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Upper and Lower α-Continuity of Soft Multifunctions

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Abstract

In this paper, we introduced the upper and lower α-continuous soft multifunctions. Also we obtain some characterizations and several properties concerning upper and lower α-continuous soft multifunctions.

Keywords: Soft sets; soft multifunction; soft continuity; soft topological spaces.

1. Introduction

There are various types of functions which play an important role in the classical theory of set topology. One of them is α-continuous functions. O. Njastad [1] introduced a weak form of open sets called α-sets. Then, A. S. Mashour et al. [2] defined the α-continuous functions and studied properties of this functions. A great deal of works on such functions has been extended to the setting of multifunctions. A multifunction is a set-valued function. The theory of multifunction was first codified by Berge [3]. In the last three decades, the theory of multifunction has advanced in a variety of ways and applications of this theory, can be found for example, in economic theory, noncooparative games, artificial intelligence, medicine, information sciences and decision theory. Papageorgiou [4], Allbrycht and Maltoba [5], Beg [6], Heilpein [7] and Butnairu [8] have started the study of fuzzy multifunctions and obtained several fixed point theorems for fuzzy mappings. On the other hand, a Russian researcher Molodtsov [9] introduced the concept of soft sets as a general mathematical tool for dealing with uncertainty and he successifully applied the soft set theory into several directions, such as smoothness of functions, theory of measurement, game theory, Riemann integration and so on. Then, Maji et al. [10] defined some operations on soft sets and some basic properties of these operations are revealed in [11]. Also, Aktaş and Çağman [12] compared soft sets with fuzzy sets and rough sets. Applications of Soft Set Theory in other disciplines and real life problems are now catching momentum.

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The topological structure of set theories dealing with uncertainties were first studied by Chang [13]. Lashin et al. [14] generalized rough set theory in the framework of topological spaces. Recently, Shabir and Naz [15] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Also they studied soft separation axioms for soft topological spaces. Zorlutuna et al. [16] introduced the concept of soft continuity of functions and studied some of its properties. Then Aygunoglu and Aygun [17] studied continuous soft functions. Recently, Kharal and Ahmad [18] defined the notion of a mapping on soft classes and studied several properties of images and inverse images of soft sets. Also they applied these concepts to the problem of medical diagnosis in medical expert systems. Then Akdag and Özkan [19] introduced the soft α-sets and soft α-continuous functions.

Akdag and Erol [20] introduced the soft multifunction and studied the properties of this multifunction. Also, they investigated the relationship between soft multifunction and information systems. We know that soft sets are a class of special information systems and both researches of soft sets and information systems study on the same formal structures. Information systems have been studies by many researchers from several domains such as knowledge engineering [21, 22], rough set theory [23] and granular computing [24, 25]. Pei and Miao [26] showed the operations of information systems are parallel to those of soft sets. They also investigated that there exist some compact connections between soft sets and information systems.

In this paper our purpose is to introduce the notion of soft upper (lower) α-continuous multifunction and to obtain some characterizations and of basic properties of this multifunction. Also we study relationship between soft continuous multifunction and soft α-continuous multifunction.

2. Preliminaries

Molodtsov [9] defined soft sets in the following manner. Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the power set of U and let $B \subseteq E$.

Definition 2.1. [9] A pair (G, B) is called a soft set over U, where G is a mapping given by $G: B \to P(U)$.

Definition 2.2. [26] For two soft sets (G, B) and (H, C) over a common universe U, (G, B) is a soft subset of (H, C), denoted by $(G, B) \subseteq (H, C)$, if $B \subseteq C$ and $\forall x \in B$, $G(x) \subseteq H(x)$.

Definition 2.3. [10] Two soft sets (G, B) and (H, C) over a common universe U are said to be soft equal if (G, B) is a soft subset of (H, C) and (H, C) is a soft subset of (G, B).

Definition 2.4. [27] The complement of a soft set (G, B), denoted by $(G, B)^c$, is defined by $(G, B)^c = (G^c, B)$. $G^c: B \to P(U)$ is a mapping given by $G^c(x) = U - G(x)$, $\forall x \in B$. G^c is called the soft complement function of G. Clearly, $(G^c)^c$ is the same as G and $((G, B)^c)^c = (G, B)$.

