

Calculating Environmental Design Loads for Floating Wind Turbine

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Abstract

Analysis of complicated structures usually requires; choice of the commercial software as well as good formulation of elements, and loads. The LS-DYNA3D explicit finite element dynamic analysis programme is believed to be good in performing structural analysis of a floating wind turbine and in simulating a full-scale model typical for moderate deep waters. The intended model for which loads are discussed is a middle size power rated floating 3 blades wind turbine elevated at about 50 m above main sea level a top a tripod lattice steel tower firmly resting on a moored floating concrete hull buoy, positioned on a concrete circular disk. The model is intended for use in moderately deep waters of up to 500m. The knowledge of floating offshore wind structures and important features of the LS-DYNA3D code are briefly revised. The theoretical basics for service loads experienced by the floating wind turbine are explored and the environmental loads are quantified and ready for use in the analysis.

Keywords: explicit finite element analysis; environmental design loads; load formulation for floating turbine.

1. Introduction

Wind energy technology has developed rapidly over the last few decades. Larger machines as well as new design trends have been introduced, which demand more sophisticated design tools, capable of providing more accurate predictions of loads. The need and interest of placing wind turbines in complex terrain areas has increased. In such sites, high wind speed, high turbulence levels and strong gusts are frequently present hence the weather conditions need careful considerations as they may seriously influence the reliability of wind turbines. In order to back-up further exploitation of wind energy it is important to provide the industry and the certifying institutions with computational tools capable of performing complete simulations of the behaviour of wind turbines over a wide range of different operational conditions.

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In the work reported herein, the LS-DYNA3D explicit code is used for analysis all loads therefore are functions of time. Important concepts and features of the code are highlighted, environmental loads quantification is proposed for use in the explicit code.

2. LS-DYNA3D Explicit Code

LS-DYNA3D is an explicit code for solving three-dimensional nonlinear solid mechanics problems which run in batch mode. The pre-processor in the updated edition of the program is capable of generating or translating any complicated geometry that might be dictated, meshing and mesh refinement and reasoning are also readily available. Upon solving for stresses and displacements in either local or global coordinates, the post-processor in the program can easily interpret the results.

LS-DYNA3D is based on a finite element discretization of the three spatial dimensions and a finite difference discretization of time. The explicit central difference method is used to integrate the equations of motion in time. The central difference method is conditionally stable, with stability governed by the ‘Courant limit’ on the time step Δt [1,2,3].

For solid elements, this limit is essentially the time required for an elastic stress wave to propagate across the shortest dimension of the smallest element in the mesh [1,2,4].

Equivalently, this maximum time step may be related to the period of the highest free vibration mode of the finite element mesh. LS-DYNA3D automatically calculates the maximum time step size at each step of the solution, thus minimizing the cost of the analysis while ensuring that stability is maintained.

LS-DYNA3D uses a lumped mass formulation for efficiency. This produces a diagonal matrix M , which renders the solution of the momentum equation.

$$Ma_{n+1} = f^{ext} - f^{int}$$

Trivial at each step, in that no simultaneous system of equations needs to be solved, f^{ext} are the applied external forces, and f^{int} are the element internal forces. The new accelerations a_{n+1} are easily found, from which the updated velocity and coordinates are calculated using the central difference integration formulas. In LS-DYNA3D the initial conditions to the transient dynamic problem are specified as initial velocities. Boundary conditions of many types ranging from constrained nodal translations and rotations to non-reflecting boundaries are prescribed easily [1,2].

In an explicit code there are many small time steps hence it is important to minimize the number of operations performed at each time step. The minimization is accomplished by using elements with one point Gauss quadrature (Gauss-Legendre Integration) for the element numerical integration. This formulation leads to spurious zero energy deformation modes the so-called “hourglass stiffness” or “hourglass viscosity” while retaining legitimate deformation modes stabilizes these spurious modes [1]. Further discussion of this and used features of the code are left to listed references [5].

