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A New Way of Looking at the Collatz Conjecture

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Abstract

The Collatz conjecture is named after a mathematician Lothar Collatz who introduced the conjecture in 1937. The Collatz conjecture which remains an unsolved problem in mathematics today, also known as the "3n + 1 conjecture", explains about a sequence defined as follows: Start with any positive integer, the starting integer can be an even number or an odd number. Consider a number *n*, if the number *n* is even, divide it by 2. If the number *n* is odd, triple it and add one. Further, it states that regardless of the choice of *n*, after some iterations of the conjecture, the number no matter what value of a positive *n* is chosen, the sequence from the number chosen, projecting between lower and peak values will eventually attains the value of 1. Once reaching the value of 1 it will cycle through the values 1, 4, 2 indefinitely. The projections of numbers involved in this conjecture is sometimes referred to as "hailstone numbers" [1] because of the different projections of each number in multiple descents and ascents before reaching the number 1. I believe that the ideas here represent an interesting new approach towards understanding the Collatz conjecture's sequences of numbers, especially the effect of the properties of even numbers divisible by 6, odd numbers divisible by 3, prime numbers and a set pattern in the projections in a certain arrangement. Following the theme, here in this article a defined pattern in the sequence of the projections which eluded mathematicians for years are discussed. This had also been computer tested and verified as true to 2^{68} in 2020.

Keywords: Prime numbers; Congruence; Collatz Conjecture; Hailstone numbers.

1. Introduction

Mathematical theorems are important building blocks for applications in mathematics and computing. Other uses are for general research, cryptography, encryption and many more applications in everyday use. No consensus exist yet for the proof of the Collatz conjecture. The search for proof are of importance to mathematics as it may contribute to a better understanding of number theory, and even help to solve other known problems and applications.

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The famous mathematician Paul Erdos said that “Mathematics may not be ready for such problems” referring to the Collatz conjecture, and Jeffrey Lagarias in [2], another famous mathematician stated in 2010, “This is an extraordinarily difficult problem, completely out of reach of present day mathematics”. A more recent attempt to prove the conjecture as true was posted in 2019 by Terrance Tau [3], a world famous mathematician interested in this unsolved problem.

For more on the Collatz conjecture see [4, 5, 6]. This article is specifically based on a novel theorem. So by adding three novel theorems to analyze the iterations of numbers in the Collatz conjecture through a 3 Column structure [see figure 1] approach which explains the interactions between 3 columns. An illustration [see figure 2] also shows a pattern in the projection of odd and even numbers within and between the 3 columns’ properties.

In this proof, the main notions which we are using are defined as follows:

To explain this conjecture, we construct a three column structure of numbers which will help us in following the projections.

The three column structure is as follows.

C1 : [1 4 7 10 13 16 19 22 25 · · ·] T , Odd and Even

C2 : [2 5 8 11 14 17 20 23 26 · · ·] T , Odd and Even

C3 : [3 6 9 12 15 18 21 24 27 · · ·] T , Odd and Even.

where T is the transpose of the array. Odd represents and odd number in the specific column. Even represents an even number in the specific columns.

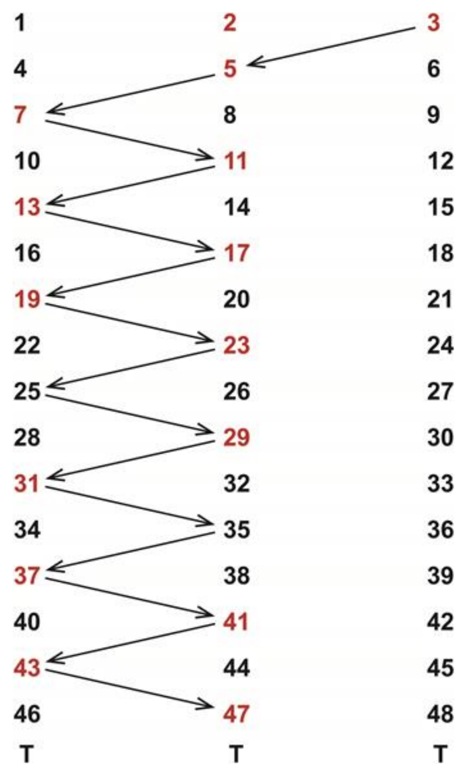

Figure 1: The 3 column structure explained.

*Note that the numbers highlighted in red represent prime numbers, present only in column C1 and C2.

where **T** is the transpose of the array.

We now fix some notations following Figure .

We set C1 to mean a number in Column 1,

- C1-0 mean an odd number in Column 1,
- C1-E mean an even number in Column 1.

Let C2 mean a number in Column 2,

- C2-0 mean an odd number in Column 2,
- C2-E mean an even number in Column 2.

Let C3 mean a number in Column 3,

- C3-0 mean an odd number in Column 3,
- C3-E mean an even number in Column 3.

2. Main section

Let us start with the three column structure of numbers [see figure 1]

C1: [1 4 7 10 13 16 19 22 25 · · ·] T , Odd and Even

C2: [2 5 8 11 14 17 20 23 26 · · ·] T , Odd and Even

C3: [3 6 9 12 15 18 21 24 27 · · ·] T, Odd and Even.

