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Application the Adomian Decomposition Method to the Dynamic Response Analysis of Cracked Beam under Moving Load

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Abstract

In this article, the Adomian method of decomposition (ADM) is used to examine the vibration of a simply supported cracked beam (SSCB) under a moving load based on the Euler Bernoulli hypothesis. the system modelled as two segments of the beam are to be connected by a massless elastic rotational spring. Bearing in mind each segment of a cracked beam a substructure that can be modeled using ADM. ADM derives for the first time on the basis of kinematic conditions and boundary conditions the characteristic / eigenvalue equation and mode shape functions of the cracked beam under a moving load. The results obtained from ADM are compared to the results obtained from the method of finite elements (FEM). ADM results show remarkable superiority compared to FEM.

Keywords: Adomian decomposition method; Euler-Bernoulli beam; Cracked beam; moving load; mode shape functions; eigenvalue equation.

1. Introduction

An important topic in the field of mechanical engineering and structural engineering is the analysis of vibrations of bridges under moving loads. A moving load will cause larger deflections and higher stresses as compared with equivalent static loads. Cracks may occur as a result of stresses on the bridge. The cracks are the main reason for the failure of the bridge structure, and when a bridge structure is subjected to cracks, its stiffness will decrease, thus reducing the bridge structure's lifetime. The depth and location of a crack is likely to be predicted by the changes in vibration parameters. From the earliest starting point of railroad development at the middle of the last century, the issue of vibrations of bridges under moving loads has pulled in numerous researchers. No bibliographic account of previous work on this subject will be given here. Here. Some selective new papers [1, 2,3,4,5,6,7] are quoted, providing additional information on the issue.

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A relatively new computational method called the Adomian Decomposition Method (ADM) [8,9,10,11,12,13] is applied in this paper to examine the problem of vibration for a cracked beam under moving load. The ADM is a powerful and useful way to solve linear and nonlinear differential equations. The aim of the ADM is to find the linear and nonlinear, ordinary, or partial differential equation solution without relying on small parameters such as the example of the perturbation method. By using the ADM as a sum of an infinite series and a quick convergence to a particular solution, the solution is considered [9,10]. The main advantages of ADM are computational flexibility and do not require any linearization or assumptions of smallness. The ADM has been recently applied to the problem of vibration of structural and mechanical systems[9,10,11,12,13] . The ADM is a well-known method of systematic solution of linear or non-linear operator equations and deterministic or stochastic operators, including normal differential equations, partial differential equations, integrated equations, integrated differential equations, etc. The ADM offers an effective methodology for computational methods and numerical simulations in the applied sciences and engineering, with real-world applications. It allows us, without unphysically restrictive assumptions, to solve both nonlinear initial value problems and boundary value problems, including the need for linearization, disturbance, ad hoc assumptions, divination of the initial term or set of basic functions, etc [14,15,16]. Using the ADM, the governing differential equation for each segment of the cracked beam becomes a recursive algebraic equation. The boundary conditions and conditions of continuity at the crack position become simple algebraic frequency equations which are appropriate for symbolic computation. In addition, after some basic algebraic operations on these frequency equations, we can simultaneously obtain the natural frequency and the corresponding mode shape solution of the closed-form series. in literature has been used this method for free vibration analysis, only in one research, Bilik and Karaçay [17] applied Modified Adomian Decomposition Method in order to analysis the vibration response of intact bridge subjected to a constant load.

2. Theoretical Model Using ADM

The ADM is an efficient method for determining the analytical solution of a large class of vibrational systems without linearization or weak assumptions of nonlinearity, which can result in a huge numerical estimation. Therefore, it is physically more accurate and practical. Although the solution obtained by the ADM is an infinite series, a n-term approach usually serves as a practical and suitable solution. In addition, an effective solution with very low values of n is often obtained[18] The solution can be improved for numerical purposes by considering further terms, and if it exists, the answer will be the exact solution to the problem. As discussed earlier, the ADM provides the solution to dynamic problems in a rapidly convergent series with easily computable terms. Because the methodology is addressed perfectly in Refs [18,19], the details of this paper will not be discussed. The ADM is used to solve the dynamic governing equation of an SSCB beam carrying a moving load based on the theory of Euler – Bernoulli beams with general elastic boundary conditions and conditions of kinematics. Consider an SSCB with a moving load shown in Fig.1 with a single-sided transverse crack whose depth is a, The crack is in the $x_{!}$ position. where $x_{"}$ and $x_{\#}$ represent end points. The width, depth and length of the beam are, respectively, b, h and l. A load F with constant velocity is moving on the beam from left edge of the beam to its right end.

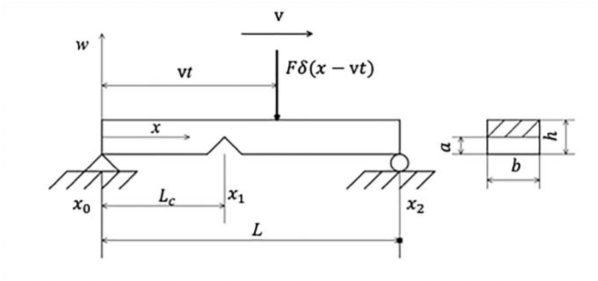


Figure 1: A simply supported cracked beam under moving load

To include the effect of crack, a massless elastic rotational spring is to connect two segments of the Euler-Bernoulli beam. This spring is the flexibility due to the crack's compliance. Euler-Bernoulli beam theory suggests that we can mention the equation of motion for forced vibration of both portions as follows:

$$EI\frac{\partial^4 w_r(x,t)}{\partial x^4}+\rho A\frac{\partial^2 w_r(x,t)}{\partial t^2}=\bar{F}\delta(x-vt), \qquad x_{r-1}<x<x_r \quad ,r=1,2 \tag{1}$$

Where $r = 1$ and 2 denoted the portion of the beam on the left of the crack and the portion of the beam on the right side of the crack respectively, E is Young's modulus of the beam material, I is the moment of inertia of the beam's cross-section, ρ is the density of material, A is the cross-sectional area of the beam, $\bar{F}$ is the moving load and $\delta(x-\mathrm{v}t)$ denoted the Dirac delta function.

