Impact of Aging and Pension System in Ecuador

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Abstract

This paper about “Impact of aging and pension system in Ecuador” will examine the effects of population ageing on the medium and long term evolution of pension systems in Ecuador, between the years 2000 to 2011. It is organized into six sections. After the introductory section, section 2 provides demographic information on ageing trends in the Latin America countries, in international context. Section 3 gives information about the institutional framework in Ecuador, then in section 4 provides indicators, data and methods of analysis, and examines the effects of ageing population structures and rising life expectancy on pension system variables: spending on pensions, the financial position, pension liabilities and the implicit rate of return; it also includes a critical analysis of the criteria about the sustainability of unfunded systems. Section 5 provide a more systematic introduction to the concept of and indicators used for implicit pension debt. In the last section contains conclusions and recommendations.

Keywords: Pension system; ageing; social security; implicit pension debt.

1. Introduction

This paper will examine the effects of population ageing on the medium and long term evolution of pension systems in Ecuador, between the years 2000 to 2011. Analyses too, effects aging population structures and rising life expectancy on pension system. Using some variables like spending on pensions, the financial position, pension liabilities and the implicit rate of return; it also includes a critical analysis of the criteria about the sustainability of unfunded systems and indicators used for implicit pension debt.

Although Latin America and Ecuador is a long way from the situation of the world’s most aged countries, it is forecast to age much more quickly than did the societies that are currently more advanced. These changes are much less protected than they are now in the more developed countries.

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Consequently, it is of the greatest importance for the countries in the region to begin assessing the different policy options available so that they can arrive at the decisions that are most appropriate to their demographic and economic conditions, now and the near future.

This paper has shown what might be the general trends in demographic indicators in Ecuador, pension spending, financial balance, pension debt and the pension system rate of return if conditions in the system, the labor market and the economy remain unchanged. It will see that it is very difficult to determine what pension liability levels are “sustainable” or “unsustainable”. If future ageing scenarios come about with system rules unchanged, Ecuador could see their expenditure, and perhaps their deficit levels, rising to meet those of the countries that are currently more advanced.

The reform has different options currently being discussed; it is worth mentioning one that involves introducing contribution-defined mechanisms into unfunded systems. This method allows for funding of contributions at a rate that matches growth in total wages, providing benefits that are actuarially fair between cohorts without the need to introduce funding.

One of the most serious problems the pensions system in Latin America [1], is the important weight of the population who does not rely on the access to the services of pension that it allows them to face his needs of consumption and saving in the oldness. Opposite to the risk of longevity, which is inevitable, there is needed that to the majority, if there is possible the totality of the adult major population, an annuity is guaranteed. For it is needed to possess the necessary sources of financing to endow a pension system under the beginning of equity, solidarity, universality, efficiency and effectiveness.

The pension model in Ecuador is founded by Bismarck, due to the fact that in it take part three fundamental actors, who are the affiliated workers, the employer on the part of the company and the State [2]. Under this model, the individuals who enjoy the services of the system, are incorporated as human or hard-working resources specially in the modern sector; Nevertheless in general in Latin America and in the Ecuador is very numerous the population who is employed at the informal sector and not have an employer then, the worker is not covered by the system of social safety. In the following sections we will examine some aspects of the reform, from the perspective of the demographic aging and his implications on the financing and the intergenerational distribution.

2. **Ageing trends in Latin America**

In Latin America the coverage of the system of social safety reaches 43 % of the economically active population (PEA). The contributing model reproduces the inequality in the access to the services of social safety, due to the existence of an authentic barrier at the entry to the affiliation for the side of the incorporation to the labor market, in view of which there is too slow the capacity of absorption of available workforce on the part of the formal or modern sector of the economy, reason for which the moment to enter to the system of social slow safety or simply it does not come; this forces to an important percentage of the economically active population to remaining out of the system of pensions.
Ageing is reached when people live longer and choose to have fewer children. Not surprisingly, therefore, patterns of declining fertility and mortality over the past two decades have led to significant shifts in the age structure of the world’s population. Although the speed most advanced in Latin America, population ageing is occurring, in all the major areas of the world (see figure 1). The share of older people (aged 60 years or older) increased from 9 per cent in 1994 to 12 per cent in 2014, and is expected to reach 25 per cent by 2050.

