American Academic Scientific Research Journal for Engineering, Technology, and Sciences

ISSN (Print) 2313-4410, ISSN (Online) 2313-4402

http://asrjetsjournal.org/

Rotor Blade Aerodynamics Forces Modelling for Dynamic Analysis

Mohamed Abdalla Almheriegh*

Associate Professor, Department of civil Engineering, Faculty of Engineering - Tripoli University, P O Box 82677, Tripoli, Libya

Email: malmherigh@gmail.com

Abstract

In order to establish a rational method for structural analysis of wind turbine rotor with respect to failure in ultimate loading, either in strength or fatigue fashion. Two famous methods in predicting the complicated nature of the two main lift and drag forces acting 'normal and tangential' forces on the rotor blade are discussed and numerical solution 'iteration' using the coefficients involved in establishing these forces is presented and, therefore are readily introduced in the analysis stage typically done by dynamics analysis codes.

Keywords: ultimate loads on rotor blades; lift and drag forces; normal and tangential forces on blades; structural analysis of wind turbine blades.

1. Introduction

Various methods are used to calculate the aerodynamic forces acting on the blades of a wind turbine. The most advanced are numerical methods solving Navier-Stokes equations for the compressible flow as well as the flow near the blades.

The two major approaches to calculating the forces are the Actuator Disk Model and the Blade Element Model. In the paper to follow a brief introduction to these methods will be presented which extracted from Freris [1,2,3,4,5,6].

----

* Corresponding author.

2. Actuator disc model

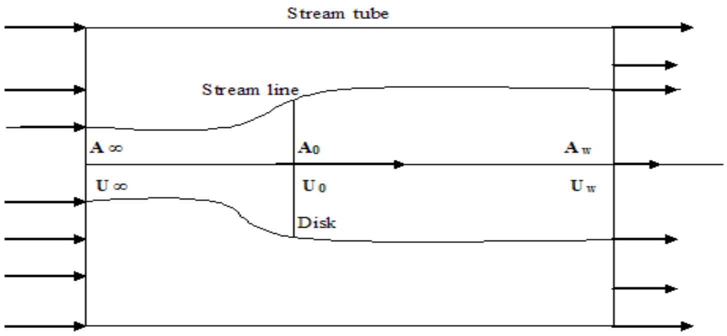

Figure 1: Actuator disk

Based on Bernoulli's equation and energy balances [5,6,4]. It is assuming that, the rotor is replaced by an actuator disc, through which the static pressure decreases discontinuously. By examining the flow through a control volume the extractable power from the turbine can be calculated Figure (1).

The stream tube has a cross-sectional area larger than the cross-sectional area for the upstream disc and a smaller area than the downstream disc. Within the stream tube, continuity is required and the rate of the mass flow must be constant.

$$\dot{m} = \left| \rho_\infty A_\infty U_\infty \right| = \left| \rho_o A_o U_o \right| = \left| \rho_w A_w U_w \right| \qquad 1$$

By introducing an axial interference factor, a, as the fractional decrease in wind velocity between the free stream and the rotor plane represented by

$$a = \frac{v}{U_\infty} \qquad 2$$

It is found that

$$U_o = U_\infty (1 - a) \qquad 3$$

The air, which passes through the disk, undergoes an overall change in velocity. The velocity multiplied by the flow rate gives the rate of change of momentum, more known as a force

$$T = \dot{L} = \dot{m}(U_\infty - U_w) \qquad 4$$

Combining the equations above with the fact that the change of momentum comes entirely from the pressure difference across the actuator disc, it is obtained that

$$p_o^+ - p_o^- = (U_\infty - U_w)\rho_o A_o (1-a) U_\infty \quad 5$$

To obtain the pressure difference, the Bernoulli's equation is applied separately to the upstream and downstream sections of the stream tube. For the upstream section it becomes

$$\frac{1}{2}\rho U_\infty^2 + p_\infty = \frac{1}{2}\rho U_o^2 + p_o^+ \quad 6$$