We know that in the Collatz conjecture, if *n* is odd, then we have two operations: magnification by 3 and then translation by one. In other words *n* odd is projected to an even number. We also know if *n* is even, then we divide it by two until it projects to a column as an odd number.

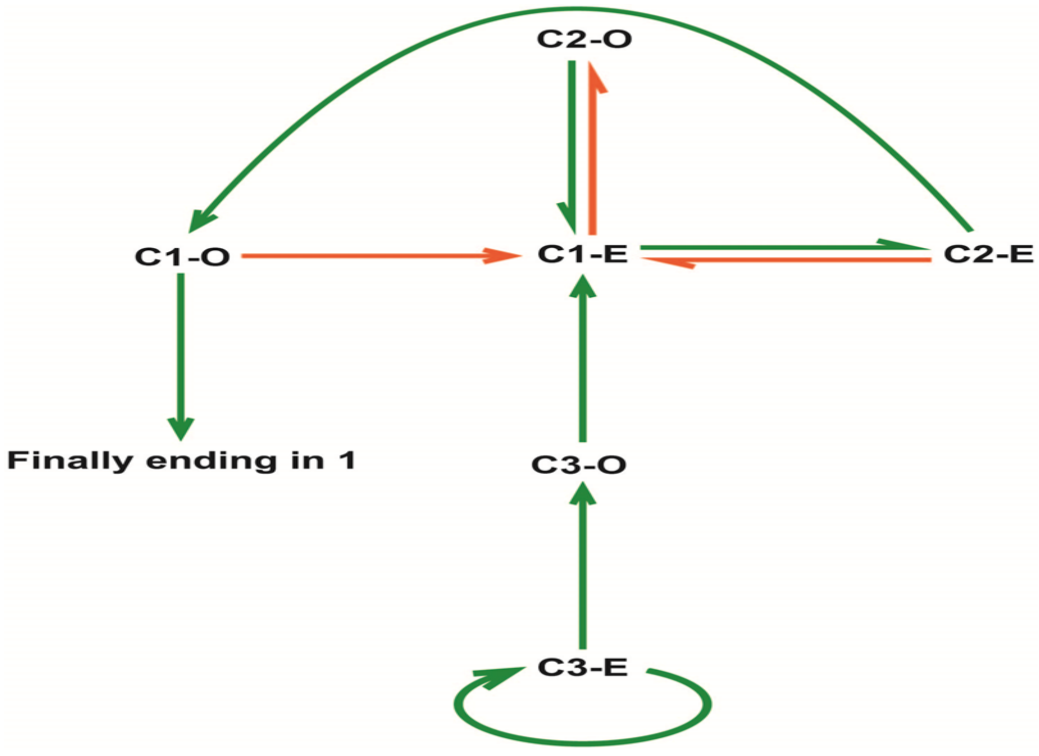

Figure 2: The rule of projection of numbers in the Collatz Conjecture.

From figure 2 we note

- Red arrows represent the 3n + 1 operations between numbers until it turns to a green even value before reaching 1 in C1-O.
- Green arrows represent the $^{n}/2$ which is the positive projections towards lowering the value of the upper

  numbers to finally attain a value of 1.

- C3 numbers, projects to themselves as even numbers and then to an odd number which finally projects to C1-

E. It also shows C 3 is now immune against receiving any projected numbers.

- Projections between C2-O and C1-E finally reach C2-E.
- Projections between C2-E and C1-E back to finally project to a C2E to C1-O which could finally attain

the value of 1, or projecting it back to C1-E for another operation between the sub-columns.

We now discuss the following six projections:

Case 1:

The Collatz number of C3-E subclass is always projected to become a C3-O subclass number with no return path to C3-E. The process may end with one iteration or multiple iterations. For example:

- 186 ∈ C3-E reaches to 93 ∈ C3-O in only iteration.
- But 24 ∈ C3-E in the first iteration reaches 12 ∈ C3-E, 12 to 6 ∈ C3-E,
- And finally 6 to 3 ∈ C3-O.

Case 2:

The Collatz number of C3-O subclass operates with magnification by 3 followed by translation of one and are always projected after only one iteration to become a C1-E number with no return to C3. For example, following the example in case 1:

- 3 ∈ C3-O is projected to 10 ∈ C1-E.
- Also, one can easily see the following one iteration projections 129 ∈ C3-O projected to 388 ∈ C1-E 219 ∈ C3-O projected to 658 ∈ C1-E 2373 ∈ C3-O projected to 7120 ∈ C1-E
- We also observe that a number of C3 class cannot ever be a projection of a number of C1 class or C2 class.

Moreover, C3 numbers are thus now isolated outside of C3 after the above iterations to receive any projections back from C1 and C2.

