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Students Difficulties of Solving Inequalities in Calculus

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Abstract

The study tries to analyze the students' difficulties and explore the errors done by the students when finding solution sets for inequalities. For these purpose a test was given to college of natural and computational science students who have taken calculus I or applied mathematics I course in Dilla University, Ethiopia. The results showed that the students are not successful in solving inequalities. The mostly observed mistake was to multiply both sides of inequality by expression that includes variable without paying attention to the sign of this expression. Moreover, significant number of procedural and technical errors is made by the students.

KeyWords: Inequalities; solving inequalities; Students' mistakes.

1. Introduction

The concept "inequality" and finding a solution set for an inequality is an important issue in Calculus as well as for all the fields of mathematics. Because, the main concern of a calculus course includes the concept of "function" and analyzing the properties of these functions. The analysis of some properties of functions requires the solutions of inequalities. For instance, in order to find the domains of the functions $f(x) = \sqrt{x^2 + 2x - 2}$ and $h(x) = ln\frac{x}{x-2}$, it is necessary to find the solution sets for the inequalities $x^2 + 2x - 2 \geq 0$ and $\frac{x}{x-2} > 0$.

Similarly, inequality solutions are required to determine the monotonicity and concavity of functions by the use of derivative (Sandor 1997). For instance, for the analyzing the monotonocity of the function $f(x) = \frac{x^2+2x+4}{2x}$ it is necessary to find the solution sets of inequalities $\frac{x^2-4}{2x^2} > 0$ and $\frac{x^2-4}{2x^2} < 0$ or $f'(x) > 0$ and $f'(x) > 0$ for the concavity the inequalities $f''(x) < 0$ and $f''(x) > 0$ which means $\frac{4}{x^3} > 0$ and $\frac{4}{x^3} < 0$.

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Therefore; Calculus text books introduce the concept of inequality before they cover functions and their properties [1], [6]. As these examples show, the students' performances in solving inequalities directly affect their success in Calculus courses. As a result, this situation triggered a considerable number of studies on the inequalities topic [9]; [7]; [8]; [3]; [2].

Moreover, due to not the problem of the concept of monotonocity or concavity of the function students fail to get the correct answer of the original problem in the course. Which is normally an error due to lack of other mathematics concept knowledge. According to Seah [5] there are two types of errors the first type of error is *procedural*. Procedural errors are those which arise from failure to carry out manipulations or algorithms despite having understood the concepts behind the problem. The second type of error is *technical* error. It refers to errors due to a lack of mathematical content knowledge in other topics or errors due to carelessness

On the basis of these considerations, this study aims at analyzing the students' difficulties and exploring the possible errors done by the students taking Calculus course in Dilla University in Ethiopia while finding solution sets for inequalities.

2. Method

The subjects of this study are 223 first year students attending Physics, Chemistry and Statistics Departments at the college of natural and computational Science of Dilla University in Ethiopia and also taking Calculus I/applied mathematics I course.

This group, are already been taught how to solve inequalities by using algebra and geometric methods. In the study a five-question test was administered.

2.1. A Test Administered to the Subjects

a) Find the solution set for the inequality $|x+1| > |x-3|$

In this question, the students are expected to find the solution by considering four situations occurred due to the properties of the absolute value.

b) Find the solution set for the inequality $\frac{2}{x-1} \geq 5$

In this question, the students are expected to find the solution for the inequality paying attention to the sign of $x-1$.

c) Find the solution set for the inequality $-2x^2+5x+3 < 0$

In this question, the students are expected to find the set where the second degree polynomials get negative values.

d) Find the solution set for the inequality $\frac{x}{2} \geq 1 + \frac{4}{x}$

In this question, the students are expected to find the solution by transforming this inequality to a rational expression after certain calculations and by considering the signs of numerator and denominator.

e) Find the solution set for the inequality $49x^2 \leq 64$

In this question, the students are expected to find the solution set by keeping the equality $\sqrt{x^2} = |x|$ in mind.

3. Results

Table 1: Students' Performances

questions	1st question	2nd question	3rd question	4th question	5th question
The percentages for the correct answers	60	30	45	20	68

60 % of the students gave the correct answer to the first question. In other words, 40 % of the students were not able to solve the inequality correctly. In this question, the students could have found the solution set by dividing the universe in to three sub sets namely: When $\boldsymbol{x} \in (-\infty, -\mathbf{1})$ **;** $\in [-\mathbf{1}, \mathbf{3})$ **;** and $\boldsymbol{x} \in [\mathbf{3}, +\infty)$ **.** When the calculations made by the students who gave the wrong answer were analyzed, consider sample student did on this question.