Definition 2.5. [10] A soft set (G, E) over U is said to be a null soft set, denoted by Φ, if $\forall x \in E$, $G(x) = \emptyset$.

Definition 2.6. [10] A soft set (G, E) over U is said to be absolute soft set, denoted by $\widetilde{U}$, if $\forall x \in E$, $G(x) = U$.

Clearly, $\widetilde{U}^c = \Phi$ and $\Phi^c = \widetilde{U}$.

Definition 2.7. [10] The union of two soft sets (G, B) and (H, C) over the common universe U is the soft set (T, D), where $D = B \cup C$ and for all $x \in D$,

$$T(x) = \begin{cases} G(x), & \text{if } x \in B \backslash C \\ H(x), & \text{if } x \in C \backslash B \\ G(x) \cup H(x), & \text{if } x \in C \cap B \end{cases}$$

This relationship is written as $(H, C)\ \widetilde{\cup}\ (G, B) = (T, D)$.

Definition 2.8. [26] The intersection of two soft sets (G, B) and (H, C) over the common universe U is the soft set (T, D), where $D = B \cap C$ and for all $x \in D$, $T(x) = H(x) \cap G(x)$. This relationship is written as $(H, C)\ \widetilde{\cap}\ (G, B) = (T, D)$.

Proposition 2.9. [15] If (G, E) and (H, E) are two soft sets over U, then

(1) $((G, E)\ \widetilde{\cup}\ (H, E))^c = (G, E)^c\ \widetilde{\cap}\ (H, E)^c$.

(2) $((G, E)\ \widetilde{\cap}\ (H, E))^c = (G, E)^c\ \widetilde{\cup}\ (H, E)^c$.

Definition 2.10. [16] Let I be arbitrary index set and $\{(G_i, E)\}_{i \in I}$ be soft sets over U.

(a) The union of these soft sets is the soft set (H, E), where $H(x) = \cup_{i \in I} G_i(x)$ for each $x \in E$.

We write $\widetilde{\cup}\ (G_i, E) = (H, E)$.

(b) The intersection of these soft sets is the soft set (M, E), where $M(x) = \cap_{i \in I} G_i(x)$ for all $x \in E$.

We write $\widetilde{\cap}\ (G_i, E) = (M, E)$.

Proposition 2.11. [16] Let I be arbitrary index set and $\{(G_i, E)\}_{i \in I}$ be soft sets over U, then

(1) $[\widetilde{\cup}\ (G_i, E)]^c = \widetilde{\cap}\ (G_i, E)^c$

(2) $[\widetilde{\cap}\ (G_i, E)]^c = \widetilde{\cup}\ (G_i, E)^c$.

Definition 2.12. [16] The soft set (G, E) over U is called a soft point in U, denoted by e_G, if for the element $e \in A$, $G(e) \neq \emptyset$ and $G(e') = \emptyset$ for all $e' \in E - \{e\}$.

Definition 2.13. [16] The soft point x_G is said to be in the soft set (H, E), denoted by $x_G\ \widetilde{\in}\ (H, E)$, if for the element $x \in E$ and $G(x) \subseteq H(x)$.

Proposition 2.14. [16] Let $x_G\ \widetilde{\in}\ \widetilde{U}$ and $(G, E) \subseteq \widetilde{U}$. If $x_G\ \widetilde{\in}\ (G, E)$, then $x_G\ \widetilde{\notin}\ (G, E)^c$.

Definition 2.15. [15] Let τ be the collection of soft sets over a universe U, then τ is called soft topology on U if

T1. U and Φ belong to τ

T2. the union of any number of soft sets in τ belongs to τ

T3. the intersection of any two soft sets in τ belongs to τ.

The triplet (U,τ,E) is called soft topological space over U. The members of τ are called soft open sets in U and complements of their are called soft closed sets in U.

Definition 2.16. [16] A soft set (T,E) in a soft topological space (U,τ,E) is called a soft neighborhood of the soft point $x_G \tilde{\in} \tilde{U}$ if there exists a soft open set (H,E) such that $x_G \tilde{\in} (H,E) \tilde{\subseteq} (T,E)$. The neighborhood system of a soft point x_G, denoted by $N_\tau(x_G)$, is the family of all its neigborhoods.