The boundary conditions requirement of a SSCB case are

$$w_1(0)=w_2(L)=0 \ , \qquad \frac{d^2 w_1(0)}{dx^2}=\frac{d^2 w_2(L)}{dx^2}=0 \tag{2}$$

While the kinematics conditions are given by

$$w_1(L_{c^-})=w_2(L_{c^+}), \ , \qquad \frac{d^2 w_1(L_{c^-})}{dx^2}=\frac{d^2 w_2(L_{c^+})}{dx^2} \ and \ \frac{d^3 w_1(L_{c^-})}{dx^3}=\frac{d^3 w_2(L_{c^+})}{dx^3} \tag{3}$$

Where the $L_{c}-$ and $L_{c}+$ symbols have indicated the positions just before and after the crack.

The crack is Supposed to be completely opened during the vibration phase, and the impact of the crack can only be seen at the position of the crack. The crack's existence induces discontinuity or shift in the beam's slope at the crack's position. that can be expressed as[20]

$$\Delta\Theta = C_m M \tag{4}$$

where M and C_m are the bending moment transmitted by the cracked section and the flexibility constant respectively.

Here

$$C_m = \frac{h}{EI} Q(\alpha,\text{cross section geometry }) \tag{5}$$

here α is the amount of crack depth indicated by

$$\alpha = \frac{a}{h} \tag{6}$$

The $Q(\alpha)$ function, which is the configuration function for the opening region of the crack, can be evaluated by fracture mechanics as[20][21]

$$Q(\alpha) = 2\left(\frac{\alpha}{1-\alpha}\right)^2 [5.93 - 19.69\alpha + 37.14\alpha^2 - 35.84\alpha^3 + 13.12\alpha^4] \tag{7}$$

We rewrite Eq. (4) as

$$\Delta\Theta = \frac{dw_2(L_{c^+})}{dx} - \frac{dw_1(L_{c^-})}{dx} = hQ(\alpha)\frac{d^2w_2(L_{c^+})}{dx^2} \tag{8}$$

A series solution of Eq. (1) can be sought in the form

According to modal analysis approach (for harmonic free vibration), the $w_r(x,t)$ can be separated in space and time:

$$w_r(x,t) = \varphi_r(x)e^{i\omega t}, i = \sqrt{-1} \tag{9}$$

Substituting the Eq. (9) into the Eq. (1) and the separating time t and space x variables, an ordinary differential equation can be obtained for two segments of the cracked beam.

$$\frac{d^4\varphi_r(x)}{dx^4} - \Psi^4\varphi_r(x) = 0, \qquad x_{r-1} < x < x_r, \qquad r = 1,2 \tag{10}$$

And

$$\Psi^4 = \frac{\rho A\omega^2}{EI} \tag{11}$$

Where $\varphi_{(r)}(x)$ is the cracked beam eigenfunctions for the region of the beam before and after the crack and ω is the normal frequency.

To derive the vibration mode shapes of cracked beam the ADM is applied to Eq. (10). First Eq. (10) is rewritten as an operator form[14]

$$H_x[\varphi_r(x)] - \Psi^4\varphi_r(x) = 0 \qquad x_{r-1} < x < x_r,\ r = 1,2 \tag{12}$$

Where $H_x = \frac{d^4}{dx^4}$,the inverse operator of H_x is then a 4-fold integral operator defined by

$$H_x^{-1} = \int_0^x \int_0^x \int_0^x \int_0^x (\cdot) dxdxdxdx \tag{13}$$

Appling H_x^{-1} on both sides of Eq. (12) yields

$$\varphi_r(x) = \varphi_r(0) + \frac{x}{1!}\varphi_r^{(1)}(0) + \frac{x^2}{2!}\varphi_r^{(2)}(0) + \frac{x^3}{3!}\varphi_r^{(3)}(0) + \Psi^4 H_x^{-1}[\varphi_r(x)], x_{r-1} < x < x_r \tag{14}$$

where $\varphi_r^{(i)}$ denotes the *ith*-order derivative with respect to *x* and sets $\varphi_r(x) = \varphi_r^{(0)}(x)$

Based on the ADM[22][2] , the solution of the vibration mode function can be expressed as a series:

$$\varphi_r(x) = \sum_{j=0}^{\infty} \varphi_r^{[j]}(x) \ , \quad x_{r-1} < x < x_r, \quad r = 1,2 \tag{15}$$

Substituting Eq. (15) into Eq. (14) by assuming $\varphi_r^{[0]}(x) = \varphi_r(0) + \varphi_r^{(1)}(0)x + \frac{x^2}{2!}\varphi_r^{(2)}(0) + \frac{x^3}{3!}\varphi_r^{(3)}(0)$ Gives

$$\sum_{j=0}^{\infty} \varphi_r^{[j]}(x) = \varphi_r^{[0]} + H_x^{-1}\left[\Psi^4 \sum_{j=0}^{\infty} \varphi_r^{[j]}(x)\right], \quad x_{r-1} < x < x_r, \quad r = 1,2 \tag{16}$$

Consequently, for each term of the series, it can be written as:

$$\varphi_r^{[0]}(x) = \sum_{u=0}^{3} \frac{x^u}{u!}\varphi_r^{(u)}(0), \tag{17}$$

as the initial term of the decomposition, and

$$\varphi_r^{[j]}(x) = \Psi^4 H_x^{-1}\left[\varphi_r^{[j-1]}(x)\right], \ j \geq 1 \ , \quad x_{r-1} < x < x_r, \quad r = 1,2 \tag{18}$$

As the recurrence relation of the decomposition. Consequently, all components of the decomposition can be identified and evaluated from Eq. (18). This is, by substituting Eq. (17) into Eq. (18), one can obtain

$$\varphi_r^{[1]}(x) = \Psi^4 H_x^{-1}\left[\varphi_r^{[0]}(x)\right] = \Psi^4 \sum_{u=0}^{3} \frac{x^{4+u}}{(4+u)!}\varphi_r^{(u)}(0), \tag{19}$$

$$\varphi_r^{[2]}(x) = \Psi^4 H_x^{-1}\left[\varphi_r^{[1]}(x)\right] = \Psi^8 \sum_{u=0}^{3} \frac{x^{8+u}}{(8+u)!}\varphi_r^{(u)}(0), \tag{20}$$

