COVID-19 Mortality Prediction: A Case Study for Istanbul

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Abstract

It is well known that it is very difficult to make predictions for the real number of deaths due to any pandemic by using SIR and similar models since the predicted solutions systematically can deviate from real data. On the other hand, death data in the long and effective pandemic period cannot reflect the real case. In order to get more correct solutions and obtain realistic predictions, the parameters of these equations must be determined more precisely. In this study, by using real data depending on all deaths in Istanbul as a case study for 2020-2022 we determined the values of the parameters of the SEIR model and obtained the solution of SEIR equations. Firstly, we show that our numerical solution has a good fit with real data of the deaths due to COVID-19 for 2020 first and second peaks and 2021 first peak. Based on this confirmation, we predicted possible the number of deaths for the 2021 second peak. Furthermore, we see that our results show the number of deaths due to COVID-19 in Istanbul. Our method strongly provides that the model can lead to correct results if the parameters of SEIR models are determined by using excess mortality approximation. Now, we extend the study to predict the number of deaths due to the pandemic effects in 2022-2023. We show that our prediction is still compatible with the number of deaths for each wave. Finally, we predict the number of deaths for the future wave of 2022-2023 and we calculate the number of infected people in Istanbul for herd immunity.

Keywords: Sars Cov-2; COVID-19; Excess Mortality; SIR and SEIR Model; Pandemic; Mortality Prediction; Basic Reproduction Number.

1. Introduction

The world has struggled to control the deadly coronavirus disease 2019 (COVID-19) which is caused by severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2). After the initial outbreak in China, the novel COVID-19 spread across the world in a short time. COVID-19 was declared a pandemic on 11th March 2020 by World Health Organization (WHO) \cite{1}. On the same day, the first case in Turkey was reported by the Ministry of Health \cite{2}.

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In order to struggle with diseases more effectively and detect their characteristics such as symptoms, spread dynamics, transmission, etc., humans first began to record the diseases. Those records paved the way to fight against the diseases. Scientists have modeled the pandemics to reveal the characteristics of the pandemic. Modeling a pandemic is very important since it may provide how to control the disease and how reduce mortality. For example, London mortality statistics were published weekly in Bills of Mortality from the 17th century to 1830s [3]. In 1662, Graunt studied mortality rates and causes in his book Natural and Political Observations Made Upon the Bills of Mortality [4].

The first mathematical model on the characteristic behaviors of contagious diseases was proposed by Bernoulli in 1766 [5]. He investigated the smallpox disease and variolation method which aimed to immunize people against smallpox with the material taken from patients. Bernoulli showed that variolation was necessary for his mathematical model [5]. In 1906, Hamer analyzed the measles epidemic and proposed a discrete-time model [6]. This model is important since it shows that the new cases of an epidemic depend on the number of patients and individuals susceptible to disease. In 1911, Ross proposed a differential equation model related to the number of cases and control of malaria [7]. In 1927, Kermack and McKendrick published their paper entitled A Contribution to the Mathematical Theory of Epidemics [8]. In their paper, they considered an interactive model for an isolated society. They proposed the SIR model. In Refs. [9, 10], they made the model more useful by adding population dynamics and developed the SIR model.

Building a mathematical model provides predictions for pandemics, effective control, and political strategies [11, 12, 13, 14, 15]. It can also provide accurate future predictions and effective struggle against the diseases [16, 17, 18]. Differential equations can be used to predict the time evolution of the pandemic. Although mathematical models are useful to predict the characteristics of a disease, they have some difficulties. It is not easy to determine the model parameters. Besides, the factors such as mutations, health infrastructure, disasters, etc. affect the evolution of a disease. Therefore, the predictions from differential equations may become harder. In order to get more correct solutions and obtain realistic predictions, the parameters of these equations must be determined more precisely.