Case 3:

The Collatz projection of C1-E subclass numbers are the numbers from C2- E subclass or C2-O subclass, and requires only one iteration. For example:

- (i) : 76 ∈ C1-E is projected to 38 ∈ C2-E,
- (ii) : 544 ∈ C1-E is projected to 272 ∈ C2-E,
- (iii) : 34 ∈ C1-E is projected to 16 ∈ C2-O,
- (iv) : 538 ∈ C1-E is projected to 269 ∈ C2-O.

Case 4:

Every number of C1-O subclass is a Collatz projection to numbers of C1-E subclass, and it takes only one iteration. For example:

- (i) 31 ∈ C1-O is projected to 94 ∈ C1-E,
- (ii) : 223 ∈ C1-O projected to 670 ∈ C1-E.

Moreover, we see that no C1-E or C1-O number ever projects to become a C3 number.

Case 5:

By the Collatz conjecture, every number of C2-E subclass is projected to a number of C1-E subclass or to a number of C1-O subclass after one iteration only. For example:

- (i) : 140 ∈ C2-E projected to 70 ∈ C1-E,
- (ii) : 788 ∈ C2-E projected to 394 ∈ C1-E,
- (iii) : 1478 ∈ C2-E projected to 739 ∈ C1-O,
- (iv) : 146 ∈ C2-E projected to 73 ∈ C1-O.

Case 6:

Using the Collatz conjecture every number of C2-O subclass is projected to a number of C1-E subclass after one iteration only. For example:

- (i) : 41 ∈ C2- O projected to 124 ∈ C1-E,
- (ii) : 143 ∈ C2-O projected to 430 ∈ C1-E,
- (iii) : 1067 ∈ C2-O projected to 3202 ∈ C-E.
- Moreover no C2-E or C2-O number ever projects to become a C3 number.

From the above discussion, we can easily state the following

2.1. Theorem

The Collatz conjecture projects every number of C3-E subclass to a number in C3-O subclass with no return path to C3-E. Also every number of C3-O subclass is projected to a C1-E subclass after one iteration only with no return to C3.

From the above theorem, we can state the following corollary:

2.2.Corollary

The C3 numbers are isolated outside of C3 after the above iterations to receive any projections from C1 or C2.

2.3. Theorem

The Collatz conjecture projects numbers of C1-E subclass to either numbers of C2-E subclass or to numbers of C2-O subclass after one iteration only. Also every number of C1-O subclass projects to stay in C1 to become a C1-E number after one iteration only. And a C1-E or C1-O number can never project to become a C3 number.

2.4. Theorem

The Collatz conjecture projects every number of C2-E subclass to a number of C1-E subclass or C1-O subclass after one iteration only. Also every number of C2-O subclass is projected to a number of C1-E subclass after one iteration only. And a C2-E or C2-O number can never project to become a C3 number. From the above case, we can easily state that because of the set properties in the 3 column structure we observe that: The number values of C3 are after a few iterations totally out of the projections by being projected to C1-E with no return to C3 by either C1 or C2 [figure 1]. Any number through the sequence of the so called "hailstorm numbers" chaos reach a final periodic orbit of { 1, 4, 2, 1 }.

3. Conclusion

It is easy through this novel approach and its observations that proof of the Collatz conjecture is not intractable as thought. Studying the projections and relationships of odd and even numbers in figure 1 and figure 2 shows us the following properties:

- All positive integers to infinity should logically follow the standard number counting sequence according to the 3 column structure in figure 1.
- Following the number counting sequence should also logically follow the set projection sequence explained in figure 2 with its unique set of properties for each of the columns 1, 2 and 3.

Figures 1 and 2 sets the stage for the proof that the Collatz conjectures' projections are now better understood.

- (i) Every C3-E and C3-O number projects to a C1-E number after one or more iterations, and that C3 after these iterations is immune to receive any further projections from C1, C2 or C3.
- (ii) C1-O numbers after one iteration project to a higher number in C1-E if not reaching the value of 1.
- (iii) C2-O numbers after one iteration only projects to a higher number in C1-E.
- (iv) C2-E numbers after one iteration only projects to a lower C1-E number.
- (v) C1-E numbers cover the full sequence of numbers in C2-E and C2-O, for example:

C1-E ***(4)*** to C2-E (2); C1-E ***(10)*** to C2-O (*5*); C1-E ***(16)*** to C2-O (8); C1-E ***(22)*** to C2-O (*11*) etc.,

- (vi) C1-O numbers projects to C1-E to cover the sequence again in (v), for example:

C1-O ***(7)*** to C1-E (22); C1-O (***13***) to C1-E (40). This process reaching a certain value such as 10 and 64 open the gates which would finally project to attain the value of 1in C1-O.

- (vii) A C2-E could project back to a C1-E for another operational repeat.

This approach indicates that the Collatz projections of any starting number are bounded to a set rule of projections in each of the 3 columns as set out in figure 2, with C1-E the heart of “the hailstorm”. Sub-class C1-E has the property as the only sub-class to receive projections from all the other sub-classes and projecting them from C1-E to exit through C2-E to C1-O value 1.

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4. Conflict of interest statement

The author declares that there is no conflict of interest in this paper. The author declare that there are no other financial relationships with any organizations that might have an interest in the submitted work.

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