Sample 1: *The solution given by one of the students who failed to give the correct answer to the first question.*

$$|x + 1| > |x - 3|$$

$$let\ x \geq 3 \Rightarrow x + 1 > x - 3 \Longrightarrow 0 > -4 \Longrightarrow S.S_1 = \mathbb{R}$$

$for\ \ let\ x < 1 \Longrightarrow -x - 1 > -x + 3 \Rightarrow 0 > 4 \Longrightarrow S.S_2 = \emptyset\ \ and\ let\ \ -1 \leq x < 3 \Rightarrow x + 1 > -x + 3 \Rightarrow x > 2 \Rightarrow S.S_3 = (2, +\infty)$

Finally he came to final solution $S.S = S.S_1 \cup S.S_2 \cup S.S_3 = \mathbb{R}$ This is normally without considering intersection of each finding with subsets of the universe. Which is a procedural error as a result of failure to carry out manipulation or algorithmic despite having understand the content behind the problem.

The second question was given a correct answer by 30 % of the students. According to this result, one might conclude that a considerable number of students failed to answer this question correctly.

The question was $\frac{2}{x-1} \geq 5$. In this question, the students are expected to find the solution for the inequality paying attention to the sign of $x - 1$.

But one of the sample students who filed to find the correct answer simply multiplied both side of $\frac{2}{x-1} \geq 5$ by $x - 1$ and obtained the inequality $2 \geq 5(x - 1)$ *and obtained* the result $x \leq \frac{7}{5}$. In this question students didn't realized the situations $x < 1$ *and* $x > 1$, so that they could not find the correct answer *i.e* $(1, \frac{7}{2}]$.

In the third question, nearly 45% of the students found the correct solution set. In other words, 55% of the students were not able to find the correct solution set.

These students mostly focused on the negative and positive values of the polynomial $-2x^2 + 5x + 3$ .they reached the incorrect answer due to the reason that they didn't realized that the equations $-2x^2 + 5x + 3 = 0$ and $2x^2 - 5x - 3 = 0$ have the same solution.

The correct solution set for the fourth question was found by 20% of the students. The majority of the students gave the wrong answer to this question. These students particularly didn't consider the variable x in the denominator in the later calculation.

Obtained

$\frac{x}{2} - \frac{4}{x} - 1 \geq 0$ from $\frac{x}{2} \geq 1 + \frac{4}{x}$ and this led them to find the inequality $\frac{x^2-2x-8}{2x} \geq 0$. This was basically acquired by making the denominator equal up to this step what they did is arithmetically correct. Next, ignoring the possibility that x may take both negative and positive values, they multiplied x by both sides of $\frac{x^2-2x-8}{2x} \geq 0$ and obtained the quadratic inequality $x^2 - 2x - 8 \geq 0$ this was led them solution set of $(-\infty, -2] \cup [4, \infty)$ this solution became considered the solution for the original inequality $\frac{x}{2} \geq 1 + \frac{4}{x}$. Unfortunately, this was the incorrect solution set.

Most of the students were successful in finding the correct answer to the fifth question 68 %. When compared with other questions, the students can be said to have been more successful in this question. It could be the reason that the inequality $49x^2 \leq 64$ has positive coefficient. But 32% of students still have a gap on this problem too.

4. Discussion

When the overall student performances are considered for all five questions, the students cannot be said to have been successful in finding the correct solution sets for inequalities. I believe that this situation might negatively affect their success in Calculus courses. According of the record of mathematics department yearly we see among students taking Calculus I or Applied mathematics I more than half students grade is less than "C" grades. Yearly an average of nearly 2000 students is register to take the above course in Dilla University.

In addition, when the calculations of all the students (those who found or failed to find the correct solution set) are analyzed, it was observed that the students only perceive the solution to the problem as a series of algebraic calculations without considering what the solution set of an inequality means. In other words, they did not check whether an "x" real number in the solution set proves the inequality. As a result of this problem they usually fail to correctly answer the question of increasing/decreasing and concavity of function. This is a procedural and technical error [5]. So here students are commuted both procedural and technical errors.

So that, if the students had followed this checking procedure, they could have found some clues to help them find the correct solution set as well as the calculus I problem as well.

5. Implication

The results of the study have shown that students mostly registering Calculus I course without proper understanding of inequality concepts. Therefore diagnosing students' level of understanding of pre-Calculus concept is vital. Mathematics instructors who are teaching this course shall understand students' gap and provide remedial on the concept of inequality before discussing related concepts in Calculus.

6. Limitation of the study

The following limitations were identified from the study:

- The study result is limited to the above mentioned students; the result could be different for other group of students.
- To determine more consistence result ,it is necessary to consider different batch;
- It only limited to solving inequality problems in relation to calculus, but other concepts like function concepts could also be sources of students' failure in calculus.
- A test is the major source of data; if it is supplemented by multiple data source more result could be obtained.

7. Recommendation

The following recommendations were made on the basis of the findings of the study:

- This study proved that the students cannot be said to have been successful in finding the correct solution sets for inequalities .therefore, calculus instructor should consider the main source of students' failure in calculus in their teaching.
- The government should revise the curriculum; so that students didn't completely understood pre calculus concepts should take remedial courses by preparing a test to separate the students in different levels.
- Students taking calculus courses should fill their gap on different pre-calculus concepts before encounter calculus course.

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