3 Explicit Analysis - Important Algorithms

3.1 General

For the simulation of the non-linear and time dependent behaviour of complex structures such as the floating wind energy converter dealt with here, such models have to provide high accuracy in the prediction of deformations and stability. The limiting factors of the computer simulation are usually the computer run time and the memory required to solve large scale problems. To overcome these problems LS-DYNA3D uses the dynamic explicit time integration procedure for the solution of the semi-discrete equations of motion both in transient as well as in the static conditions (dynamic relaxation). In an explicit analysis neither the element nor the system matrices are built (the solution is element based) and consequently the memory requirements are insignificant. Unfortunately, explicit methods are only conditionally stable and hence the time step size has to be smaller than a critical value, this is directly dependent on the largest frequency of the finite element discretization (smallest element). As a result, in large scale problem such as the one dealt with here, extremely short time steps occur which increase computer run time. Contrary to implicit schemes the generation and fabrication of the system matrices, which are time and memory consuming, are avoided by explicit one where lumped mass and damping matrices are employed. Working with system vectors (instead of system matrices), which may be added up by the finite element contributions for the computation of the state variables it is possible to increase the number of degrees of freedom and thus large problems can be tackled. This is the main driving reason favouring explicit algorithms for large scale and nonlinear problems. Further to that, the use of explicit schemes provides the opportunity to create a uniform software concept both for the solution of static and dynamic problems. To this end a static problem has to be transformed into a dynamic one by adding an artificial acceleration and an artificial damping. This method is known as dynamic relaxation. When static problems are solved by dynamic relaxation both the mass and damping matrices lose their physical background and become fictitious quantities which control the iteration process.

3.2 Dynamic relaxation for static initialization

LS-DYNA3D contains a limited capability for performing quasistatic analysis using a dynamic relaxation algorithm. This feature is primarily intended to be used to generate a static stress solution as an initial condition for a transient dynamic analysis, but it has been applied with some success to the solution of more general static problems [1,2,3].

The dynamic relaxation method is based on the observation that the long time limit of a damped dynamic solution is the quasistatic solution. Damping is introduced through a 'dynamic relaxation factor' (default = 0.995) which multiplies the velocities computed at each step of dynamic relaxation solution. This factor can be adjusted by the user if required; increasing the factor decreases the effective damping while decreasing this factor increases the effective damping [1,2,3].

During the dynamic relaxation solution process, 'time' is really just a parameter to describe the solution process, and does not correspond to physical time. The current implementation uses a dynamic relaxation time step equal

to the standard dynamic time step. Thus, if it is desired to slowly apply the static loads to minimise overshoot in the solution, then a short trial dynamic run can be made to determine the time step size. The static loads to be applied during the dynamic relaxation solution can then be applied over some number of time steps (typically 2000-5000 but problem dependent) Jerry [1], and this determines the time points to be used on the load curve controlling the static loads.

Current implementation of dynamic relaxation in LS-DYNA3D is susceptible to dynamic overshoot if static loads are applied too quickly. If only history-independent material models (such as elasticity) are used, then the resulting solution will still be correct and this overshoot behaviour is of little consequence [1].

If history-dependent material (such as plasticity) is used, however this dynamic overshoot can cause yielding which is erroneous, and therefore an incorrect static solution is obtained. Thus, the dynamic relaxation static solution capability can be used with confidence for elastic initialisation, but must be carefully used with slowly applied loads to prevent overshoot and inaccuracy in history-dependent static problems [1,2]. Dynamic relaxation is not used in this work to limit computer run time and because loads are applied gradually, therefore the forward discussion aimed at enhancing this important capability.

3.3 Dynamic relaxation for stresses in rotating bodies

In many applications such as flywheel design, machine tool safety, or turbine engine containment, it is important to solve a transient dynamic problem beginning with a stress state induced by rotational motion. This problem is easily solved in LS-DYNA3D using the dynamic relaxation option for computing the initial stresses. The part of the model which is rotating should be identified, by material as receiving a body force load due to a prescribed angular velocity. These body force loads should reference a load curve which begins at zero and increases to a value coinciding with the rotational velocity, and remains constant at that value to some large time. This load curve should be marked as active for static initialization only. Initial velocities for these rotating bodies computed on their post-initialization deformed geometry may be generated by specifying on the control card the number of materials to initialize for rotational motion, and then listing these materials in their proper location, Jerry [1]. This approach will allow a smooth transition from the body-force-based calculation of the initial stresses into the transient dynamic phase where the bodies actually rotate in space. During the transient portion of the analysis, the rotating bodies may be allowed to rotate freely, or may have rotational velocities prescribed for them using load curves flagged to be active for transient dynamic analysis only. Also, other loads may be also added to the rotational body during the transient dynamic phase, such as impact with stationary object, Jerry [1].