Older people are the world fastest-growing age group. In 2014 the annual growth rate for the population aged 60 years or older will be almost triple the growth rate for the population as a whole. In absolute terms, the number of people aged 60 years or older will be almost triple the growth rate for the population as a whole. In absolute terms, the number of people aged 60 years or older has almost double between 1994 and 2014, and people in this age group now outnumber children under the age or 5. The growth in the number of older people was fastest in Latin America and the Caribbean.

The average of the life expectancy on having been born already has reached the brand of seventy five years, and the life expectancy at the age of 60 is almost 25 years in the region in his set.

From the five year period of 1950-1955 up to the current importance (2010-2015), an inhabitant of the region won in mean more than 23 years in the duration of his life - an increase of almost 4 years in every past decade-, up to reaching the average current life expectancy of almost 75 years.

In the region, a great heterogeneity exists between countries with regard to the aging: some of them have, and they will continue having many years more, age distributions are young enough (for example, Nicaragua, Honduras, Bolivia). An important number is in the intermediate stages of the demographic transition and there is projected that they will age significantly during the next decades (including two more settlements, Brazil and
Mexico), whereas others are already relatively aged, as Uruguay, Argentina and some of the Caribbean. Countries like Chile, Costa Rica and Panama are a bit less aged, since they have limited levels of mortality, but not such a low fecundity still.

The intergenerational transfers realized by means of familiar / community mechanisms or of market [6], they have been traditionally important, and continue it being so much in the developing countries as developed. In Latin America, some countries had begun the institutionalization of his systems of pensions at the beginning of the 20th century (as Argentina, Chile and Uruguay), but the majority of the national systems became more generalized in the period of the postwar period.

![Figure 2: World ageing outlook, by region, 1990-2050](image)

Figure 2: World ageing outlook, by region, 1990-2050 (Population aged 65 and over/population aged 15 to 64). Source [7].

The systems suffered also a series of problems. There have been difficulties to reach the ideal one of the universal coverage, to assure a complete and opportune collection of the contributions, to protect the royal value of the reserve funds, and to realize adjustments in the contributions and the benefits needed by the demographic changes and the ripeness of the systems [7]. The problems accumulated with the years and there were accentuated by the economic crisis of the eighties in the region, which made more evident and visible the administrative and financial deficiencies.

These changes are considerable enough: while in 1990 the relation of dependence of the oldness (population of 65 and more years with regard to that of 15 to 64 years) was changing between 6 % and 18 % between regions (world average = 10 %), there is projected that in 2030 all the regions, except Africa, they will have relations of dependence that will range between 16 % and 35 % (world average = 17 %). The population of Latin America and the Caribbean will age with major rapidity that the average, since there is projected that in her the relation of dependence of the oldness will reach the world average in 2030, but from a number lower than the average (8 %) in 1990, and that in 2050 the relation of Latin-American dependence will treble fully to come to near 27 %, approximately three percentage points on the world average.
In the measure that is advanced in the demographic transition, it begins and the process of aging hastens and the relations of dependence of the major persons start rising. In 2010 already it had increased in several countries of the region: the Argentina, Chile, Cuba and the Uruguay.

The new demographic age is characterized by a rapid increase of the relations of dependence of the major persons in all the countries [8]. It will overcome 60 % in Cuba and will approach 40 % in Chile and the Uruguay, and will have increased even in Guatemala. But the impressive change of the demographic panorama in this new age will be estimated clearly in 2070, when it passes to a distribution of the relations of dependence diametrically opposite to the one that was observed in 1950. This year all the countries will line up in very low levels of relations of young dependence - a homogenization of this feature-, and will reach high levels of the relation of dependence of major persons, though with certain variability.

Nevertheless, in general terms, a clear trend will happen to the homogenization of this phenomenon, since also one has seen in all the other demographic analyzed components.

In Latin America, since in the rest of the world, the majorities of the public systems of pensions are of distribution and possess relatively scanty reservations. The public expenditure in social safety, on a worldwide scale has reached in average between 8 % and 10 % of the GDP in the Organization Economic of Cooperation and Development (OECD) and in the countries of Eastern Europe [9]. In many industrial countries, the expense appears between two or three principal expenses of government. The projections of the OECD [10] foresee that in the 21st century the payments of pensions might fluctuate between 5 % and 20 % of the GDP, and that the deficits might fluctuate between 0 % and 10 % of the GDP, if substantial adjustments are not realized in the next years.