Similarly, downstream

$$\frac{1}{2}\rho U_w^2 + p_\infty = \frac{1}{2}\rho U_o^2 + p_o^- \quad 7$$

Subtracting equation (6) from equation (7) yields

$$p_o^+ - p_o^- = \frac{1}{2}\rho (U_\infty^2 - U_w^2) \quad 8$$

Equations (8) and (5) give

$$U_w = (1-2a) U_\infty \quad 9$$

Substituting (9), (3), and (1) into (4) obtains the force, T, which gives

$$T = 2\rho A_o U_\infty^2 a(1-a) \quad 10$$

Combining (3), and (9) and the rate of work done by the force; $P = T U_o$ the power extraction from the air is obtained as:

$$P = 2\rho A_o U_\infty^3 a(1-a)^2 \quad 11$$

Or, by introducing the dimensionless power coefficient, $C_p = 4a(1-a)^2$

$$P = \frac{1}{2}\rho A_o U_\infty^3 C_p \quad 12$$

The power coefficient represents the efficiency of the turbine, which depends on variables like the wind speed, the rotor speed and the pitch angle. The coefficient shows how much of the kinetic energy in the air stream that is transformed into mechanical energy. The maximum C_p as a function of a is $C_p = 16/27 \cong 0.59$, at $a = 1/3$ (obtained by taking the first derivative of the power coefficient $C_p = 4a(1-a)^2$ with respect to 'a' and equating it to zero).

The maximum value of C_p = 0.59 is called the Betz limit and applies to all types of wind turbines. Intuitively there must be a wind speed change at which the conversion efficiency is maximum. If there were no change in wind speed, no energy would be extracted, and the power of the wind turbine would be zero. If the air were brought completely to rest, all its energy would dissipate. However, a rotating wind turbine will not completely prevent the flow of air, so it can only extract a proportion of the kinetic energy in the wind. Hence in terms of force exerted by wind on rotor area based on this assumption pressure is underestimated on blades. Therefore, its use for predicting these forces is in decline. Modern wind turbines operate at performance coefficient of about (0.4)

3. Blade Element Theory

For the use of aeroelastic codes in design calculations, the aerodynamic method has to be very time efficient. The Blade Element Momentum (BEM) theory has been shown to give good accuracy with respect to time cost.

In this method, the turbine blades are divided into a number of independent

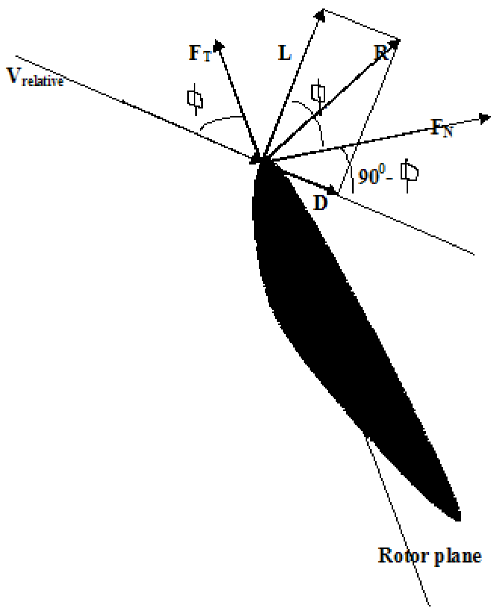

Figure 2: Load forces on blade section

elements along the length of the blade. At each section, a force balance is applied involving 2-D section lift and drag with the thrust and torque produced by the section. At the same time, a balance of axial and angular momentum is applied. This produces a set of non-linear equations, which can be solved numerically for each blade section, the presented discussion follows Andres [5] and Det Norske [4], only the force in the flow direction was regarded. The BEM theory, also takes notice of the tangential force due to the torque in the shaft. The left force L per unit length is perpendicular to the relative speed V_{rel} of the wind and equals:

$$L = \frac{\rho c}{2} V_{rel}^2 C_L \qquad 13$$

Where c is the blade chord length. The drag force D per unit length, which is parallel to V_{rel} is given by

$$D = \frac{\rho c}{2} V_{rel}^2 C_D \qquad 14$$

Since the interest only in the forces, normal to and tangential to the rotor-plane, the lift and drag are projected on these directions, Figure (2)

$$F_N = L\cos\phi + D\sin\phi \qquad 15$$

And

$$F_T = L\sin\phi - D\cos\phi \qquad 16$$

The application of this theory requires information about the lift and drag air foil coefficients C_L and C_D. Those coefficients are generally given as functions of the angle of incidence, Figure (3)

$$\alpha = \phi - \theta \qquad 17$$

Further, it is seen that

$$\tan\phi = \frac{(1-a)U_\infty}{(1+a')\omega r} \qquad 18$$

In practice, the coefficients are obtained from a 2D wind tunnel tests. If α exceeds about 15^0, the blade will stall. This means that the boundary layer on the upper surface becomes turbulent, which will result in a radical increase of drag and a decrease of lift. The lift and drag coefficients need to be projected onto the normal and tangential directions.

$$C_N = C_L\cos\phi + C_D\sin\phi \qquad 19$$

And

$$C_T = C_L \sin\phi - C_D \cos\phi \qquad 20$$

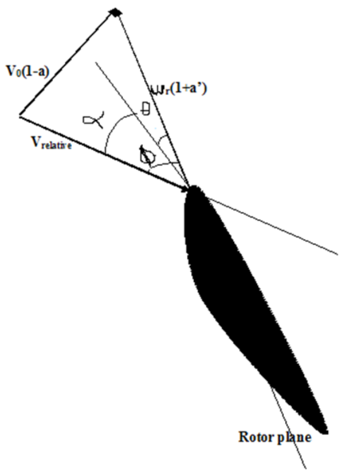

Figure 3: Velocities at rotor plane

Rotor Blade Chosen RISΦ-1 airfoil

Further, a solidify σ is defined as the fraction of the annular area in the control volume, which is covered by the blades

$$\sigma(r) = \frac{c_r B}{2\pi r} \qquad 21$$

Where B denotes the number of blades.

The normal force and the torque on the control volume of thickness dr since $F_N and F_T$ are forces per length

$$dT = NF_N dr = \frac{1}{2}\rho N \frac{U_\infty^2 (1-a)^2}{\sin^2\phi} cC_N dr \qquad 22$$

And

$$dQ = rNF_T dr = \frac{1}{2}\rho N \frac{U_\infty (1-a)\omega r(1+a')}{\sin\phi\cos\phi} cC_L rdr \qquad 23$$

Finally, the two influence factors are declared by

$$a = \frac{1}{\frac{4\sin^2\phi}{\sigma C_N} + 1} \qquad 24$$

And

$$a' = \frac{1}{\frac{4\sin\phi\cos\phi}{\sigma C_T} - 1} \qquad 25$$

These two factors are the key to establish a value for the forces normal and tangential to rotor plane using blade element method 'BEM'. These factors are partially empirical and solution could be attained via iteration for each value of r/R.

To increase accuracy tip loss correction factor need be applied, this is to allow for the velocities and forces not being circumferentially uniform due to the rotor having a finite number of blades. This factor is expressed as:

$$F = \frac{2}{\pi}\arccos(\exp(\frac{B}{2}\frac{R-r}{r\sin\phi})) \qquad 26$$

This reduction factor is called Prandtl's tip loss factor Det Norske [4].

This is yields:

$$a = \frac{1}{\frac{4F\sin^2\phi}{\sigma C_N} + 1} \qquad 27$$

$$a' = \frac{1}{\left(\frac{4F\sin\phi\cos\phi}{\sigma Cr} - 1\right)} \qquad 28$$

All terms as defined before.