And

$$\varphi_r^{[j]}(x) = \Psi^{4j} H_x^{-1}\left[\varphi_r^{[j-1]}(x)\right] = \Psi^{4j} \sum_{u=0}^{3} \frac{x^{4j+u}}{(4j+u)!} \varphi_r^{(u)}(0), j \geq 1 \tag{21}$$

In practice, the solution will be the n-term approximation

$$\varphi_r^n(x) = \sum_{j=0}^{n-1} \varphi_r^{[j]}(x) = \sum_{j=0}^{J-1} \Psi^{4j} \sum_{u=0}^{3} \frac{x^{4j+n}}{(4j+n)!} \varphi_r^{(u)}(0) \tag{22}$$

and from the convergence of ADM[22] , φ_r^n approaches $\varphi_r^{[n]}(x)$ as $n \to \infty$

$$\varphi_r(x) = \lim_{n\to\infty} \varphi_r^n(x) = \sum_{j=0}^{\infty} \varphi_r^{[j]}(x) \tag{23}$$

For simplicity, the constants are set as $a_r = \varphi_r(0)$, $b_r = \varphi_r^{(1)}(0)$, $c_r = \varphi_r^{(2)}(0)$, $d_r = \varphi_r^{(3)}(0)$

here a_r, b_r, c_r and d_r are constants associated with the *r-th* segments (*r* =1,2).

The final vibration mode shape function has been shown in Eq. (23). One can consider as many terms as needed to obtain a more accurate deflection function. If the series converges, the j-term partial sum will be the approximate closed-form solution of the differential equation[22]

By substituting the final vibration mode shape function into the four boundary conditions and kinematics conditions of the cracked beam, a system of eight equations with unknown constants is given as

$$T_{s1}^n(\Psi^n)a_r + T_{s2}^n(\Psi^n)b_r + T_{s3}^n(\Psi^n)c_r + T_{s4}^n(\Psi^n)d_r = 0 \tag{24}$$

Where $T_{s1}^n(\Psi^n)$, $T_{s2}^n(\Psi^n)$, $T_{s3}^n(\Psi^n)$ and $T_{s4}^n(\Psi^n)$ are the coefficients of unknown constants a_r, b_r, c_r, and d_r corresponding to the n-term solution series as a function of Ψ^n. Also, *s* changes from one to four with respect to each boundary condition.

These constants in the second segment (a_2, b_2, c_2 and d_2) are related to those in the first segment (a_1, b_1, c_1 and d_1) through the compatibility conditions in Eqs. (3) and (4) and cab be expressed as

$$\begin{Bmatrix} a_2 \\ b_2 \\ c_2 \\ d_2 \end{Bmatrix} = \begin{bmatrix} z_{11}^n(\Psi^n) & \cdots & z_{14}^n(\Psi^n) \\ \vdots & \ddots & \vdots \\ z_{41}^n(\Psi^n) & \cdots & z_{44}^n(\Psi^n) \end{bmatrix} \begin{Bmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \end{Bmatrix} = Z_{4\times4} \begin{Bmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \end{Bmatrix} \tag{25a}$$

Where $Z_{4\times4}$ is a 4×4 transfer matrix which depends on eigenvalue Ψ^j

After applying the boundary conditions, the following equation can be obtained as:

$$\begin{Bmatrix}0\\0\end{Bmatrix} = T_{2\times4}\begin{Bmatrix}a_2\\b_2\\c_2\\d_2\end{Bmatrix} = T_{2\times4}\,Z_{4\times4}\begin{Bmatrix}a_1\\b_1\\c_1\\d_1\end{Bmatrix} = Q_{2\times4}\begin{Bmatrix}a_1\\b_1\\c_1\\d_1\end{Bmatrix} \tag{25b}$$

Where

$$\text{where}\quad T_{2\times4} = \begin{bmatrix} t_{11}^n(\Psi^n) & t_{12}^n(\Psi^n) & t_{13}^n(\Psi^n) & t_{14}^n(\Psi^n) \\ t_{21}^n(\Psi^n) & T_{22}^n(\Psi^n) & t_{23}^n(\Psi^n) & t_{24}^n(\Psi^n) \end{bmatrix} \tag{25c}$$

$$Q_{2\times4} = T_{2\times4}\,Z_{4\times4} = \begin{bmatrix} q_{11}^n(\Psi^n) & q_{12}^n(\Psi^n) & q_{13}^n(\Psi^n) & q_{14}^n(\Psi^n) \\ q_{21}^n(\Psi^n) & q_{22}^n(\Psi^n) & q_{23}^n(\Psi^n) & q_{24}^n(\Psi^n) \end{bmatrix} \tag{25d}$$

Thus, the existence of nontrivial solution requires

$$\begin{vmatrix} q_{11}^n(\Psi^n) & q_{13}^n(\Psi^n) \\ q_{21}^n(\Psi^n) & q_{23}^n(\Psi^n) \end{vmatrix} = 0 \tag{26}$$

Equation (26) leads to the eigenvalues equation solving Ψ^n, which is the *n-th* eigenvalue related with the *n-th* natural frequency of the beam. Substituting Ψ^n into Eq. (26) and choosing an arbitrary constant, other three unknown constants can be evaluated. Then Eq. (23) gives the vibration mode shapes function for both span segments (the region before and after crack).

