i^{-z_0} P_l^{z_1} P_c^{z_2} P_t^{z_3}\right)$$

Equation (25) can be written as

$$W = Q^{\alpha q} B\left(P_i^{-z_0} P_l^{z_1} P_c^{z_2} P_t^{z_3}\right) \tag{30}$$

Taking the natural logarithm of both sides of equation (30), gives

$$\text{In} W = In\left\lfloor Q^{\alpha q} B\left(P_i^{-z_0} P_l^{z_1} P_c^{z_2} P_t^{z_3}\right)\right\rfloor$$

$$InW = InB + \alpha qInQ - z_0 InP_i + z_1 InP_l + z_2 InP_c + z_3 InP_t + \varepsilon \tag{31}$$

where z_i = the coefficient to be estimated for i=0, 1, 2, 3.

Therefore, equation (31) is log-linear simply because a dependent variable and the independent variables have been transformed into natural logarithmic form. In this case, the coefficients of a log-linear equation had been explained as production elasticities.

3. Results and Discussion

3.1 Descriptive statistics from 2003 to 2014

Table 2: Statistical analysis of primary data.

Variable	Obs	Mean	Std. Dev	Min	Max
Output (Q)-Tshs/ha	12	337266.7	152117.8	178200	570000
Netprofit (Tshs/ha)	12	62933.33	98542.83	-41000	218000
Idemand (W)	12	63000	9671.89	54500	82000
Labour (Wages)-Tshs	12	176333.3	40207.04	124000	220000
Transport (Tshs)-T	12	12500	6908.493	5000	20000
Cultivating costs (Tshs/ha)	12	22500	5838.742	15000	30000
Av-input price (P_i)	12	281200	60341.63	204650	359250
Total costs (Tshs)	12	274333.3	59802.45	198500	352000

Table 3: Statistical analysis of secondary data from 2003 to 2014

Variable	Obs	Mean	Std. Dev	Min	Max
Idemand (W)	12	62466.67	8226.27	47500	78500
Output (Q)-Tshs/ha	12	218203.3	230068.5	41280	693000
Labour (Wages-L)-Tshs/ha	12	207375	54557.36	185000	340000
Transport (Tshs/cart)-T	12	11041.67	4579.889	5000	20000
Cultivating costs (Tshs/ha)-C	12	22500	5838.742	15000	30000
Av-input price-Tshs/ha	12	226581.2	230078.2	47280	700500
Total costs (Tshs)	12	343383.3	69911.79	252500	468500
Netprofit (Tshs)	12	-125180	170417.5	-264620	270000

From the primary data in Table 2 it is observed that the average output per hectare is 337267 Tanzanian shillings and the average total costs used to produce the cotton output is 274333Tanzanian shillings. The net profit obtained due to difference between cotton output and total costs used for production is averaged equal to 62933Tshs. The net profit earned by cotton farmers in producing cotton per year is too little amount. This proves that the cotton farmers incur high costs for cotton production. Moreover, the average costs for inputs demanded by cotton farmers in cotton production is 63000 per hectare while costs for labour force, transport and cultivating costs are 176333Tshs, 12500Tshs and 22500Tshs respectively, contributing to increase costs for cotton production to the farmers.

In this case, the data presented in Table3 have shown that the average cost for inputs demanded by cotton farmers per hectare is 62467. However, labour force costs, cultivation costs and transport costs are 207375Tshs, 11042Tshs, and 22500Tshs per hectare respectively. When the average output is 218203Tshs and the total cost is 343383Tshs, its net profit is negative 125180Tshs which indicates that the cotton farmers have received loss in cotton production.

Therefore, from the two analyses in Table 2 and Table 3, it can be concluded that the cotton farmers incur high costs for cotton production as Table 2 displays small amount of net profit while net profit in Table 3 shows negative to mean that cotton farmers had received loss amount in cotton production. The costs of cotton production continue to increase as what Tanzania Cotton Board reported in 2008/9 season, that the income received by cotton farmers when compared with total costs of 174.2 dollars which is equivalent to 265655Tshs per hectare is almost negligible. Furthermore, loss of single marketing channel pushed up the costs of marketing chemicals and leads to collapse in supply and distribution. So farmers could not able to afford to purchase chemicals at market prices. The average costs for pesticides increased from 1600Tshs a kilogram in 1993/94 to 5000Tshs in 1998/99 which demonstrates the rate of increase was about 25 percent per year in nominal terms [14].

3.2 Regression results

A mathematical equation was generated as total demanded inputs function in respect to costs, which includes the average input price, labour force costs, transport costs, and cultivating costs. Where an average input price has obtained from the fertilizers, seeds, pesticides, sprayers and animal fertilizers as all these are the inputs required for cotton production.

The primary data were collected through distribution of 35 questionnaires to cotton farmers who have real knowledge about cotton production in Tanzania. Also, secondary data were collected from the Tanzania Cotton Board and Ukiliguru Agriculture Training Institute.

From Table 4, the variables that have been narrated in terms of natural logarithm were estimated by using Ordinary least Squares (OLS) as indicated in a mathematical model. Thus, from the results the adjusted R^2= 0.9975, it means that 99.75% of input demanded costs per hectare in cotton production is perfectly fit to the model for the data obtained in survey in Shinyanga-Bariadi of 2015. So the model can be written as

$$InW = -5.31 + 0.04 InQ + 4.013 InP_i - 2.45 InP_l - 0.304 InP_c - 0.21 InP_t$$

The estimated coefficient for input price (P_i) is positive at 1% level. The coefficient obtained is equal to 4.013. This indicated that input demand for costs in cotton production was infinitely elastic and hence cotton farmers were seriously sensitive to the changes in input prices. Therefore, there is positive relationship between input price and input demand costs function for cotton production. The coefficients for labour force, cultivation, and transport are also negative at 1% level, which shows that input demand cost function with labour force; cultivating and transport are complementary inputs. Where 1% increases in labour force, cultivating and transport will contribute to reduce the costs for input demand in cotton production by 2.45%, 0.304% and 0.21% respectively. Since the coefficient of output is 0.04, which is positive. Thus, there is positive relationship between cotton output and input demand for costs and this demonstrates that as cotton output grows by 1% will lead to change input demand for costs per hectare by 0.04% which is statistically significant at 1% level.