Definition 2.17. [16] A soft set (T,E) in a soft topological space (U,τ,E) is called a soft neighborhood of the soft set (G,E) if there exists a soft open set (H,E) such that $(G,E) \tilde{\subseteq} (H,E) \tilde{\subseteq} (T,E)$.

Theorem 2.18. [16] The neighborhood system $N_\tau(x_G)$ at x_G in a soft topological space (U,τ,E) has the following properties:

(a) If $(H,E) \in N_\tau(x_G)$, then $x_G \tilde{\in} (H,E)$,

(b) If $(H,E) \in N_\tau(x_G)$ and $(H,E) \tilde{\subseteq} (M,E)$, then $(M,E) \in N_\tau(x_G)$,

(c) If $(H,E),(M,E) \in N_\tau(x_G)$, then $(G,E) \tilde{\cap} (M,E) \in N_\tau(x_G)$,

Definition 2.19. Let (U,τ,E) be a soft topological space and let (G,E) be a soft set over U.

(a) [15] The soft closure of soft set (G,E) in (U,τ,E) is the soft set $cl(G,E) = \tilde{\cap}\, \{(S,E) : (S,E)\ is\ soft\ closed\ and\ (G,E) \tilde{\subseteq} (S,E)\}$.

(b) [16] The soft interior of soft set (G,E) in (U,τ,E) is the soft set $int(G,E) = \tilde{\cup}\, \{(S,E) : (S,E)\ is\ soft\ open\ and\ (S,E) \tilde{\subseteq} (G,E)\}$.

$cl(G,E)$ is soft closed and the smallest soft closed set containing (G,E), in the sense that it is contained in every soft closed set containing (G,E). Similarly, by property T3 for soft open sets, $int(G,E)$ is soft open and the largest soft open set contained in (G,E).

Corollary 2.20. Let (U,τ,E) be a soft topological space and let (G,E) be a soft set over U. Then

(a) [15] (G,E) is soft closed iff $(G,E) = cl(G,E)$.

(b) [16] (G,E) is soft open iff $(G,E) = int(G,E)$.

Theorem 2.21. [16] A soft set (G,E) is soft open if and only if for each soft set (H,E) contained in (G,E), (G,E) is a soft neighborhood of (H,E).

Proposition 2.22. Let (U,τ,E) be a soft topological space and let (G,E) and (H,E) be soft sets over U.

(a) [15] If $(G,E) \tilde{\subseteq} (H,E)$, then $cl(G,E) \tilde{\subseteq} cl(H,E)$.

(b) [16] If $(G,E) \tilde{\subseteq} (H,E)$, then $int(G,E) \tilde{\subseteq} int(H,E)$.

Theorem 2.23. [16] Let (U,τ,E) be a soft topological space and let (G,E) be a soft set over U.

(a) $(cl(G,E))^c = int((G,E)^c)$.

(b) $(int(G,E))^c = cl((G,E)^c)$.

3. α-Continuity of Soft Multifunction

Let $\boldsymbol{Y}$ be an initial universe set and $\boldsymbol{E}$ be the non-empty set of parameters.

Definition 3.1. [20] A soft multifunction $\boldsymbol{F}$ from an ordinary topological space $(\boldsymbol{X},\boldsymbol{\tau})$ into a soft topological space $(\boldsymbol{Y},\boldsymbol{\sigma},\boldsymbol{E})$ assings to each $\boldsymbol{x}$ in $\boldsymbol{X}$ a soft set $\boldsymbol{F}(\boldsymbol{x})$ over $\boldsymbol{Y}$. A soft multifunction will be denoted by $\boldsymbol{F}: (\boldsymbol{X},\boldsymbol{\tau}) \to (\boldsymbol{Y},\boldsymbol{\sigma},\boldsymbol{E})$. $\boldsymbol{F}$ is said to be onto if for each soft set $(\boldsymbol{G},\boldsymbol{E})$ over $\boldsymbol{Y}$, there exists a point $\boldsymbol{x} \in \boldsymbol{X}$ such that $\boldsymbol{F}(\boldsymbol{x}) = (\boldsymbol{G},\boldsymbol{E})$.