4. Load Cases

An important part of the design process for a wind turbine is its ability to withstand the various loads it will experience during its expected life. The prime purpose for this assessment is to quantify loads that the turbine will be able to withstand with sufficient safety margin. This task will be systemised by analysing the developed model for a number of relevant load cases including combinations of environmental, operational and external

conditions incorporated in it. In the design of a wind turbine, it is important to identify all load cases which are relevant. Based on evaluation of the analysis, like in this work, structural response of the structure in terms of strength, serviceability and fatigue behaviour will show any drawbacks in load assessment. Lack of statistical data, knowledge of the properties of the used materials, extreme load combinations, method of analysis etc, all could be sources of errors that will lead to failure and financial losses. Design loads for floating wind turbine is broadly discussed in Mohamed [5,6,8,9,10,11], rich list of references are also reported in these references as well.

5. Service Conditions

This must reflect and quantify the most significant conditions that a wind turbine is likely to experience such as;

- Normal operation and power production conditions
- Cut-in, cut-out and standstill conditions
- Transportation, installation and assembly
- Faults, maintenance, testing loads, loss of mooring conditions

All these conditions must be quantified and reflected in the design loads together with main structural, mechanical and environmental loads the turbine might face. Design load cases to be used in analysis are constructed by a combination of a relevant design situations and external environmental conditions. Such combinations reflect:

- Normal low speed standstill conditions
- Normal operation and normal external conditions
- Normal operation and extreme external environmental conditions
- Standstill condition under severe surviving environmental conditions.

Formulating the quantification of these conditions to incorporate them in the analysis phase will be the focus of the next sections.

6. Wind Field Presentations

It is very important for the wind industry to accurately describe the wind conditions. Turbine designers need the information to optimise the design of their turbines and turbine investors need the information to estimate their income from electricity generation. As is well known, the highest wind velocities are generally found on hilltops, exposed coasts and offshore. Various parameters need to be known concerning the wind, including the mean wind speed, directional data, and variations about the mean in the short term (gusts), daily, seasonal and annual variations, and variations with height. These parameters are highly site specific and can only be determined with sufficient accuracy by measurement at a particular site over a sufficiently long period.

From the point of view of wind energy, the most striking characteristic of wind resource is its variability. The wind is changing both geographically and temporally. Furthermore, this variability persists over a wide range of time scales, both in space and in time, and the importance of this is amplified by the cubic relationship to the available power. The values suggested in this paper will be presented later.

7. Load Types

The external loads acting on a wind turbine are mainly wind loads. As a wind turbine consists of slender elements such as blades and tower, inertia loads will be generated in addition to the gravity loads that act on these elements. Loads due to operation such as centrifugal forces, Coriolis forces and gyroscopic forces can be of significant effect.

In most cases loads on offshore wind turbine can be classified as:

- Aerodynamic blade loads due to wind.
- Gravity loads on all turbine parts.
- Centrifugal forces due to rotation.
- Gyroscopic loads due to yawing.
- Hydrodynamic loads on the supporting hull.

Gravity loads on turbine blades can cause bending moments in blades in the edge wise direction. In pitch-controlled blades, gravity loads will result in moments in the flap wise direction. Due to the rotational nature of blades, these gravitational moment effects will be cyclic. Eventually, the larger the rotor diameter, the greater will be the root moment, typically the blade root bending moment will follow a fourth-power law in blade diameter. Considering that the rotor area follows a quadratic power law in rotor diameter, this forms one of the challenges in making the wind turbine larger.