Latin America shows a global level of expense in pensions similar to that of Asia, of the order of 2 % to 3 % of the GDP in average. The comparison of this level added of expense with the indicators of aging mentioned before reveals an important gap between the Latin-American countries and the regions and the most developed countries, which have indicators of aging that duplicate or treble those of Latin America, but that spend four or five times more in pensions, in relation with his GDP. We will take again the analysis of these differences in the section. Nevertheless, there are several differences between both regions: in Asia, the public systems are in general of more recent creation, and tend to cover a minor proportion of the labor force and of the persons of age [11].

Full capitalization is necessarily the only solution or the most suitable for the problems of the systems of pensions, since some observers raise. Nowadays, the majority of the analysts agree in the studies and complete evaluations of politics must consider different options, which include diverse degrees of distribution and capitalization, systems defined by benefits and by contributions, and combinations of public and private participation [12], taking it in counts the different national, both demographic and economic and political conditions. The following sections of this study examine the trends of medium term of the systems from a demographic wide perspective, analyzing first several indicators of the systems of pensions and his relation with the aging. Later they are considered to be the obligations linked to the pensions in the systems of distribution,
and the paper that they recover in them the age structure of the population and the mortality in advanced age. Here in after there are approached the fiscal costs come from the transitions of the system of distribution to that of capitalization, then to conclude with final observations.

Almost all the indicators of the systems of pensions are influenced by demographic variables, especially and in more direct form by the aging of the population. In this article we have chosen four variables to illustrate our analysis of the effects of the aging on different aspects of the systems of pensions of distribution: i) the annual expense in pensions, expressed as percentage of the GDP; ii) the annual balance sheet of the system of pensions - is to say, income fewer expenses-, that it can express as a fraction of the income of the system or of the GDP; iii) the obligation pensional implicit named also debt pensional implicit, and iv) the implicit profitability (or intern) of the system, which corresponds to the flows of the payments of contributions and of the perceived sums for concept of pensions during the life of a cohort. In the annex figure more details on the concepts, measures and information used for his calculation. The most standard indicators, as the annual expense in pensions or the financial balance sheet of the system, are periodic measurements (it is to say, annual flows) that they serve to quantify the gravitation of the system of pensions on the national economy, but that give little information about the financial sustainability of medium or long term of the system or of his distributive performance, especially in different generational groups. The latter aspects catch them better, though in a partial and imperfect way, other indicators summarize of the flows of contributions and benefits across the life cycle.

We have chosen d, the relation of dependence of the oldness, that in forward we will be call simply a relation of dependence, as the indicator of the demographic aging for the analysis of the indicators selected of the system of pensions.

In the next graphic the expense in pensions as percentage of the GDP, estimated on the base of the information compiled by the OIT of the national expense in social safety in the period 1991-1993. These estimations refer to the old age pensions, survival and invalidity of the public and private sector, and exclude the transfers done to other programs.

There is no a narrow manifest relation between the degree of aging and the expense in pensions. This reflects partly the problems of comparability of the information of expense, but probably what is more important, you differ in the coverage of the population of the system of pensions, the rates of price and replacement, the conditions of eligibility and the degree of ripeness. These two types of limitations are present.

Nevertheless, the information, opposite to the proportion of the 60-year-old population and more, suggests a couple of things. First, since already it had been foreseen, a great number of countries have an expense of the order of 2 % of the GDP or less; between them, a valuable number of Caribbean and Central American countries, Ecuador and Bolivia. The remaining countries have an expense that goes from a minimum near to 3 % in Colombia to a maximum of more than 8 % of the GDP in Cuba; in an intermediate level countries are as Argentina, Panama and Chile. Considered in his set, this information reveals low levels of expense to medium, according to the international standards.
The surplus balance sheet is the norm, especially in the countries with more incipient systems. The exceptions are small deficits of Panama and the most substantial of Chile and Cuba.

In sum, the information newly analyzed gives general useful information about the indicators and the relations of interest, but they offer possibilities rather limited for his more detailed study and specific. In order to isolate and to analyze the effects of the aging on the systems of pensions, the most viable and productive approach consists of using models of national level, but disintegrated according to age, which the key parameters of the system bear in mind. The following model, who needs to do abstraction of several of the details and peculiarities of the systems (see the annex), gives approximate orders of magnitude of the indicators of interest, and not precise estimations. The cost that supposes the simplification provides as compensation, additional, the possibility of studying a more wide range of levels of aging (beyond the observed ones till now in Latin America) and to examine other important variables of the system, as the obligations for concept of pensions and the profitability intern of the system, which they are extremely difficult or impossible to calculate directly only on the base of the existing information [13].