Definition 3.2. [20] For a soft multifunction $\boldsymbol{F}: (\boldsymbol{X},\boldsymbol{\tau}) \to (\boldsymbol{Y},\boldsymbol{\sigma},\boldsymbol{E})$, the upper inverse $\boldsymbol{F}^+(\boldsymbol{G},\boldsymbol{E})$ and the lower inverse $\boldsymbol{F}^-(\boldsymbol{G},\boldsymbol{E})$ of a soft set $(\boldsymbol{G},\boldsymbol{E})$ over $\boldsymbol{Y}$ are defined as follows: $\boldsymbol{F}^+(\boldsymbol{G},\boldsymbol{E}) = \{\boldsymbol{x} \in \boldsymbol{X}: \boldsymbol{F}(\boldsymbol{x}) \tilde{\subseteq} (\boldsymbol{G},\boldsymbol{E})\}$ and $\boldsymbol{F}^-(\boldsymbol{G},\boldsymbol{E}) = \{\boldsymbol{x} \in \boldsymbol{X}: \boldsymbol{F}(\boldsymbol{x}) \tilde{\cap} (\boldsymbol{G},\boldsymbol{E}) \neq \boldsymbol{\Phi}\}$. Moreover, for a subset $\boldsymbol{M}$ of $\boldsymbol{X}$, $\boldsymbol{F}(\boldsymbol{M}) = \tilde{\cup} \{\boldsymbol{F}(\boldsymbol{x}): \boldsymbol{x} \in \boldsymbol{X}\}$.

Proposition 3.3. [20] Let $\boldsymbol{M}$ be a subset of $\boldsymbol{X}$. Then the follows are true for a soft multifunction $\boldsymbol{F}: (\boldsymbol{X},\boldsymbol{\tau}) \to (\boldsymbol{Y},\boldsymbol{\sigma},\boldsymbol{E})$:

(a) $M \subset F^+(F(M))$. If F is onto $M = F^+(F(M))$.

(b) $M \subset F^-(F(M))$. If F is onto $M = F^-(F(M))$.

Proposition 3.4. [20] Let $(\boldsymbol{G},\boldsymbol{E})$ be a soft set over $\boldsymbol{Y}$. Then the followings are true for a soft multifunction $\boldsymbol{F}: (\boldsymbol{X},\boldsymbol{\tau}) \to (\boldsymbol{Y},\boldsymbol{\sigma},\boldsymbol{E})$:

(a) $F^+((G,E)^c) = \tilde{X} - F^-(G,E)$

(b) $\boldsymbol{F}^-((\boldsymbol{G},\boldsymbol{E})^c) = \tilde{\boldsymbol{X}} - \boldsymbol{F}^+(\boldsymbol{G},\boldsymbol{E})$.

Proposition 3.5. [20] Let (G_i, E) be soft sets over Y for each $i \in I$. Then the follows are true for a soft multifunction $F: (X, \tau) \rightarrow (Y, \sigma, E)$:

(a) $F^{-}(\tilde{\cup}\,(G_i, E)) = \tilde{\cup}\,(F^{-}(G_i, E))$.

(b) $F^{+}(\tilde{\cap}\,(G_i, E)) = \tilde{\cap}\,(F^{+}(G_i, E))$.

Definition 3.6. Let (X, τ) be an ordinary topological space and (Y, σ, E) be a soft topological space. Then the soft multifunction $F: (X, \tau) \rightarrow (Y, \sigma, E)$ is said to be;

(a) soft upper α-continuous at a point $x \in X$ if for each soft open (G, E) such that $F(x) \tilde{\subseteq} (G, E)$, there exists an α-open neighborhood $P(x)$ of x such that $F(z) \tilde{\subseteq} (G, E)$ for all $z \in P(x)$.

(b) soft lower α-continuous at a point $x \in X$ if for each soft open (G, E) such that $F(x) \tilde{\cap} (G, E) \neq \Phi$, there exists an α-open neighborhood $P(x)$ of x such that $F(z) \tilde{\cap} (G, E) \neq \Phi$ for all $z \in P(x)$.