More practical implementation of this method is further detailed in the literature, while results of the iteration done using the MathCAD software to arrive at linear pressure profile for typical one case blade parameters is shown next to this paragraph:

4. MathCAD sheet for blade wind load typical iteration

$\omega := 2.31 \mathrm{rad} \cdot \mathrm{sec}^{-1}$

$$\phi := \mathrm{atan}\left[\frac{(1-a)\cdot V_0}{(1+a_1)\cdot \omega \cdot r}\right]$$

Input parameters

$\phi = 28.419 \mathrm{deg}$

$c := 0.604 \cdot \mathrm{m}$

Pitch Angle $\alpha := \phi - \theta$ $\alpha = 17.819 \mathrm{deg}$

$r := 10 \cdot \mathrm{m}$

Using alpha, calculate Lift and Drag coefficients:

$\theta := 10.6 \cdot \mathrm{deg}$

$\alpha lpha_1 := 3 \cdot \mathrm{deg}$ $\alpha lpha_2 := 5 \cdot \mathrm{deg}$ $\alpha lpha_3 := 10 \cdot \mathrm{deg}$ $\alpha lpha_4 := 15 \cdot \mathrm{deg}$

$\alpha lpha_5 := 20 \cdot \mathrm{deg}$ $\alpha lpha_6 := 25 \cdot \mathrm{deg}$

$Re := 1.6 \cdot 10^6$

$CL_1 := 0.6$ $CL_2 := 0.94$ $CL_3 := 1.21$ $CL_4 := 1.25$ $CL_5 := 1.18$ $CL_6 := 0.98$

$CD_1 := 0$ $CD_2 := 0$ $CD_3 := 0.03$ $CD_4 := 0.085$ $CD_5 := 0.16$ $CD_6 := 0.32$

$C_L := \mathrm{linterp}(\alpha lpha, CL, \alpha)$ $C_L = 1.211$

Iteration No 1

$C_D := \mathrm{linterp}(\alpha lpha, CD, \alpha)$ $C_D = 0.127$

Normal Force Coefficient $C_N := C_L \cdot \cos(\phi) + C_D \cdot \sin(\phi)$

Tangential Force Coefficient $C_T := C_L \cdot \sin(\phi) - C_D \cdot \cos(\phi)$

$C_N = 1.125$

$C_T = 0.464$

$$\sigma_r := \frac{c \cdot B}{2 \cdot \pi \cdot r} \qquad \sigma_r = 0.029$$

$$F := \frac{2}{\pi} \cdot \mathrm{acos}\left(e^{-B \cdot \frac{R - r}{2 \cdot r \cdot \sin(\phi)}}\right)$$

$F = 0.999$

$$a := \frac{1}{\frac{4 \cdot F \cdot \sin(\phi)^2}{\sigma_r \cdot C_N} + 1} \qquad a_1 := \frac{1}{\frac{4 \cdot F \cdot \sin(\phi) \cdot \cos(\phi)}{\sigma_r \cdot C_T} - 1}$$

$a = 0.035$ $a_1 = 8.069 \times 10^{-3}$

Iteration No 2

$$\phi := \operatorname{atan}\left[\frac{(1-a)\cdot V_0}{\left(1+a_1\right)\cdot\omega\cdot r}\right] \qquad \phi = 27.394\,\mathrm{deg}$$

Pitch Angle $\alpha := \phi - \theta$ $\qquad \alpha = 16.794\,\mathrm{deg}$

Using alpha, calculate Lift and Drag coefficients

$\alpha lpha_1 := 3\cdot\mathrm{deg}$ $\quad \alpha lpha_2 := 5\cdot\mathrm{deg}$ $\quad \alpha lpha_3 := 10\cdot\mathrm{deg}$ $\quad \alpha lpha_4 := 15\cdot\mathrm{deg}$

$\alpha lpha_5 := 20\cdot\mathrm{deg}$ $\quad \alpha lpha_6 := 25\cdot\mathrm{deg}$