In order to determine the COVID-19 characteristics, COVID-19 has been widely studied in the literature for two years [16, 19, 20, 21, 22, 23, 24]. In order to predict the trend of COVID-19 in Italy, Wangping and his colleagues used the extended SIR model [16]. They also estimated the basic reproductive number $R_0$ from Markov Chain Monte Carlo methods. Danon and his colleagues adapted an existing national-scale metapopulation model to get the spread of COVID-19 in England and Wales [19]. They found that the size and spread rate of the epidemic highly depends on seasonal transmission. Fanelli and Piazza analyzed the temporal dynamics of the COVID-19 outbreak in China, Italy, and France [20]. Roosa and his colleagues [21] reported a short-term estimation of the cumulative number of reported cases of the COVID-19 epidemic in Hubei province and other provinces in China as of February 9, 2020. Using the data until February 9, 2020, they also estimated the number of reported cases between 37,415 and 38,028 in Hubei Province and 11,588–13,499 in other provinces by February 24, 2020. In Ref. [22], Li and his colleagues studied the transmission process of COVID-19 based on official data. Based on Ref. [23], Mangoni and Pistilli [24] developed a generalized SEIR model to estimate the dynamics of COVID-19. Additionally, several chaotic models have been proposed to estimate the
evolution of the spreading of COVID-19 by Mangiarotti and his colleagues [25], Danca [26] and Postavaru and his colleagues [27].

Many deadly pandemics such as the plague, Spanish flu etc. occurred in history. Apart from those deadly diseases, COVID-19 has the best statistics and data. However, recording exact and reliable data include serious difficulties such as medical, political, and economical issues. They may affect the reliability of these data. On the other hand, the mortality records of a country are more reliable, and excess mortality data are independent of political, economic, and medical issues.

As before mentioned, the differential models provide to determine the behaviors of an epidemic despite its disadvantages. SEIR is one of these models [11]. In this study, we use the SEIR model to predict the COVID-19 waves and basic reproduction number $R_0$ for Istanbul. To the best of our knowledge, we calculate the parameters of the SEIR model from excess mortality methods for the first time. Since SEIR model parameters depend on the excess mortality method, which is independent of medical, political etc. issues, we believe our findings are reliable.

The paper is organized as follows: In Section 2, we review the excess mortality method, SIR, and SEIR models. In Section 3, we present the details of determining the parameters $\beta, \eta, \epsilon$ and $R_0$ of the SEIR model by using excess mortality methods for COVID-19. By using the excess mortality method, we present the number of deaths due to COVID-19 for each wave. We estimate the total number of deaths for the future wave in 2022-2023. In Section 4, we finally discuss our results and estimate the number of infected people for herd immunity.

2. Model

2.1. Mathematical models of the pandemic

In this study, we use the SEIR model [11] which is the modification of the SIR model. Just like the SIR model, SEIR is also a set of coupled differential equations. It is possible to construct many diseases spread models from SIR and modified SIR models [8,9,10,15,16,23,24]. Recently, much attention has been paid to these models due to COVID-19. Because these models provide to estimate numbers of dead, recovered, and infected individuals for local and global regions during a pandemic. These models completely depend on the parameters and give reliable results. However, contagious diseases may be regarded as catastrophic events. Because stochastic factors such as vaccines, mutations, curfews or quarantine precautions may make the process more chaotic. Therefore, it is difficult to make accurate predictions. On the other hand, it is not impossible to make accurate predictions for the short term since all stochastic effects can be expressed in macroscopic forms in the limit cases. SIR equations are given by [8]

$$\frac{dS}{dt} = -\beta S I,$$

$$\frac{dI}{dt} = \beta S I - \eta I,$$

$$\frac{dR}{dt} = \eta I,$$

(1)
Where $S$, $I$, and $R$ correspond to susceptible, infected, and recovered individuals, respectively. The total probability is $S + I + R = 1$. Let us explain $S$, $I$ and $R$, respectively. $S$ represents the individuals that may have a tendency to get a disease. $I$ represents the individuals that are infected and spread the disease. $R$ represents the individuals that have not got any possibility to get a disease. Individuals who recovered or died from a disease can be considered in $R$. The parameter $\beta$ corresponds to disease spread speed while the parameter $\eta$ is related to illness duration. We define the parameter basic reproduction number as $R_0 = \frac{\beta}{\eta}$. It is important for the disease spread dynamics.