(c) soft upper (lower) α-continuous if F has this property at every point of X.

Proposition 3.7. A soft multifunction $F: (X, \tau) \rightarrow (Y, \sigma, E)$ is soft upper α-continuous if and only if for all soft open set (G, E) over U, $F^{+}(G, E)$ is α-open in X.

Proof. First suppose that F is soft upper α-continuous. Let (G, E) is be soft open set over Y and $x \in F^{+}(G, E)$. Then, from Definition 3.6 we know that there exists an α-open neighborhood $P(x)$ of x such that for all $z \in P(x)$, $F(z) \tilde{\subseteq} (G, E)$ which means that $F^{+}(G, E)$ is α-open as claimed. The other direction is just the definition of soft upper continuty of F.

Proposition 3.8. $F: (X, \tau) \rightarrow (Y, \sigma, E)$ is soft lower α-continuous multifunction if and only if for every soft set (G, E) over Y, $F^{-}(G, E)$ is open set in X.

Proof. First assume that F is soft α-lower continuous. Let (G, E) soft open set over Y and $x \in F^{-}(G, E)$. Then there is an α-open neighborhood $P(x)$ of x such that $F(z) \tilde{\cap} (G, E) \neq \Phi$ for all $z \in P(x)$. So $P(x) \subseteq F^{-}(G, E)$ which implies that $F^{-}(G, E)$ is α-open in X.

Now suppose that $F^{-}(G, E)$ is α-open. Let $x \in F^{-}(G, E)$. Then $F^{-}(G, E)$ is an α-open neighborhood of x and for all $z \in F^{-}(G, E)$ we have $F(z) \tilde{\cap} (G, E) \neq \Phi$. So F is soft lower α-continuous.

Definition 3.9. [20] Let (X, τ) be an ordinary topological space and (Y, σ, E) be a soft topological space. Then a soft multifunction $F: (X, \tau) \rightarrow (Y, \sigma, E)$ is said to be;

(a) soft upper continuous at a point $x \in X$ if for each soft open (G, E) such that $F(x) \tilde{\subseteq} (G, E)$, there exists a open neighborhood $P(x)$ of x such that $F(z) \tilde{\subseteq} (G, E)$ for all $z \in P(x)$.

(b) soft lower continuous at a point $x \in X$ if for each soft open (G,E) such that $F(x) \tilde{\cap} (G,E) \neq \Phi$, there exists an open neighborhood $P(x)$ of x such that $F(z) \tilde{\cap} (G,E) \neq \Phi$ for all $z \in P(x)$.

(c) soft upper (lower) continuous if F has this property at every point of X.

Proposition 3.10. [20] For a soft multifunction $F:(X,\tau) \rightarrow (Y,\sigma,E)$ the followings are true

(a) F is soft upper continuous for each soft open set (G,E) over Y, $F^{+}(G,E)$ is open in X.

(b) F is soft lower continuous for each soft open set (G,E) over Y, $F^{-}(G,E)$ is open in X.

Remark 3.11. Let F be soft upper (lower) α-continuous then F is soft upper (lower) continuous, but this inverse is not true in general by which following example.

Example 3.12. Let $\tau = \{\emptyset, X, \{a\}, \{a,c\}\}$ be a topology on $X = \{a,b,c\}$ and let $\sigma = \{\Phi, \tilde{Y}, (G,E), (H,E)\}$ be a soft topology over $Y = \{y_1, y_2, y_3\}$ where $E = \{e_1, e_2, e_3\}$, $G(e_1) = \{y_1\}$, $G(e_2) = \{y_3\}$ and $G(e_3) = \emptyset$, $H(e_1) = \{y_1, y_2\}$, $H(e_2) = \{y_3\}$ and $H(e_3) = Y$. Then the multifunction $F:(X,\tau) \rightarrow (Y,\sigma,E)$ given by $F(a) = (G,E)$, $F(b) = (H,E)$ and $F(c) = Y$ is soft upper α-continuous multifunction but is not soft upper continuous multifunction. Because $F^{+}(G,E) = \{a\}$ and $F^{+}(H,E) = \{a,b\}$ are α-open sets but $\{a,b\}$ is not open sets in X.