2. Forced responses

The original equation of motion (Eq. (1)) can be expressed as

$$\frac{\partial^4 w_r(x,t)}{\partial x^4} + \frac{\rho A}{EI}\frac{\partial^2 w_r(x,t)}{\partial t^2} = \frac{\bar{F}}{EI}\delta(x - \mathrm{v}t), \qquad r = 1,2 \tag{27}$$

Using the modal expansion theory, the forced response $w_r(x,t)$ can be expressed as

$$w_r(x,t) = \sum_{j=1}^{n} \varphi_r^n(x)\ \eta_r^n(t), \qquad r = 1,2 \tag{28}$$

where $\varphi_r^n(x)$ are eigenfunctions of the cracked beam and which are obtained from the above section (section 2),$\eta_r^n(t)$ are generalized coordinates and *n* is the number of terms used to approximate the solution. Substitute Eq. (28) into Eq. (27), multiplying by $\varphi_r(x)$, and integrating from 0 to *l* leads to

$$\frac{d^2\eta_r(t)}{dt^2} + \omega^2\eta_r(t) = F_s\varphi_r(\mathrm{v}t), \qquad r = 1,2 \tag{29}$$

$$\text{where} \qquad F_s = \frac{\bar{F}}{\rho A}$$

To derive the generalized coordinates of cracked beam the ADM is applied to Eq. (29). First Eq. (29) is rewritten as an operator form.

$$G_t[\eta_r(t)] + \omega^2 \eta_r(t) = F_s \varphi_r(\mathrm{v}t), \qquad r = 1,2 \tag{30}$$

Where $G_t = \frac{d^4}{dt^4}$,the inverse operator of ${G_t}^{-1}$ is then a 2-fold integral operator defined by

$${G_t}^{-1} = \int_0^t \int_0^t (\cdot) dt dt \tag{31}$$

Appling ${G_t}^{-1}$ on both sides of Eq. (30) yields

$$\eta_r(t) = \eta_r(0) + t\eta_r^{(1)}(0) - {G_t}^{-1}\left[\omega_j^2 \eta_r(t) - F_s \varphi_r(\mathrm{v}t)\right] \tag{32}$$

Based on the ADM, the solution of the vibration mode function can be expressed as a series:

$$\eta_r^n(t) = \sum_{j=0}^{\infty} \eta_r^{[j]}(t) \tag{33}$$

Consequently, for each term of the series, it can be written as:

$$\eta_r^{[0]}(t) = \sum_{u=0}^{1} \left[\frac{t^u}{u!} \eta_r^{(u)}(0)\right] + {G_t}^{-1}[F_s \varphi_r(\mathrm{v}t)] \tag{34}$$

Substituting Eq. (22) into Eq. (34) gives

$$\eta_r^{[0]}(t) = \sum_{u=0}^{1} \left[\frac{t^u}{u!} \eta_r^{(u)}(0)\right] + F_s {G_t}^{-1}\left[\sum_{j=0}^{J-1} \Psi^{4j} \sum_{u=0}^{3} \frac{(\mathrm{v}t)^{4j+n}}{(4j+n)!} \varphi_r^{(u)}(0)\right]$$

$$= \sum_{u=0}^{1} \left[\frac{t^u}{u!} \eta_r^{(u)}(0)\right] + F_s \left[\sum_{j=0}^{J-1} \Psi^{4j} \sum_{u=0}^{3} \frac{\mathrm{v}^{4j+n}.\, t^{4j+n+2}}{(4j+n+2)!} \varphi_r^{(u)}(0)\right] \tag{35}$$

as the initial term of the decomposition, and

$$\eta_r^{[j]}(t) = -\omega^{2j} G_t^{-1}\left[\eta_r^{[j-1]}(t)\right], \; j \geq 1\,, \qquad r = 1,2 \tag{36}$$

As the recurrence relation of the decomposition. Consequently, all components of the decomposition can be identified and evaluated from Eq. (36). This is, by substituting Eq. (35) into Eq. (36), one can obtain

$$\eta_r^{[1]}(t) = -\omega^2 G_t^{-1}\left[\eta_r^{[0]}(t)\right]$$

$$= -\omega^2\left[\sum_{u=0}^{1}\left[\frac{t^{u+2}}{(u+2)!}\eta_r^{(u)}(0)\right] + F_s\left[\sum_{j=0}^{J-1}\Psi^{4j}\sum_{u=0}^{3}\frac{\mathrm{v}^{4j+n}.\,t^{4j+n+4}}{(4j+n+4)!}\varphi_r^{(u)}(0)\right]\right], \quad (37)$$

$$\eta_r^{[2]}(t) = -\omega^4 G_t^{-1}\left[\eta_r^{[1]}(t)\right]$$

$$= \omega^4\left[\sum_{u=0}^{1}\left[\frac{t^{u+4}}{(u+4)!}\eta_r^{(u)}(0)\right] + F_s\left[\sum_{j=0}^{J-1}\Psi^{4j}\sum_{u=0}^{3}\frac{\mathrm{v}^{4j+n}.\,t^{4j+n+6}}{(4j+n+6)!}\varphi_r^{(u)}(0)\right]\right], \quad (38)$$

$$\eta_r^{[3]}(t) = -\omega^4 G_t^{-1}\left[\eta_r^{[2]}(t)\right]$$

$$= -\omega^6\left[\sum_{u=0}^{1}\left[\frac{t^{u+6}}{(u+6)!}\eta_r^{(u)}(0)\right] + F_s\left[\sum_{j=0}^{J-1}\Psi^{4j}\sum_{u=0}^{3}\frac{\mathrm{v}^{4j+n}.\,t^{4j+n+8}}{(4j+n+8)!}\varphi_r^{(u)}(0)\right]\right], \quad (39)$$

and

$$\eta_r^{[m]}(t) = -\omega^{2m} G_t^{-1}\left[\eta_r^{[m-1]}(t)\right] =$$

$$-\omega^{2m}\left[\sum_{u=0}^{1}\left[\frac{t^{u+2m}}{(u+2m)!}\eta_r^{(u)}(0)\right] + F_s\left[\sum_{j=0}^{J-1}\Psi^{4j}\sum_{u=0}^{3}\frac{\mathrm{v}^{4j+n}.\,t^{4j+n+2m+2}}{(4j+n+2m+2)!}\varphi_r^{(u)}(0)\right]\right], m \geq 1 \quad (40)$$