There are some disadvantages of the SIR model. For example, the SIR model does not include the incubation period of a disease. A more realistic model is the modification of the SIR model, i.e., SEIR model [11]. In the SEIR model, there is an extra parameter $\epsilon$ which represents the number of individuals exposed to COVID-19. When people are exposed to coronavirus, they do not immediately become ill. There is a time period between getting the virus and transmitting it to humans. This period is called the incubation period. The incubation period changes the date and value of the peak. SEIR equations are given by [11]

\[
\frac{dS}{dt} = -\beta SI, \\
\frac{dE}{dt} = \beta SI - \epsilon E, \\
\frac{dI}{dt} = \epsilon E - \eta I, \\
\frac{dR}{dt} = \eta I, 
\]

(2)

where the parameter $\epsilon$ is related to the incubation period of the epidemic.

### 2.2. Determining the model parameters

In order to determine the $S$, $E$, $I$ and $R$, we first define the SEIR equations in the Matlab program. We consider the $\epsilon$ parameters between two and fourteen days, i.e., the incubation period of COVID-19. On the other hand, we consider the $\eta$ parameters between two and twenty-eight days, i.e., illness duration of COVID-19 [28]. In order to determine the $\beta$ parameter, we consider the $R_0$ between 0.1 and 14. By data fitting, we find $S$, $E$, $I$ and $R$ parameters which have the lowest percentage of error. In Figs. (11), (12), (13) and (14), we show the percentage of error for $\beta$ and $\eta$ parameters. In Table 1, we present the parameters that have the lowest percentage of error for all waves. In order to produce Figs. (3), (4), (5) and (9), we choose the results that are the best compatible with data in Table 1.
Figure 1: Numbers of reported death for 2015-2019 (yellow), 2020 (blue), 2021 (red), and 2022 (green).

Excess mortality determines the number of deaths from all reported deaths during a crisis and expected deaths for the same time in the absence of the crisis [29,30]. The excess mortality method was used to determine the moralities of the Great Plague of London in 1665 [31], the influenza epidemic in London in 1875 [32,33], influenza pandemics of 1918, 1957, 1968, 2009 [34,35], as well as seasonal influenza epidemics [36]. Excess mortality is given by [29]

\[
\text{Excess Mortality} = \text{Reported Deaths} - \text{Expected Deaths} \tag{3}
\]

This method is more reliable to detect the number of deaths from COVID-19 [37]. Using the excess mortality method, WHO has obtained the number of deaths from seasonal influenza, pandemics, and other public health threats since 2009 [38].

Figure 2: Using excess mortality method, numbers of death due to COVID-19 in 2020 (blue), 2021 (red), and 2022 (green).
Table 1: The averages of ten results have the lowest percentages of error.