$CL_1 := 0.55$ $\quad CL_2 := 0.9$ $\quad CL_3 := 1.2$ $\quad CL_4 := 1.25$ $\quad CL_5 := 1.2$ $\quad CL_6 := 1.0$

$CD_1 := 0$ $\quad CD_2 := 0$ $\quad CD_3 := 0.03$ $\quad CD_4 := 0.085$ $\quad CD_5 := 0.18$ $\quad CD_6 := 0.31$

$C_L := \mathrm{linterp}(\alpha lpha, CL, \alpha)$ $\qquad C_L = 1.232$

$C_D := \mathrm{linterp}(\alpha lpha, CD, \alpha)$ $\qquad C_D = 0.119$

Normal Force Coeffiecient $\qquad C_N := C_L\cdot\cos(\phi) + C_D\cdot\sin(\phi)$ $\qquad C_N = 1.149$

Tangential Force Coefficient $\qquad C_T := C_L\cdot\sin(\phi) - C_D\cdot\cos(\phi)$ $\qquad C_T = 0.461$

$$\sigma_r := \frac{c\cdot B}{2\cdot\pi\cdot r} \qquad \sigma_r = 0.029$$

$$F := \frac{2}{\pi}\cdot\operatorname{acos}\left(e^{-B\cdot\frac{R-r}{2\cdot r\cdot\sin(\phi)}}\right) \qquad F = 0.999$$

$$a := \frac{1}{\frac{4\cdot F\cdot\sin(\phi)^2}{\sigma_r\cdot C_N} + 1} \qquad a_1 := \frac{1}{\frac{4\cdot F\cdot\sin(\phi)\cdot\cos(\phi)}{\sigma_r\cdot C_T} - 1}$$

$$a = 0.038 \qquad a_1 = 8.213\times 10^{-3}$$

Iteration No 3

$$\phi := \operatorname{atan}\left[\frac{(1-a)\cdot V_0}{(1+a_1)\cdot\omega\cdot r}\right] \qquad \phi = 27.316\deg$$

Pitch Angle $\alpha := \phi - \theta$ $\alpha = 16.716\deg$

Using alpha, calculate Lift and Drag coefficients**:**

$\alpha lpha_1 := 3\cdot\deg$ $\alpha lpha_2 := 5\cdot\deg$ $\alpha lpha_3 := 10\cdot\deg$ $\alpha lpha_4 := 15\cdot\deg$

$\alpha lpha_5 := 20\cdot\deg$ $\alpha lpha_6 := 25\cdot\deg$

$CL_1 := 0.55$ $CL_2 := 0.9$ $CL_3 := 1.2$ $CL_4 := 1.25$ $CL_5 := 1.2$ $CL_6 := 1.0$

$CD_1 := 0$ $CD_2 := 0$ $CD_3 := 0.03$ $CD_4 := 0.085$ $CD_5 := 0.18$ $CD_6 := 0.31$

$C_L := \mathrm{linterp}(\alpha lpha, CL, \alpha)$ $C_L = 1.233$

$C_D := \mathrm{linterp}(\alpha lpha, CD, \alpha)$ $C_D = 0.118$

Normal Force Coeffiecient $C_N := C_L\cdot\cos(\phi) + C_D\cdot\sin(\phi)$ $C_N = 1.149$

Tangential Force Coefficient $C_T := C_L\cdot\sin(\phi) - C_D\cdot\cos(\phi)$ $C_T = 0.461$

$$\sigma_r := \frac{c\cdot B}{2\cdot\pi\cdot r} \qquad \sigma_r = 0.029$$

$$F := \frac{2}{\pi}\cdot\operatorname{acos}\left(e^{-B\cdot\frac{R-r}{2\cdot r\cdot\sin(\phi)}}\right) \qquad F = 0.999$$

$$a := \frac{1}{\frac{4\cdot F\cdot\sin(\phi)^2}{\sigma_r\cdot C_N} + 1} \qquad a_1 := \frac{1}{\frac{4\cdot F\cdot\sin(\phi)\cdot\cos(\phi)}{\sigma_r\cdot C_T} - 1}$$