In practice, the solution will be the n-term approximation

$$\eta_r^n = \sum_{m=0}^{n-1}\eta_r^{[m]}(t)$$

$$= \sum_{m=0}^{n-1}\left[-\omega^{2m}\left[\sum_{u=0}^{1}\left[\frac{t^{u+2m}}{(u+2m)!}\eta_r^{(u)}(0)\right] + F_s\left[\sum_{j=0}^{J-1}\Psi^{4j}\sum_{u=0}^{3}\frac{\mathrm{v}^{4j+n}.\,t^{4j+n+2m+2}}{(4j+n+2m+2)!}\varphi_r^{(u)}(0)\right]\right]\right], m \geq 1 \quad (41)$$

and from the convergence of ADM [ ], η_r^n approaches $\eta_r^{[m]}(t)$ as $n \to \infty$

$$\eta_r(t) = \lim_{n\to\infty} \eta_r^n(t) = \sum_{m=0}^{\infty} \eta_r^{[m]}(t), \quad r = 1,2 \tag{42}$$

The final generalized coordinates function for two segments (the region before and after the crack) has been shown in Eq. (42) One can consider as many terms as needed to obtain a more accurate deflection function. If the series converges, the j-term partial sum will be the approximate closed-form solution of the differential equation. The constants have been found by applying the four boundary conditions and kinematics conditions of the cracked beam.

The dynamic response of cracked beam under moving load can be found from substituting Eq. (23) and Eq. (42) into Eq. (28).

4. Numerical results and discussion

4.1. Validation

In this subsection, the accuracy and efficiency of *ADM* solutions are investigated through a comparison with those obtained from FEM. The *ADM* model developed in this article is applied to a SSCB under a moving load. The numerical values of geometrical and material parameters of the Euler Bernoulli beam are: length L = 5(m), Young's modulus E = 2.1×1011(Gpa), cross-sectional second moment of inertia $I = \frac{b\times h^3}{12}(m^4)$,width b = 0.5(m), height h = 1(m), mass density ρ = 7860(kg/m^3) (Mahmoud, 2001). To obtain the dynamic response of cracked beams under a moving load, Equations (23), (28) and (42) is used for an ADM analysis, the corresponding numerical results are denoted by ADM. On the other hand, for an FEM analysis, the total number of finite elements used in the analysis is varied in order to improve the numerical accuracy; and the corresponding results are denoted by FEM. The dynamic transverse deflections against time response are normalized relative to the value, $\frac{FL^3}{48EI}$, which is the related static deflection was applied to the mid-span beam due to load amplitude F.

4.2 Moving load with constant velocity

For example, a moveable load with a constant load amplitude F = 2000(N), a constant velocity v = 80(m/s) and initial zero conditions 0 are taken into consideration. Fig.2, indicates the normalized deflection for mid-point of the beam. In this figure, $\frac{vt}{L}$ indicate to the normalized position of moving load once over the beam. Fig. 2 shows that the results predicted by FEM model would converge to those predicted by the present ADM method provided that the total number of elements used in FEM could increase beyond 100. In Fig. 2, it is seen that the results of ADM are in good agreement with those of exact solution and closed to FEM solution. This could verify the higher accuracy of the present ADM method. The percent proportional error (PPE) is defined in Eq. (43) and shows the numerical error of ADM and FEM methods in comparison with exact solution and Table 1 shows this ratio of the error.

$$\text{PPE}(\%) = \frac{\text{Exact solution } - \text{ADM or FEM response}}{\text{Exact solution}} \times 100 \quad (43)$$

Although the numerical error would be diminished by increasing the number of elements, it could be seen that the efficiency of using ADM is much bigger than that using numerous elements in FEM. Section 4 assumes that the initial conditions for numerical examples are zero, whereas the moving load crosses the beam for once and it could be a pseudo initial condition for a Euler Bernoulli beam. Pseudo initial speed situation means a pulse function that limits Dirac delta function at a not so small-time interval That might mimic the function of theoretical impulses but is very close to it. Fig.3 shows a normalized dynamic response for different load speeds with ADM and FEM (100 elements) at the mid-span of a cracked beam. As predicted, an increase in load velocity will cause the maximum dynamic deflection to correspondingly increase. Fig.4 shows the effect of moving load amplitude on the dynamic deflection response at the midpoint of a cracked beam at constant velocity v = 70 (m/s) under moving load. It can be seen that by increasing the load amplitude from 1000 (N) to 2000 (N), the mid-point deflection of cracked beam is increased.

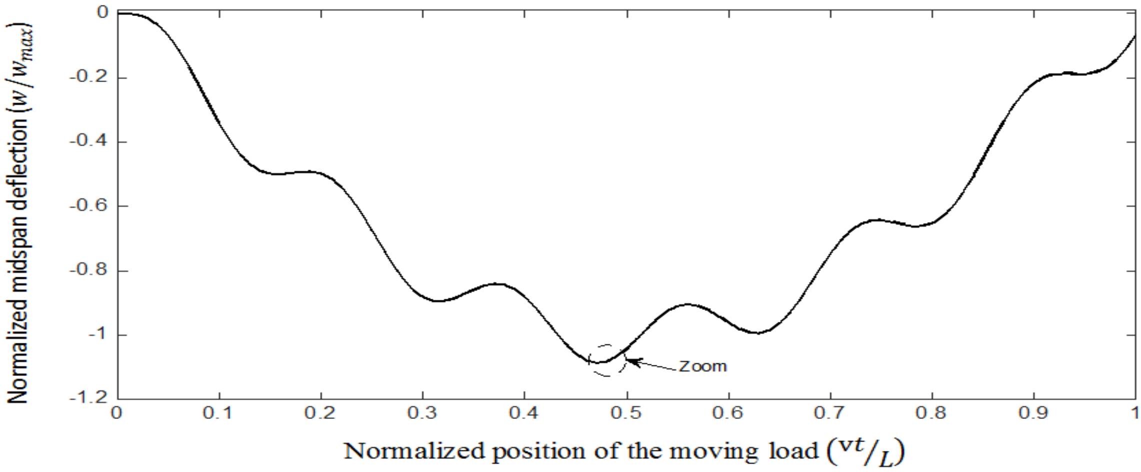
Fig.5 shows the difference between the mid-point deflection of the cracked beam and the same one for an intact beam (uncracked beam). It is seen that the maximum mid-span deflection of the beam with open crack is greater than that of the healthy beam. In addition, the time response of mid-span deflection causes a horizontal shift-in-time. Such vertical and horizontal shifts in the response time signals will occur because the stiffness of the cracked beam is lower than that of the healthy beam. The good agreement between the results from ADM and from those of transfer matrix method (Gopalakrishnan and his colleagues 1992) , lead to the utilization of ADM for this case study. Maximum normalized relative error deflection (MNRED) is defined in Eq. (44) and shows the numerical error of ADM and FEM methods in comparison with the transfer matrix method. These numerical error ratios are given in Table 2. The efficiency of using ADM in comparison with FEM could be seen again in this table.