<table>
<thead>
<tr>
<th>Wave</th>
<th>$R_0 = \beta / \eta$</th>
<th>$R_f$</th>
<th>$\beta$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>First wave of 2020</td>
<td>3.168</td>
<td>95.1%</td>
<td>0.231420</td>
<td>0.073068</td>
</tr>
<tr>
<td>Second wave of 2020</td>
<td>1.624</td>
<td>65.5%</td>
<td>0.230520</td>
<td>0.141947</td>
</tr>
<tr>
<td>First wave of 2021</td>
<td>1.609</td>
<td>64.7%</td>
<td>0.228682</td>
<td>0.142129</td>
</tr>
<tr>
<td>Second wave of 2021</td>
<td>2.110</td>
<td>82.4%</td>
<td>0.100476</td>
<td>0.047619</td>
</tr>
<tr>
<td>Third wave of 2021</td>
<td>1.310</td>
<td>43.3%</td>
<td>0.087919</td>
<td>0.067114</td>
</tr>
<tr>
<td>First wave of 2022</td>
<td>1.540</td>
<td>60.8%</td>
<td>0.220000</td>
<td>0.142857</td>
</tr>
<tr>
<td>Second wave of 2022</td>
<td>0.980</td>
<td>&lt; 1%</td>
<td>0.054444</td>
<td>0.055556</td>
</tr>
<tr>
<td>Third wave of 2022</td>
<td>2.410</td>
<td>88.1%</td>
<td>0.114762</td>
<td>0.047619</td>
</tr>
<tr>
<td>Prediction 1</td>
<td>1.400</td>
<td>51.2%</td>
<td>0.077800</td>
<td>0.055600</td>
</tr>
<tr>
<td>Prediction 2</td>
<td>1.200</td>
<td>31.4%</td>
<td>0.120000</td>
<td>0.100000</td>
</tr>
</tbody>
</table>

We take the numbers of daily deaths in Istanbul from 2015 to 2019 [39] to predict the expected number of deaths in 2020-2022. In the calculation of expected deaths, 40% deaths in 2019, 30% deaths in 2018, 20% deaths in 2017, 5% deaths in 2016, and 5% deaths in 2015 were taken. In order to make our time series more reliable, we consider the following average over seven days.

\[
d = \frac{d_i + d_{i-1} + d_{i-2} + d_{i-3} + d_{i-4} + d_{i-5} + d_{i-6}}{7}
\]  

In Fig. (1), we present the reported numbers of deaths in 2015-2022. The yellow line represents the average number of deaths between 2015 and 2019. Note that the value of the yellow line is between 200 and 250. Furthermore, the numbers of reported death for 2020 (blue line), 2021 (red line), and 2022 (green line) reach bigger values than the yellow line. It is clearly obvious that the number of deaths dramatically increased during the COVID-19 pandemic. In Fig. (2), using the excess mortality method for the data between 2015-2019, we show the numbers of death in 2020, 2021, and 2022 due to COVID-19.

3. Istanbul Covid-19 Waves

3.1. The first wave of 2020

The first case in Turkey was reported on March 2020 and COVID-19 quickly spread across the country. The first wave in Istanbul occurred in the spring of 2020. The pandemic started on 15th March 2020 and 1,653 people died from COVID-19 until the pandemic’s first peak on 10th April 2020. Then it started to decrease. During the decrease of the first wave, 2,798 people died from COVID-19.

The first wave in Istanbul reached its peak in 26 days, then it decreased in 43 days. The total time of the first wave is 69 days. 4,451 people died from COVID-19 during the first wave. In Fig. (3), we show daily and total deaths. It can be seen that the positive slope of graphics is bigger than the negative slope. This implies that the value of $R_0$ is high.
In order to detect the parameters $\beta$, $\eta$, and $\epsilon$ of the first wave, we must obtain the numerical solutions. First of all, we define the SEIR model equations and data in the Matlab program. The parameters and their percentages of error can be found in Fig.(11). Comparing SEIR model with data gives more than 10,000 results within 5 percent of error. We use the results of the first wave of 2020 in Table 1 to produce Fig. (3).

3.2. The second wave of 2020

The second wave started on 22nd October 2020. It peaked on 26th November 2020 and ended on 14th January 2021. Mortality increased for 35 days and decreased for 49 days. The second wave lasted 84 days. 4,754 people died from COVID-19 until the peak. During the decrease of the second wave, 6,433 people died. 11,187 people died from COVID-19 during the second wave.

Comparing the first and second waves, we can easily see differences between them. The second wave is more symmetrical than the first wave. It means that the value of $R_0$ is smaller than the first wave’s. The number of deaths for the second wave is higher than the first wave’s.

Figure 3: Number of deaths for the first wave in 2020 (a) (Left) Daily (b) (Right) Total.