The demographic aging can transform the surplus of the initial balance sheet when the populations are "young" (d minor that 0.3), said Bravo, in considerable deficits, up to of the order of two times the income of the system, or of 6 % of the GDP, when the populations become very aged (d about 0.8) under average constant parameters of the Latin-American systems.

3. Impact of ageing on the expenditure and financial balance of the system in Ecuador

The performing projections of the financial situation of pensions systems consists of using accounting identities to infer expenditures and revenues. In the case of expenditures the behavior of some agents and the functioning of markets is not explicitly modeled.

By definition, pension expenditures are given by the following equation:

\[ \text{Pension expenditures} = \text{Number of pensions} \times \text{Average pension} \]

Typically the number of pensions are projected using population and employment forecasts over the future, where, for earnings related pension schemes, employment years are converted into pension entitlements. Similarly using some macroeconomic scenario for wage growth, average pensions into the future are computed by, first, computing the rate of growth of the benefits of new and current retirees and, secondly, taking out the benefits corresponding to exits due to deaths. There are many dimensions in which population and macroeconomic scenarios are enriched to improve the forecasting of pensions system expenditures.

The main elements needed to perform this type of exercises, that is, demograpic and macroeconomic projections. But before discussing them, it is noteworthy that, under the same approach, there is an alternative, simpler way of making proyections of pension system expenditures. By scaling expenditures with respecto to GDP [14]:

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It follows that the ratio of pension expenditures to GDP is just the product of three factors: i) a demographic factor – the ratio of the retired population to working age population [15], ii) a labor market factor – the inverse of the employment rate, and iii) an institutional – economic factor – the ratio of average pension per retiree to average labor productivity, the very much depends on rules about computation of pension benefits and indexation of pensions, and other provisions determining how wage growth gets translated into the rate growth of the average pension. Using this, the uncertainty about the evolution of pensions expenditures is reduced since i) the age structure of the population is easier to predict that just the size of each cohort, ii) the employment rate has a lower range of variation that total employment, and iii) the ratio of average pension per retiree to average labor productivity is also easier to foresee as it depends on precise rules which can be specified under different scenarios (See table 1).

Table 1: Ecuador: Pension expenditure. Calculus like percentage of GDP. Source [18].

<table>
<thead>
<tr>
<th>Years</th>
<th>Retired Population / working age population (1)</th>
<th>1/employment rate(2)</th>
<th>Average pension per retiree (3) US$</th>
<th>Average labor productivity (4)</th>
<th>Pension expend. /GDP <em>(1)</em>(2)*(3)/(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>0,0262</td>
<td>1,60</td>
<td>777,31</td>
<td>3,016,95</td>
<td>1,077%</td>
</tr>
<tr>
<td>2006</td>
<td>0,0342</td>
<td>1,38</td>
<td>2,721,05</td>
<td>6,239,55</td>
<td>2,064%</td>
</tr>
<tr>
<td>2009</td>
<td>0,0392</td>
<td>1,53</td>
<td>3,376,21</td>
<td>9,382,56</td>
<td>2,155%</td>
</tr>
<tr>
<td>2011</td>
<td>0,0445</td>
<td>1,61</td>
<td>4,164,42</td>
<td>11,931,04</td>
<td>2,495%</td>
</tr>
</tbody>
</table>

Figure 3 illustrates the tendencies forecast for these two variables as the population ages. Isolates the demographic effect in the usual way by setting the value of other accompanying factors and examining the change in the indicators that concern us within a plausible range for our ageing variable dependency ratio (d). Four reference points are graphed for each system indicator. The four points, from left to right, are obtained...
from the demographic profile (adult age structure and mortality schedule) of Ecuador in 2000, 2006, 2009 and 2011. In addition, to provide an idea of the way pension system indicators may change over the longer term.

The chart shows the positive relationship between pension system spending and ageing (indexed by \( d \)), other things being equal, that is embedded in the model (Annex) conditions. The values for the data confirm the fact, that when system cover a fraction of a demographically “young” working-age population, pension spending may be just 1% or 2% of GDP, but that this can reach levels of close to 10% when the country’s population reaches and advanced state of ageing, even if coverage remains as incomplete as it is at present in Latin America [16]. Spending levels can be even higher in practice when coverage is wider or universal: pension spending in Italy is already about 15% of GDP and, as has already been noted, it is expected to go even higher than this in some OECD countries over the twenty-first century.