Theorem 3.13. The followings are equivalent for a soft multifunction $F:(X,\tau) \rightarrow (Y,\sigma,E)$:

(a) F is soft upper α-continuous.

(b) for each soft closed set (G,E) over Y, $F^{-}(G,E)$ is α-closed in X.

(c) for each soft set (G,E) over Y, $\alpha cl(F^{-}(G,E)) \subseteq F^{-}(cl(G,E))$.

(d) for each soft set (G,E) over Y, $F^{+}(Int(G,E)) \subseteq \alpha Int(F^{+}(G,E))$.

Proof. (a)⇒(b) Let (G,E) be any closed soft over Y. Then $(G,E)^{c}$ is soft open and by proposition 1 implies $F^{+}((G,E)^{c}) = X - F^{-}(G,E)$ is α-closed set. So, $F^{-}(G,E)$ is α-closed in X.

(b)⇒(c) Let (G,E) be any soft set over Y. Then $cl(G,E)$ is soft closed set. By (b), $F^{-}(cl(G,E))$ is α-closed in X. Hence $\alpha cl(F^{-}(G,E)) \subseteq \alpha cl(F^{-}(cl(G,E))) = F^{-}(cl(G,E))$.

(c)⇒(d) Let (G,E) be any soft set over Y. Then $F^{+}(Int(G,E)) = F^{+}(X - (X - Int(G,E))) = X - F^{-}(X - Int(G,E)) = X - F^{-}(cl(X - (G,E)))$. By (c), we have $\alpha cl(F^{-}(X - (G,E)))) \subset F^{-}(cl(X - (G,E)))$ then $X - (F^{-}(cl(X - (G,E)))) \subset X - \alpha cl(F^{-}(X - (G,E))))$. $X - \alpha cl(F^{-}(X - (G,E)))) \subset \alpha Int(X - F^{-}(X - (G,E)))) = \alpha Int(F^{+}(X - (X - (G,E)))) = \alpha Int(F^{+}(G,E))$.

(d)⇒(a) Let (G,E) be any soft set over Y. By (d),

$\alpha Int(F^{+}(G,E)) \subseteq F^{+}(G,E) = F^{+}(Int(G,E)) \subseteq \alpha Int(F^{+}(G,E))$. This shows that $F^{+}(G,E)$ is open set. Then by Proposition 3.7 F is soft upper α-continuous.

Theorem 3.14. The following are equivalent for a soft multifunction $F:(X,\tau) \rightarrow (Y,\sigma,E)$:

(a) F is soft lower α-continuous.

(b) for each soft closed set (G,E) over Y, $F^{+}(G,E)$ is α-closed in X.

(c) for each soft set (G,E) over Y, $\alpha cl(F^{+}(G,E)) \subseteq F^{+}(cl(G,E))$.

(d) for each soft set (G,E) over Y, $F^{-}(Int(G,E)) \subseteq \alpha int(F^{-}(G,E))$.

Proof. It is similar to that of Theorem 3.13.

Definition 3.15. [20] For a soft multifunction $F:(X,\tau) \rightarrow (Y,\sigma,E)$, the graph multifunction $G_F: X \rightarrow X \times Y$ of F is defined as $G_F(x)$=the soft set $x_1 \times F(x)$, for every $x \in X$. over $X \times Y$, where x_1 is the soft set over X. We shall write $\{x\} \times F(x)$ for $x_1 \times F(x)$.

Lemma 3.16. [20] For a soft multifunction $F:(X,\tau) \rightarrow (Y,\sigma,E)$, the followings are hold:

(a) ${G_F}^{+}(M \times (H,E)) = M \cap F^{+}(H,E)$

(b) ${G_F}^{-}(M \times (H,E)) = M \cap F^{-}(H,E)$

Theorem 3.17. Let $F:(X,\tau) \rightarrow (Y,\sigma,E)$ be a soft multifunction. If the soft graph function of F is soft lower (upper) α-continuous, then F is soft lower (upper) α-continuous.