Figure 4: Number of deaths for the second wave in 2020 (a) (Left) Daily (b) (Right) Total.
The numbers of deaths for the second wave are given in Fig. (4). We calculate the parameters for the second wave. We present the percentages of error for the parameters β and η in Fig. (12). In Fig. (12.a), one can see that the parameter β has a low percentage of error between 0.215 and 0.235. Its percentage of error has the lowest value near 0.23. In Fig. (12.b), the parameter η has a low percentage of error between 0.13 and 0.15. It has the lowest percentage of error near 0.14. We use the results of the second wave of 2020 in Table (1) to produce Fig.4.

3.3. The first wave of 2021

The first wave of 2021 started on 11th March 2021. It peaked on 20th April 2021 and ended on 8th June 2021. Mortality increased for 41 days and decreased for 49 days. 3,886 people died until the peak. During the decrease of the wave, 4,422 people died. 8,308 people died during the first wave of 2021.

![Figure 5: Deaths numbers of the first wave in 2021 (a) (Left) Daily (b) (Right) Total.](image)

The number of deaths is higher than the number of deaths in the first wave of 2020. The first wave of 2021 is more symmetrical than the first wave of 2020. The number of deaths is given in Fig. (5). The percentages of error for parameters β and η are given in Fig. (13). In Fig. (13.a), it can be seen that the parameter β has a low percentage of error between 0.21 and 0.24. Its percentage of error is the lowest near 0.23. In Fig (13.b), one can see that the parameter η has a low percentage of error between 0.13 and 0.15. In Table 1, its percentage of error is the lowest near 0.14. We list the parameters which have the lowest error percentage. We use the results of the first wave of 2021 in Table (1) to produce the Fig. (5).

3.4. Effects of vaccination on COVID-19

Up to now, we have modeled COVID-19 by using SEIR and found the model parameters β, η, and ϵ. These parameters determine the behaviors of the pandemic. The parameter ϵ does not affect the trend of the epidemic but changes the peak date of the epidemic.
Vaccination in Turkey began with CoronaVac on January 14, 2021, and then has continued with Turkovac and Pfizer–BioNTech [40, 41]. In Fig. (6), we give the percentage of vaccination for Turkey [42]. The red curve shows the vaccinated percentage of people who received at least one vaccine dose while the green curve presents the fully vaccinated percentage of people who received all doses prescribed by the vaccination protocol. Increased vaccination rates may dramatically affect the characteristic behaviors of the main waves. As can be seen in Figs. (7) and (8), five smaller waves appear instead of the previous three main waves. In Table 1, we present the results that have the lowest percentage of error for five smaller waves and use these results in Tables 1 to produce the Fig. (9). One can find the number of deaths, η, and β parameters in Figs. (9) and (14), respectively. In Table 2, we summarize the information about all waves.

Figure 6: Vaccinated and fully vaccinated rates in Turkey.

Figure 7: Number of deaths for three COVID-19 waves. Dashed and dotted black lines correspond to predictions without the effects of vaccination.
As can be seen in Table 1, the values of $R_0$ are 3.168 for the first wave of 2020, 1.624 for the second wave of 2020, and 1.609 for the first wave of 2021. The second and third values of $R_0$ are lower than the first value. It means that the spread speed decreased. Because taking precautions such as curfew, and quarantine against COVID-19 is very effective. We present our predictions with black dashed and dotted lines. We predicted the second wave of 2021 in the previous version of this manuscript. We assumed that the parameter value of the second wave of 2021 is nearly close to the last two waves. As can be seen in Fig. (7), our predictions do not match the data. Because the vaccination rate was not enough during the previous three waves. Therefore, our first prediction does not take into account the effects of vaccination. We estimated lower and upper bounds for the new wave since we supposed the parameters of the second wave of 2021 are close to the previous two waves. In Fig. (7), we show the number of deaths for 2020 and 2021. It can easily be seen that our model (black line) is fits with 2020 (blue line) and 2021 (left part of the red line) data.