$$\text{MNRED}(\%) = \frac{\text{Transfer matrix response } - \text{ADM or FEM response}}{\text{response Transfer matrix response}} \times 100 \quad (44)$$

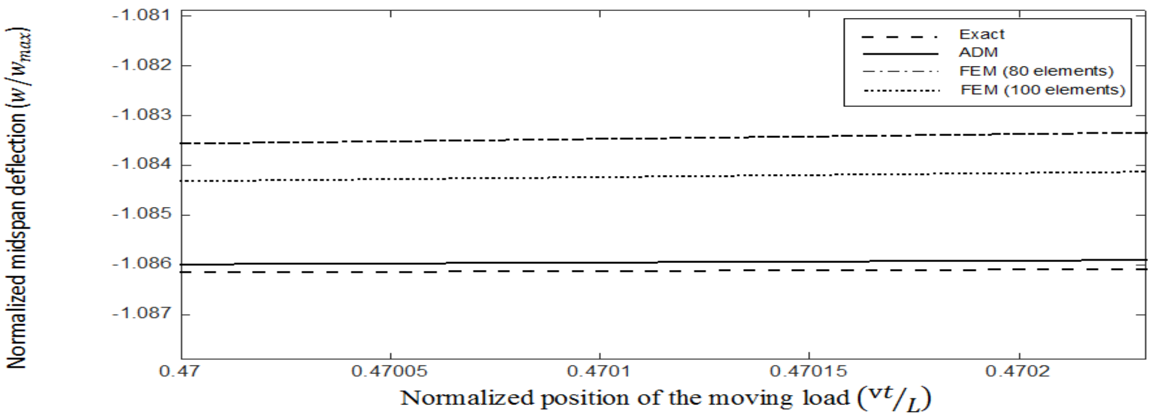
The effect on the mid-point deflection of the beam is shown in Fig. 6. In this case, both the vertical and the horizontal changes are extended to the higher values of magnitude and time for the beam's midpoint deflection, respectively. The horizontal shift-in-time phenomenon demonstrates the cracked beam's fundamental period-lengthening event corresponding to the deficiency in stiffness. This is more pronounced with higher levels of crack depth. In Eq. 45, The factor of crack magnification (FCM) displays the numerical ratio of the average deflection between cracked and intact beams subjected to moving load with constant velocity, v =70 (m/s). Table 3 shows these ADM numerical results.

$$\text{FCM} = \frac{\text{maximum mid} - \text{point cracked beam deflection}}{\text{maximum mid} - \text{point intact beam deflection}} \times 100 \tag{45}$$

The effect of an open crack location on the dynamic response of a SSCB under moving load with constant velocity, v =70 (m/s), is investigated in Fig. 7 by ADM. It seems that getting the crack location away from the beam's midpoint would decrease the midpoint deflection of the beam until it becomes nil at the beam's two ends.



(a)



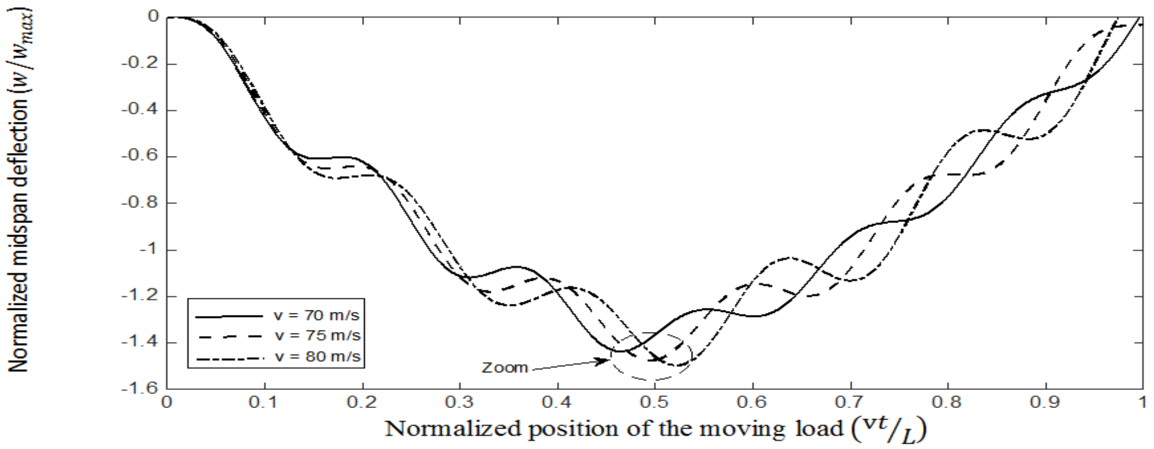
(b)

Figure 2: (a) Comparison between the normalized deflection at mid-span of a SSCB subjected to moving load with constant velocity v=80 (m/ s), by using ADM, FEM and exact solution (80 and 100 elements) (b) zoom-in subfigure (a) shows the comparison between these three methods