Proof. For a subset V of X and (G,E) a soft set over U, we take $(V \times (G,E))(z,y) = \begin{cases} \emptyset, & if\ z \notin V \\ G(y), & if\ z \in V \end{cases}$

Let $x \in X$ and let (G,E) be soft open set such that $x \in F^{-}(G,E)$. Then we obtain that $x \in G\_\{F\}^{-}(X \times (G,E))$ and $X \times (G,E)$ is a soft set over Y. Since the soft graph multifuntion $G\_\{F\}$ is soft lower α-continuons, it follows that there exists an α-open set P containing x such that $P \subseteq G\_\{F\}^{-}(X \times (G,E)$. From here, we obtain that $P \subseteq F^{-}(G,E)$. Thus F is soft lower α-continuous.

The proof of the soft upper α-continuity of F is similar to the above.

Theorem 3.18. Let $F:(X,\tau) \rightarrow (Y,\sigma,E)$ be a soft multifunction and M be an α-open set of X. Then the restriction $F|_M$ is soft upper α-continuous if F is soft upper α-continuous.

Proof. Let (H,E) be any soft open set over Y such that $F|_M(x) \subseteq (H,E)$. Since F is soft upper α-continuous and $F(x) = F|_M(x) \subseteq (H,E)$, then there exists α-open set $U \subseteq X$ containing x such that $F(z) \subseteq (H,E)$ for all $z \in U$. Put $U_1 = U \cap M$ then we have U_1 is α-open set in M containing x and $F(U_1) = F|_M(U_1) \subseteq (H,E)$. This shows that $F|_M$ is soft upper α-continuous.

Theorem 3.19. Let $F: (X, \tau) \rightarrow (Y, \sigma, E)$ be a soft multifunction and M be an α-open set of X. Then F is soft lower α-continuous if and only if the restriction $F|_M$ is soft lower α-continuous.

Proof. Let (H, E) be any soft open set over Y such that $F|_M(x) \cap (H, E) \neq \Phi$. Since $F(x) = F|_M(x)$, then $F(x) \cap (H, E) \neq \Phi$. Also since F is soft lower α-continuous there exists α-open set $U \subseteq X$ containing x such that $F(z) \cap (H, E) \neq \Phi$ for all $z \in U$. Put $U_1 = U \cap M$ then we have U_1 is α-open set in M containing x and $F(U_1) \cap (H, E) \neq \Phi$.Therefore $F|_M(U_1) \cap (H, E) \neq \Phi$. This shows that $F|_M$ is soft lower α-continuous.

Corollary 3.20. Let $F: (X, \tau) \rightarrow (Y, \sigma, E)$ be a soft multifunction and $\{F|_{M_i}: i \in I\}$ be an α-open cover set of X. The followings are hold:

(a) F is soft lower α-continuous if and only if the restriction $F|_{M_i}$ is soft lower α-continuous for every $i \in I$.
(b) F is soft upper α-continuous if and only if the restriction $F|_{M_i}$ is soft upper α-continuous for every $i \in I$.

Proof. (a): Let $x \in X$ and $x \in M_i$ for an $i \in I$. Let (G, E) be a soft closed set over Y with $F|_{M_i}(x) \cap (G, E) \neq \Phi$. Since F is soft lower α-continuous and $F(x) = F|_{M_i}(x)$, there exists an α-open set P containing x such that $P \subseteq F^{-}(G, E)$. Take $P_1 = P \cap M_i$. Then P_1 is an α-open set in $M_i$containing x. We have $P_1 \subseteq \left(F|_{M_i}\right)^{-}(G, E)$. Thus $F|_{M_i}$ is soft lower α-continuous.

Conversely, let $x \in X$ and (G, E) be a soft closed set over Y with $F(x) \subseteq (G, E)$. Since $\{M_i: i \in I\}$ is an α-open cover set of X, then $x \in M_{i_0}$for an $i_0 \in I$. We have $\left(F|_{M_{i_0}}\right)^{-}(x) = F(x)$ and hence $x \in \left(F|_{M_{i_0}}\right)^{-}(G, E)$. Since $\left(F|_{M_{i_0}}\right)^{-}$ is soft lower α-continuous, there exists an α-open set $B = D \cap M_{i_0}$ in M_{i_0} such that $x \in B$ and $F^{-}(G, E) \cap M_{i_0} = \left(F|_{M_{i_0}}\right)^{-}(G, E) \supset B = D \cap D \cap M_{i_0}$, where D is an α-open set in X. We have $x \in B = D \cap D \cap M_{i_0} \subset \left(F|_{M_{i_0}}\right)^{-}(G, E) = F^{-}(G, E) \cap M_{i_0} \subset F^{-}(G, E)$. Hence F is soft lower α-continuous.