The adoption of the free yaw system (rotation about vertical axis) in the floating wind turbine, believed to render the gyroscopic forces (in the form of a moment) of minor significance especially for 3-blades turbine Det Norske [6], the even distribution of loads about the global vertical axis will help support this belief and is hence applied herein.

Aerodynamic, forces believed to be the most significant and will be calculated according to the ‘Beam Element Theory’ sometimes abbreviated as BEM and Momentum Theory and will be applied as nodal forces on blades while segment pressure value is calculated and applied to nacelle shell face opposing the wind direction as segment pressure as will be discussed.

Hydrostatic water pressure is applied all round to the floating hull and addressed by the code due to body loads and water structure interaction. Wave and current effects will be superimposed linearly in the horizontal flow direction, as a kinematics velocity. Hydrodynamic forces due to wave, current and water pressure will be calculated using “Morison’s” equation. It is used to calculate the hydrodynamic loads accordingly, as the sum of inertia forces and drag forces. The resulting force will be applied to the supporting hull as a vertical linear

pressure profile, acting in the direction of flow, coincident with wind flow and normal to hull vertical axis decreasing with depth. Further formulation of Morison's equation will follow in this paper. Meanwhile quantification of hydrodynamic loads proposed here is detailed later.

8. Aerodynamic Loads on Rotor

Once all necessary equations have been derived from the Momentum Theory and the Beam Element theory using the approach discussed in [5]. Then the different control volumes (airfoil) are assumed to be independent and each strip of the blade may be treated separately and therefore the results for one radius can be computed before solving for another radius. For each control volume, the algorithm can be derived into ten steps "Blade Element Theory" equations as developed in [5], and are listed in attached Appendix to this paper:

1. Initialise a and a' typically $a = a' = 0$
2. Compute the flow angle ϕ , equation A1
3. Assuming local pitch angle θ relative to rotor blade compute the local angle of attack α , equation A2
4. From the assumed (used) airfoil data knowing α read $C_L(\alpha)$ and $C_T(\alpha)$ these values are empirical and based on wind tunnel tests on the airfoil they are either provided by manufactures or ready in standards a typical value of Reynolds' number (R_E) is needed
5. Compute C_N and C_T coefficients, equations A3, A4
6. Calculate solidity σ and correction factor F using equation, equations A5, A6
7. Calculate a and a' , equations A7, A8
8. If a and a' has changed more than a certain tolerance relative to values assumed: go to step 2 starting with values attained for a and a' else continue.
9. If $a > 0.3$ then the simple momentum theory breaks down i.e. ' a ' must be corrected as follows:

If $a > a_{critical} = a_c \cong 0.2$ then Glauert's correction apply and hence:

$$a = \frac{1}{2} \left(2 + K(1 - 2a_c) - \sqrt{(K(1 - 2a_c) + 2)^2 + 4(Ka_c^2 - 1)} \right) \text{ in which}$$

$$K = \frac{4F \sin^2 \phi}{\sigma C_N}, \text{ calculated corrected value of } a \text{ replaced previous value}$$

10. Finally forces normal and tangential to rotor plane are calculated using:

$$F_N = \frac{1}{2} \rho \frac{V_0^2 (1-a)^2}{\sin^2 \phi} c C_N = \text{Normal force per unit length of blade and}$$

$$F_T = \frac{1}{2} \rho \frac{V_0(1-a)\omega \times r(1+a')}{\sin \phi \cos \phi} c C_T = \text{Tangential force per unite length of blade}$$

With the terms as defined later of this paper, these values are assumed acting normal and tangential to rotor swept plane orthogonal to wind flow direction. Therefore for coned blades such as is the case here, forces are transformed and applied at the cone angle of 10 degrees (in this model) abiding with the wind direction.

This is in principle the least complicated combined formulation of the Beam Element and Momentum Theories, for wind turbine blades load calculation. In order to get better results the method need to be extended and corrections some times are relevant. Simplifications and idealisations are the source of simplicity of the method over other existing methods.