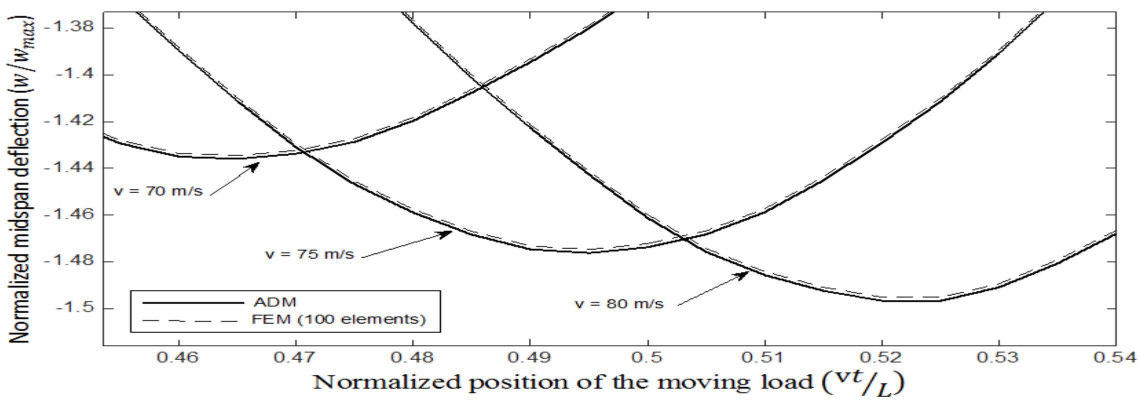
Table 1: Difference between maximum normalized defection at the mid-point of a simply- supported beam for ADM, FEM, and exact solution, v = 80 (m/s)

Method	Maximum normalized deflection of mid-span	PPE (%)
Exact solution	-1.0361	-

ADM	-1.0860	0.0092
FEM (80 elements)	-1.0836	0.2302
FEM (100 elements)	-1.0843	0.1657



(a)



(b)

Figure 3: (a) moving load velocity (70, 80, 90(m/s)) effects on the normalized deflection at mid-span of a SSCB by using ADM and FEM (100 elements) (b) zoom-in subfigure (a) shows the comparison between these two method

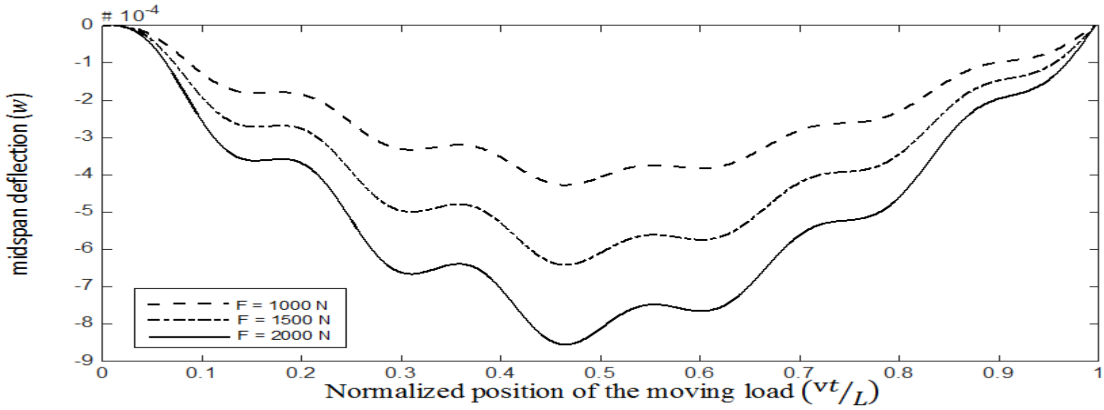
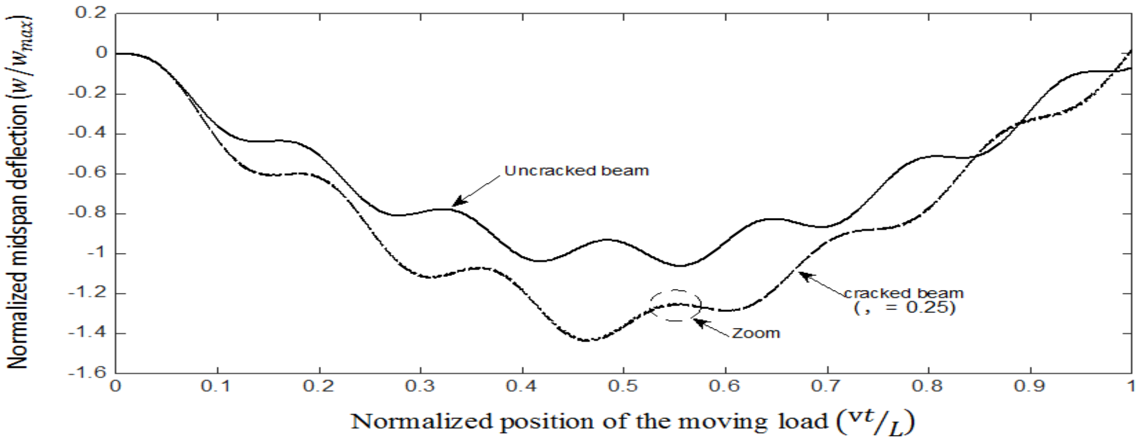
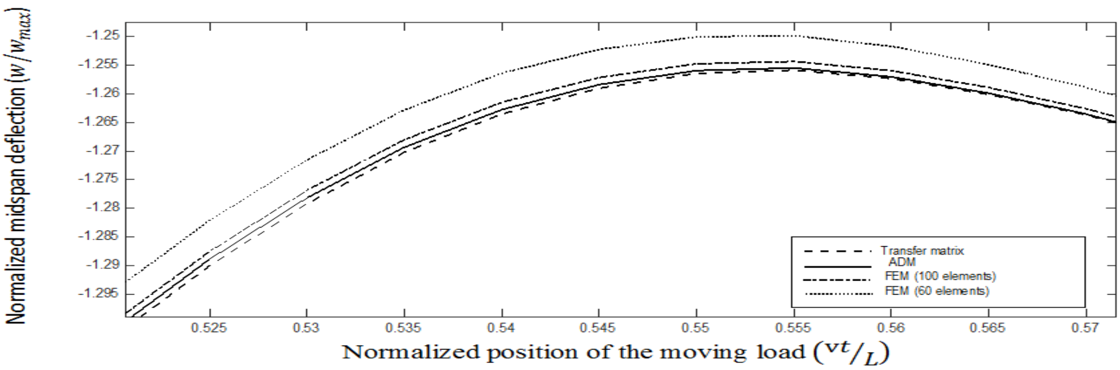