In Fig. (8), we show our estimation for the next wave. As can be seen in the figure, we present the total number of deaths for a minimum and maximum case. We choose $R_0 = 1.4$, $\beta = 0.0778$ and $\eta = 0.0556$ for Prediction 1. The total number of deaths is 4910 for Prediction 1. On the other hand, we choose $R_0 = 1.2$, $\beta = 0.12$ and $\eta = 0.1$ for Prediction 2. This case corresponds to 3313 total deaths for Prediction 2.

The value of $R_0$ slightly increases for the second wave of 2021, i.e, $R_0 = 2.11$. Seasonal effects, mutations, loosening the precautions after vaccination, etc. may have caused the increase of $R_0$. The values of $R_0$ for the third wave of 2021 and the first wave of 2022 are smaller than the previous values. $R_0$ for the second wave of 2022 has the lowest value, i.e., $R_0 = 0.98$ implies the decrease of COVID-19. Although $R_0$ for the third wave of 2022 has the second biggest value, the total number of death for this wave is the lowest value. As can be seen in Table 2, the number of deaths due to COVID-19 tends to decrease.

![Figure 8: Number of deaths for COVID-19 waves. Dashed black lines correspond to predictions for the future COVID-19 wave.](image-url)
4. Conclusions and discussions

In this study, we try to predict real death data of Istanbul due to COVID-19 for 2020-2023 years by using the SEIR model. The main problem of mathematical models is that the solutions easily deviate from the real data in the field depending on chosen parameter values. In order to get more precise results, the parameters of the model must be determined accurately. Based on this motivation, we determined the values of the parameters \( \beta \), \( \eta \), \( \epsilon \), and \( R_0 \) of the SEIR model by using excess mortality since mortality records are more reliable. We numerically obtained the solution of SEIR equations. We showed that our numerical solution has a good fit with real data of the deaths due to COVID-19 for each wave. In this period, we see that our results show the number of deaths due to COVID-19 in Istanbul. Additionally, we extend the study to estimate the number of deaths in 2022-2023. We show that our prediction method is compatible with the number of deaths for each wave.

Interestingly, we have found that \( R_0 \) may have the tendency to decrease for precautions and vaccination. As can be seen in Table 1, \( R_0 \) decreases for the first three COVID-19 waves. Taking strict precautions such as curfew, and quarantine may affect the spread speed of COVID-19 for the first three waves. \( R_0 \) increases for the second wave of 2021. This increase may be related with loosen the precautions. Then, \( R_0 \) tends to decrease until the third wave of 2022. As for the third wave of 2022, it again increases, namely, has the biggest second value of all waves, but the total number of deaths is the lowest. As can be seen in Table 1, the total number of deaths usually tends to decrease.

### Table 2: COVID-19 waves.

<table>
<thead>
<tr>
<th></th>
<th>Start date</th>
<th>Finish date</th>
<th>Period</th>
<th>Total deaths</th>
<th>Average daily deaths</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st wave of 2020</td>
<td>15.03.2020</td>
<td>22.05.2020</td>
<td>69</td>
<td>4451</td>
<td>64.51</td>
</tr>
<tr>
<td>2nd wave of 2020</td>
<td>22.10.2020</td>
<td>13.01.2021</td>
<td>84</td>
<td>11187</td>
<td>133.18</td>
</tr>
<tr>
<td>1st wave of 2021</td>
<td>11.03.2021</td>
<td>8.06.2021</td>
<td>90</td>
<td>8308</td>
<td>92.31</td>
</tr>
<tr>
<td>2nd wave of 2021</td>
<td>20.06.2021</td>
<td>26.09.2021</td>
<td>99</td>
<td>5538</td>
<td>55.94</td>
</tr>
<tr>
<td>3rd wave of 2021</td>
<td>27.09.2021</td>
<td>10.01.2022</td>
<td>106</td>
<td>8575</td>
<td>80.9</td>
</tr>
<tr>
<td>1st wave of 2022</td>
<td>11.01.2022</td>
<td>3.03.2022</td>
<td>52</td>
<td>4548</td>
<td>87.46</td>
</tr>
<tr>
<td>2nd wave of 2022</td>
<td>4.03.2022</td>
<td>10.06.2022</td>
<td>99</td>
<td>3186</td>
<td>32.18</td>
</tr>
<tr>
<td>3rd wave of 2022</td>
<td>21.06.2022</td>
<td>14.10.2022</td>
<td>116</td>
<td>3063</td>
<td>26.41</td>
</tr>
<tr>
<td>Prediction 1</td>
<td>01.12.2022</td>
<td>15.03.2023</td>
<td>105</td>
<td>4910</td>
<td>46.76</td>
</tr>
<tr>
<td>Prediction 2</td>
<td>01.12.2022</td>
<td>15.03.2023</td>
<td>105</td>
<td>3313</td>
<td>31.55</td>
</tr>
</tbody>
</table>