Table 4: Regression analysis

Dependent variable: *InW*				
Independent variable	Coefficient	Std. Error	t-statistic	Prob
B	-5.31	1.00	-5.29	0.002
InQ	0.04	0.32	1.23	0.264
InP_i	4.013	0.29	13.84	0.000
InP_l	-2.45	0.17	-14.77	0.000
InP_c	-0.304	0.03	-10.48	0.000
InP_t	-0.21	0.02	-9.99	0.000
Cross-sectional fixed(dummy variables)				
R-squared	0.9986			
Adjusted R-squared	0.9975			
F-statistic	871.97			
Prob	0.000			
Number of obs	12			
Statistically Significant at 1% level				

Therefore, negative sign of the coefficients in the regression analysis from primary data indicates that labour force, cultivating, transport and input prices have effects in evaluation of input demand function for costs in relation to the cotton output as what Mumtaz [8] said in 2009 for his study that the coefficients for inputs like cultivation, fertilizers, seeds, and pesticides have been noted to be statistically significant at 1% level in producing cotton.

From the findings of Table 5, the variables of a mathematical model were estimated by using Ordinary least Squares (OLS). Thus, from the results the adjusted R^2= 0.48, which makes sense for 48% of input demand per hectare in cotton production is perfectly fit to the model. Thus the model should be addressed as

$$InW = 5.42 - 2.3InQ + 2.39InP_i + 0.85InP_l + 0.196InP_c + 0.15InP_t$$

At 1% level, the estimated coefficient for input price is positive to indicate that input demand was elastic. This is simply because the coefficient of input price is 2.39. Thus the cotton farmers were sensitive to the changes of input price. Moreover, labour force, cultivation, and transport have positive coefficients of 0.85, 0.196, and 0.15 respectively at 1% level. This means that 1% increase in labour force; cultivation and transport will lead to increase the input demand for costs by 0.85%, 0.196% and 0.15% correspondingly. In this case, cotton farmers will get loss in the sense that there is no any net profit received from cotton production per hectare as shown with the coefficient of output is negative, which is equivalent to -2.30.

Table 5: Regression results of the data from the Tanzania Cotton Board and Ukiliguru Agriculture Training Institute

Dependent variable: *InW*				
Independent variable	Coefficient	Std. Error	t-statistic	Prob
B	5.42	4.37	1.24	0.262
InQ	-2.30	1.91	-1.20	0.274
InP_i	2.39	2.09	1.14	0.298
InP_l	0.85	0.47	0.18	0.861
InP_c	0.196	0.38	0.51	0.626
InP_t	0.15	0.13	1.19	0.280
Cross-sectional fixed(dummy variables)				
R-squared	0.7165			
Adjusted R-squared	0.4802			
F-statistic	3.03			
Prob	0.1047			
Number of obs	12			
Statistically Significant at 1% level				

The regression analysis in Table 5 gives evidence for what Tanzania Cotton Board [2] reported in Match 2010 that the net profit earned by cotton farmers is almost negligible when compared with the total costs used in cotton production per hectare.

Therefore, the findings of Table 4 and Table 5 provides proof that cotton farmers in Tanzania incur high costs in cotton production per hectare in such way that net profit is almost insignificant.

4. Conclusions and Recommendations

4.1 Conclusions

The cotton production system was investigated with high concentration to the input demand in respect to its costs. The input demand for costs required in cotton production per hectare was estimated by using primary and secondary data from 2013 to 2014. From findings the cotton farmers have received output in the range of 218200 to 337200 Tanzanian shillings per hectare, which is too little amount compared with the total costs used per hectare that ranges from 274300 to 343300 Tanzanian shillings. Thus, on comparison it shows that a farmer receives small net profit and sometimes gets loss in producing cotton. The input prices observed to have power to influence the input demand function and the cotton output as both coefficients for primary and secondary data in all mathematical models are 4.03 and 2.39 which are narrated to be positive respectively. This means that the cotton farmers are really sensitive to the changes of input prices that contribute whether to have profitable output or output with loss when compared to the total costs used per hectare. Also the interpretation can be stated as higher input prices, then will lead to lower input demand and thus lower the cotton output and its visa versa is true.

4.2 Recommendations

Since cotton production is so important for the national economy as well as alleviation of poverty to the cotton farmers in Tanzania. From the findings of this paper, it can be recommended and suggested that education to the cotton farmers should be emphasized and be given priority in order to increase effectiveness and efficient on using cotton inputs in cotton production. This can help to raise productivity of cotton for life sustainability of the people.

Also Tanzania Cotton Board should work together with government in order to establish good policy that can be specifically for cotton production and should be implemented in the physical sense for reasonable time. This policy should be addressed in such a way that initiation of textile and clothes industries should be given priority so that to accommodate cotton quantity produced by the farmers. The profitability of initiation of textile and clothes industries, the government can help to provide and solve problems of cotton market and thus the farmers can receive motivated price.

Moreover, the government policy should state specifically on the provision of incentives for cotton inputs to the farmers so that they will be motivated intrinsically to produce cotton in the reasonable costs.

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