Another idea of the aggregate financial performance of the system is obtained from the balance that is the difference between the system’s annual revenue from contributions and its yearly pension payments.

**Figure 3:** Ecuador, expenditure and dependency. Source [19].

Figure 4 show this indicator: the balance as a percentage of GDP. It is also linear function of \( d \), in accordance with the conditions of the model. It suggest that, if the system variables (contribution rates, replacement rates and membership conditions) are no adjusted, population ageing can turn a surplus in the initial balance when population are “young” \( (d \text{ less than 0.3}) \) into substantial deficits, which may amount to as much as twice system revenue or 6% of GDP when population ageing is very advanced \( (d \text{ of around 0.8}) \).

4. **Sustainability of unfunded systems and intergenerational equity**

Define “sustainable” level of pension spending or an “unsustainable” deficit level, in practice, involves many simultaneous variables [17]. Figure 5 plots the implicit rate of return of the pension systems \( (\rho) \), which reflects the net lifetime benefit or participating in the system for a given individual or cohort. What \( \rho \) measures is the
excess proportionate present value that the individual or cohort obtains in pension benefits over and above the value of the contributions paid into the system over their working life, an indicator analogous to the one used to measure the rate of return on financial investments. As in the case of the variables plotted previously, this indicator is calculated in accordance with the general assumptions of the model (Annex), with two alternative scenarios now being considered: firstly, the case where the system variable remain constant throughout the ageing process and the general government budget absorbs any imbalance that may arise in the pension system and, secondly, the case where the ratio between contributions and benefits is planned in such a way that it adjusts to changes in the demographic profile and financial balance is obtained at all times.

Figure 4: Ecuador, financial balance and dependency. Source [19].

Figure 5: Ecuador, rate of return and dependency. Source [19].

The line shows that when the contributions / benefits ratios are relative constant, the greater adult longevity that occurs in the ageing process leads firstly to a slight fall in ρ. This happens because the greatest declines in adult mortality are obtained first for the “young adult” or working ages, while later declines mainly occur at more advanced (retirement) ages. This means that the ratio between the number of years lived as a pensioner and the
number lived as a worker first remains constant or declines somewhat, then rises sharply.

The implicit pension debt is a useful summary indicator and, as the following sections show in some detail, is essential for understanding and measuring the fiscal cost of closing an existing pay as you go system. In some cases, a more consistent diagnosis can be obtained when spending, the financial deficit and the pension debt are all substantial, as for example in Ecuador, a low dependency system to have a big implicit debt and at the same time, a permanent surplus as demonstrated by the model. In fact, Ecuador, has a big pension debt, had a modest operating surplus in the 2000s.

**Figure 6:** Ecuador, liabilities toward actives and dependency. Source [19].

When all the indicators point in the same direction, the need to make major adjustments to the system becomes more obvious. In figure 6 plots the pension liabilities (“debt” owed by the Government for pensions) towards the economically active population (Da) expressed, as a proportion of GDP. Then figure 7 plots the pension liabilities towards the retires (Dr). Then in figure 8 plots the implicit pension debt in Ecuador for years: 2000, 2006, 2009 and 2011.

**Figure 7:** Ecuador, liabilities toward retireed and dependency. Source [19].
Uncertainty about the future, particularly in the medium to long term, means that in any specific instance these assessments have to be interpreted with care, since the accuracy of the results depends directly on the validity of the assumptions made.

Figure 8: Ecuador, implicit pension debt and dependency. Source [19].

5. Conclusions

Population ageing in Ecuador’s case is a key underlying cause of the system’s financial problems. This paper has shown what might be the general trends in pension spending, financial balance, pension debt and the pension system rate of return if conditions in the Ecuadorian economic system, the labor market remain unchanged. The analysis suggests that, if future ageing scenarios come about with system rules unchanged, the Ecuador could see their expenditure, and perhaps their deficit levels, rising to meet those of the countries that are currently more advanced. It was seen, though, that it is very difficult to determine what pension liability levels are “sustainable” or “unsustainable”, both in general, except in a few rather extreme cases. Rising expenditure, due in part to long term population ageing, should not be regarded as an imminent crisis or bankruptcy. However, as it reflects a natural reallocation of resources within the life cycle owing to the increase in life expectancy among individuals and cohorts. The same is true in the case of pension liabilities, which naturally tend to be higher in countries with older populations and with mature systems that have greater coverage. If systems have to be adapted, parametric changes to offset the effects of population ageing may often be enough. Then if reform options need to be evaluated, in economics like Ecuador, in the light of numerous possible permutations of public, private and mixed systems and funded or unfunded schemes.