Before finishing the paper, we give some comments on the number of infected people to gain herd immunity in Istanbul. Solving Eq. (1), one can determine how many individuals get infected with COVID-19 until the end of the pandemic. Using

\[
S + I + R = 1 \tag{5}
\]

with the initial conditions \( S(0) = 1 \) and \( R(0) = 0 \), one finds

\[
S(t) = e^{-R_0 R(t)} \tag{6}
\]

At the end of COVID-19, the value of I is zero. Employing Eq. (6) in Eq. (5), we obtain

\[
S(t) = e^{-R_0 t} \tag{7}
\]
\[ R_f + e^{-R_0R_f} = 1 \]

where \( R_f \) is the number of recovered people at the end of COVID-19. We show numerical solutions of this equation in Fig. (10). In Fig. (10), one can see that all people nearly get infected for \( R_0 = 5 \). For example, varying the value of \( R_0 \) from 7 to 5 does not affect the number of infected people. However, varying the value of \( R_0 \) from 5 to 3 affects the number of infected people.

We give the percentage of infected people for herd immunity in Table (1). One can easily see that \( R_f \) depends on \( R_0 \). If \( R_0 \) increases (decreases), \( R_f \) increases (decreases). For the first wave of 2020, 95.1% of people in Istanbul should get infected to gain herd immunity. The rates for the two next waves decrease. Therefore, the number of infected people for herd immunity decreases. Then, \( R_f \) decreased or increased depending on the values of \( R_0 \) for five smaller waves.

As a result, we show in this case study that the SEIR model can yield more correct predictions if the parameters are determined by using excess mortality approximation. Therefore, we would like to emphasize that such well parameterized models can be used in various epidemic predictions.

5. Appendix: The tables and figures of waves
Figure 9: Number of deaths for the waves between 20.06.2021-14.10.2022. (a) (Left) Daily (b) (Right) Total.
Figure 10: The behavior of $R_f$ with respect to $R_0$.

Figure 11: Percentages of error for the first wave in 2020 (a) (Left) The parameter $\beta$ (b) (Right) The parameter $\eta$ for the first wave.

Figure 12: Percentages of error for the second wave in 2020 (a) (Left) The parameter $\beta$ (b) (Right) The parameter $\eta$ for the second wave.
Figure 13: Percentages of error for the first wave in 2021 (a) The parameter $\beta$ (b) The parameter $\eta$ for the first wave.
Figure 14: Percentages of error for the waves between 20.06.2021-14.10.2022. (a) (Left) The parameter $\beta$ (b) (Right) The parameter $\eta$.

Acknowledgments

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6. Data availability

The datasets are available as a supplementary file and generated from the website

7. Declarations

Ethics Declarations

This research did not contain any studies involving animal or human participants, nor did it take place on any private or protected areas.

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Author Contributions

E. Y. collected and analyzed data and plotted figures. E. Y. and E. A. interpreted the results. All authors wrote and revised the manuscript.

Competing Interests

The authors declare no competing interests.

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