(b): It is similar to the proof (a).

Definition 3.21. [20] Let $F: (X, \tau) \rightarrow (Y, \sigma)$ be a multifunction and let $G: (Y, \sigma) \rightarrow (Z, \vartheta, E)$ be a soft multifunction. Then the soft multifunction $G \circ F: (X, \tau) \rightarrow (Z, \vartheta, E)$ is defined by $(G \circ F)(x) = G(F(x))$.

Proposition 3.22. [20] Let $F: (X, \tau) \rightarrow (Y, \sigma)$ be a multifunction and let $G: (Y, \sigma) \rightarrow (Z, \vartheta, E)$ be a soft multifunction. Then we have

(a) $(G \circ F)^{+}(H, E) = F^{+}(G^{+}(H, E))$

(b) $(G \circ F)^{-}(H, E) = F^{-}(G^{-}(H, E))$

Definition 3.23. [3] Let (X, τ) and (Y, σ) be two ordinary topological spaces. Then a multifunction $F: (X, \tau) \rightarrow$

(Y, σ) is said to be;

(a) upper continuous if for each open set V in Y, $F^{+}(V)$ is an open set in X.

(b) lower continuous if for each open set V in Y, $F^{-}(V)$ is an open set in X.

Theorem 3.24. Let $F: (X, \tau) \rightarrow (Y, \sigma)$ be a multifunction and let $G: (Y, \sigma) \rightarrow (Z, \vartheta, E)$ be a soft multifunction. If F is upper (lower) α-continuous and G is soft upper (lower) continuous then $G \circ F$ is soft upper (lower) α-continuous.

Proof. Let (H, E) be any soft open set over Z. Since G is soft upper continuous then $G^{+}(H, E)$ is open in Y. Since F is upper α-continuous then $F^{+}(G^{+}(H, E)) = (G \circ F)^{+}(H, E)$ is α-open in X. Therefore $G \circ F$ is soft upper α-continuous.

Definition 3.25. [16] A family Ψ of soft sets is a cover of a soft set (G, E) if $(G, E) \tilde{\subseteq} \tilde{\cup} \{(G_i, E): (G_i, E) \in \Psi, i \in I\}$. It is soft open cover if each member of Ψ is a soft open set. A subcover of Ψ is a subfamily of Ψ which is also cover.

Definition 3.26. [16] A soft topological space (Y, σ, E) is said to be soft α-compact if each soft α-open cover of Y has a finite soft subcover.

Theorem 3.27. Let $F: (X, \tau) \rightarrow (Y, \sigma, E)$ be an soft upper α-continuous surjective multifunction and X is α-compact then Y is soft compact.

Proof. Let $F: (X, \tau) \rightarrow (Y, \sigma, E)$ be an onto soft multifunction and let $\Psi = \{(G_i, E): i \in I\}$ be a cover of Y by soft open sets. Then since F is soft upper α-continuous, the family of all open sets of the form $F^{+}(G_i, E)$, for $(G_i, E) \in \Psi$ is an α-open cover of X which has a finite subcover. However since F is surjective, then it is easily seen that $F(F^{+}(G_i, E)) = (G_i, E)$ for any soft set (G_i, E) over Y. There is the family of image members of subcover is a finite subfamily of Ψ which covers Y. Consequently (Y, σ, E) is soft compact.

3. Conclusion

Our purpose in this paper is to define upper and lower soft $\boldsymbol{\alpha}$-continuous multifunctions and study their various properties. Moreover, we obtain some characterizations and several properties concerning such multifunctions. We expect that results in this paper will be basis for further applications of soft mappings in soft sets theory and corresponding information systems.

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