$a = 0.038$ $a_1 = 8.231\times 10^{-3}$

FINAL FORCES

$$K := \frac{4 \cdot F \cdot \sin(\phi)^2}{\sigma_r \cdot C_N} \qquad a_c := 0.2$$

$$a := \text{if}\left[a < 0.2, a, 0.5 \cdot \left[2 + K \cdot (1 - 2 \cdot a_c) - \sqrt{\left[K \cdot (1 - 2 \cdot a_c) + 2\right]^2 + 4 \cdot \left(K \cdot a_c^{\,2} - 1\right)}\right]\right]$$

$$\rho := 1.025 \cdot \text{kg} \cdot \text{m}^{-3} \qquad r = 10\,\text{m}$$

$$F_N := 0.5 \cdot \rho \cdot \frac{V_0^{\,2} \cdot (1 - a)^2}{\sin(\phi)^2} \cdot (c \cdot C_N)$$

$$F_N = 244.45\text{N} \cdot \text{m}^{-1}$$

$$F_T := 0.5 \cdot \rho \cdot \frac{V_0 \cdot (1 - a) \cdot \omega \cdot r \cdot (1 + a_1)}{\sin(\phi) \cdot \cos(\phi)} \cdot c \cdot C_T \qquad F_T = 98.102\text{N} \cdot \text{m}^{-1}$$

$$\lambda := 10 \cdot \text{deg}$$

Resolve Forces

$$F_{res} := \sqrt{F_N^{\,2} + F_T^{\,2}} \qquad F_{res} = 263.4\text{N} \cdot \text{m}^{-1}$$

$$FORCE_N := F_N \cdot \cos(\lambda) + F_T \cdot \sin(\lambda) \qquad FORCE_N = 257.771\text{N} \cdot \text{m}^{-1}$$

$$FORCE_T := -F_N \cdot (\sin(\lambda)) + F_T \cdot \cos(\lambda) \qquad FORCE_T = 54.163\text{N} \cdot \text{m}^{-1}$$

$$F_{res} := \sqrt{FORCE_N^{\,2} + FORCE_T^{\,2}}$$

$$F_{res} = 263.4\text{N} \cdot \text{m}^{-1}$$

$$Forces_0 := FORCE_N$$

$$Forces_1 := FORCE_T$$

Forces =

	0
0	257.771
1	54.163

$\text{N} \cdot \text{m}^{-1}$

5. Conclusions

Theoretical investigation based on blade element method theory (BEM) to derive left and drag forces thus calculating wind pressure acting on rotor blade of wind turbine, the mathematical formula developed is solved via iteration using developed Mathcad sheet and forces need be applied to rotor during finite element analysis are readily calculated.

References

[1] Freris L. L "Wind Energy Conversion Systems" Printice Hall 1996.

[2] Walker F. J and Jenkins N "Wind Energy Technology" John Wiley & Sons UK 1997.

[3] DET NORSKE VERITAS (DNV), Offshore Standard, DNV-OS-E301, Position Mooring, 2004, http://www.dnv.com, October 2004.

[4] Det Norske Veritas "Guidelines for Design of Wind Turbines" second edition 2002, printed by Judsk Centraltrykkeri, Denmark ISBN 87-550-2870-5

[5] Anders Ahlström "Simulating Dynamic Behaviour of Wind Power Structures" Licentiate Thesis Royal Institute of Technology, Department of Mechanics Stockholm2002. http://www2.mech.kth.se/~andersa/thesis/Licthesis.pdf, January 2003.

[6] David M. Eggleston and Forrest S. Stoddard "Wind Turbine Engineering Design" 1987 Van Nostrand Inc. New York, ISBN 0-442-22195-9.