Figure 4: The effect of the amplitude of the moving load on the mid-span deflection of an SSCB undergoing a moving load with constant velocity v=70 (m/s) and α=0.25 using ADM



(a)



(b)

Figure 5: (a) Comparison between the normalized deflection at mid-span of a simply supported cracked and intact beam subjected to moving load with constant velocity v=70 (m/ s), by using Transfer matrix, ADM and FEM (60 and 100 elements) (b) zoom-in subfigure (a) shows the comparison between these three methods

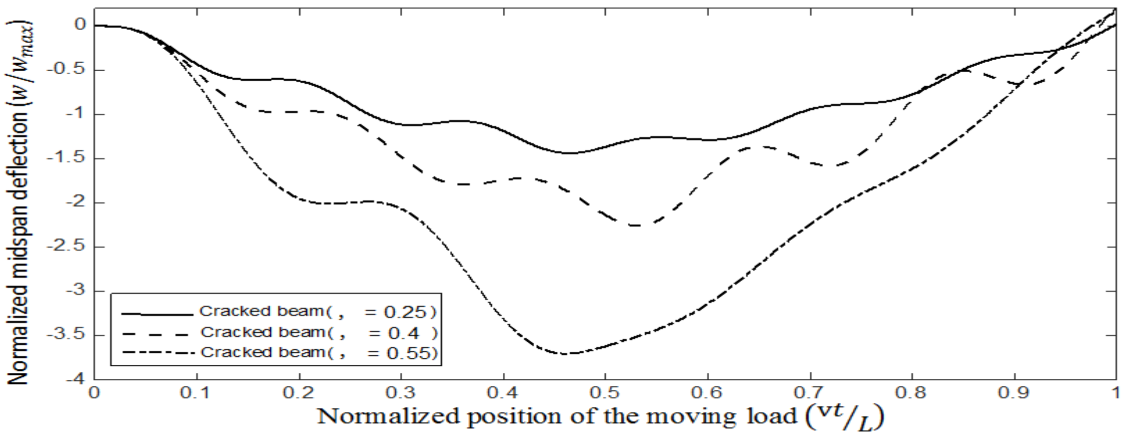


Figure 6: Crack depth ratio effects on the deflection at mid-span of a SSCB subjected to moving load with constant velocity(v=70m/s) by using ADM

Table 2: Difference between maximum normalized defection in mid-point of simply supported beam in ADM, FEM and transfer matrix method, v=70 (m/s) and $\alpha = 0.25$

Method	Maximum normalized deflection of mid-span	MNRED (%)
Transfer matrix	-1.4269	1
ADM	-1.4268	0.007
FEM (30 elements)	-1.4128	0.9882
FEM (60 elements)	-1.4139	0.9111
FEM (90 elements)	-1.4166	0.7218

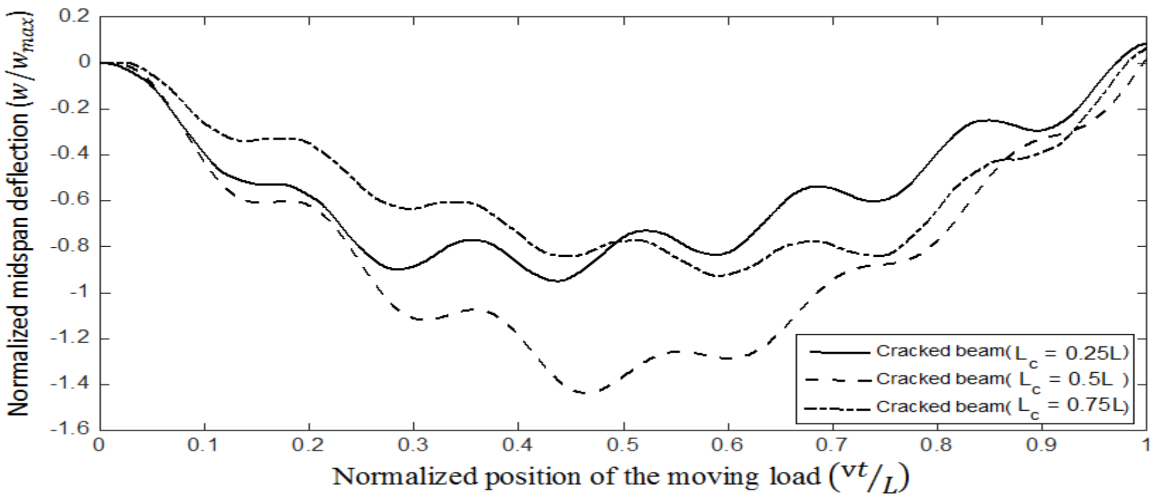


Figure 7: Crack location effects on the normalized deflection at mid-span of a SSCB subjected to moving load with constant velocity(v=70m/s) and α=0.25, by using ADM

Table 3: Effect of crack size on the ratio between maximum normalized deflections of cracked and healthy beams under moving load with constant velocity, v=70 (m/s), by using ADM

α	Maximum normalized deflection of mid-span	FCM
0	-1.061	1
0.15	-1.1726	1.105
0.25	-1.4268	1.345
0.35	-1.8977	1.789
0.45	-2.537	2.391
0.55	-3.6196	3.412

4. Conclusions

In this article, the vibrations of cracked Euler Bernoulli beams under moving load are analyzed using ADM. The open crack was modeled by a massless rotational spring. The main characteristics of ADM were its higher accuracy and its superiority comparison with other methods, such as FEM. It was shown that the present ADM could provide extremely accurate solutions, as compared with those obtained from an FEM having even one hundred elements, the following results were observed: Vertical and horizontal shifts of the (both healthy and cracked) beam mid-point deflection to the higher values were a result of

• increase of load movement speed. (Move load quicker)

• Increase the moving load amplitude. (Greater moving load)

• Increased the height ratio of the crack depth to a beam. (Cracks deeper)

The reduction of the maximum deflection at the midpoint of the cracked Euler Bernoulli beam could be the result of shifting crack location away from the center.

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