The used model (annex), which has had to leave out a number of the details and peculiarities of pension systems, gives rough orders of magnitude for the indicators we are concerned with, and not exact calculations.

The Ecuadorian empirical data are not conclusive either, except for a few specific aspects. This paper examined one direct effect of switching to a funding system, the need to make the implicit pension debt explicit, and the
fiscal costs that this entails. The model used considers the age structure of the population and shows the important role that this plays. It also shows the lesser effect of old-age mortality and the more substantial one of the system’s population coverage, labor market factors and other system parameters such as contribution and replacement rates. Ecuador that is at an intermediate stage of ageing and has systems with narrow population coverage show high levels of implicit pension debt, which is smaller than their annual GDP (79.7%).

Among the different reform options currently being discussed, it is worth mentioning one that involves introducing contribution define mechanisms into unfunded systems. For example in Ecuador nowadays the domestic employment could pay a symbolic contribution (USD 10 dollars) each month, during 30 years; in exchange they will be able to accede to the system of pensions.

There is recommended that the State, with initiative of the government it generate incentives to take advantage of the demographic bond, of such form the following generations are less struck by the aging of the Ecuadoran population. For example optimizing the investments financed with the surpluses that generate the system of pensions now or increasing the retirement age.

References


conferencias No.2 Seminario Responsabilidades Fiscales en Sistemas de Previsión, Santiago, Chile, ECLAC, 2-3 Sept. 1999, pp. 383-393.


[13] At first, another approach would be to realize longitudinal analyses, for countries with chronological reliable series of information on expense and revenue. This needs statistical consistent and complete series that they include at least four or five decades.


[15] This is the “dependency ratio”. If the coverage rate of the pension system –the number of pensions to the eligible population-, remains constant, then this dependency ratio is just the quotient between the ratio of the number of pensions to employment and the coverage rate.


Annexe:

**Model equations that link the ageing with pensions system**

The following set of equations is used in the paper of Bravo (2000). This model allows studying the relationship between a standard ageing index in the analysis of pension systems, the old age dependency ratio \( d \), i.e., the ratio or retirement age to the working age population, and selected pension system indicators. The equations are based on the assumption that the contribution rate \( c \), the replacement rate \( r \) and the ratio of covered workers to the waged work force \( k \); in the Ecuadorian case won’t be constant, because change with the information to the annual household surveys (ENEMDU). Assuming additionally that pensions are calculated as a proportion of real wages, it follows that:

1. Pension system expenditure \( (E) \), expressed as a fraction of annual GDP \( (Y) \) is:

\[
E = r \times s \times k \times d
\]

Where \( s \) is the share of the wage mass in GDP.

2. Since system revenue from contributions is \( C = c \times s \times k \), the system’s financial balance \( (C-E) \), expressed as a ratio to GDP is:

\[
F_i = s \times k \times (c - r d)
\]

and the balance as a fraction of revenue is:

\[
F_{ii} = 1 - \left( \frac{r}{c} \right) d
\]

3. The implicit pension debt (see Bravo and Uthoff, 1999) is given by:

\[
D = s \times k \times (c \times A_{a} + r \times d \times A_{r})
\]

Where \( A_{a} \) is the discounted average number of years of contribution by the economically active at a given moment in time, and \( A_{r} \) is the discounted average expected lifetime of all those in retirement ages at the given moment in time.

4. The pension system’s implicit rate of return \( (\rho) \), when the system rules are fixed throughout the cohort’s adult lifetime (Bravo 1996, p. 127), can be written as:

\[
\rho_i = \frac{1}{(A_{R} - A_{W})} \left[ \ln \left( \frac{r}{c} \right) + \left( \frac{LR}{LW} \right) \right] + \sigma
\]
Where $A_R$ is the mean age at retirement, $A_W$ the mean working age, LR is the average number of years lived in retirement, LW the number of working years lived by the individual or cohort of interest, and $\sigma$ is the growth rate of wages. When the financial equilibrium rule holds (Bravo 1996:127), then

$$\rho_t = \frac{\ln\left(\frac{LR}{LW}\right) - \ln(\sigma)}{(A_R - A_W)} + \sigma$$

(6)