From the above discussion given Reynolds' number, lift and drag coefficients for the used airfoil then for each strip at r from the blade centre knowing θ and guessing a and a', iteration till convergence is attained (a process could only be feasible through a computer-based subroutine using for example MathCAD or Fortran as developed in [5]). Assuming that convergence is attained this will give pressure values normal and tangential to the rotor swept plane and for each (r/R) chord radius ratio for the blade. Thus, for different ratios of r/R the iterative operation is repeated and the corresponding pressure value is gained. Therefore, blade airfoils must be chosen for which iteration is performed and resulting wind thrust will be specific for a blade with the assumed airfoil at the given strip. The final pressure or thrust profile normal to the blade and parallel to wind direction is believed to be in the form similar to that shown in Figure (1) but are boundary and airfoil dependent. This thrust profile is assumed to be acting uniformly over the whole rotor area, hence applying it to blades will coincide with the assumption of the Beam Element Method used for development of the approach. When this profile is reached applying it to the model could be done through creating a load curve for each r/R ring as well as the node set for all nodal forces located around this ring in the 3 blades. The thrust then applied as nodal forces for each corresponding radius.

Detailed quantification of these aerodynamic forces is developed in [5].

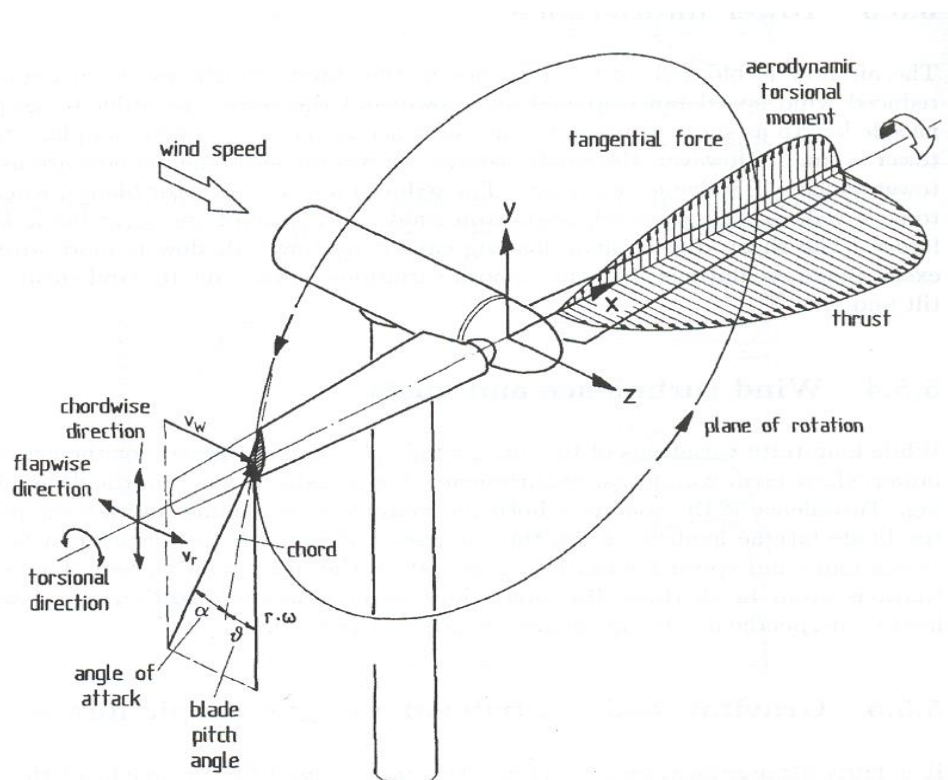


Figure 1: Modelling wind loads on blades.

9. Current and Wave Forces

Waves and currents cause distributed forces on structures placed in their field. The three main categories are:

1. Drag forces caused by pressure differences between front side and rear side of the exposed structural component. The pressure differences caused by the friction between the water and the structure that may trigger separation of the boundary layer into a turbulent wake.
2. Inertial mass forces occurring in accelerating flows partly as the forces that would have accelerated the replaced water volume in an undisturbed flow and partly the force required to make the change of the flow pattern to fit with the presence of the structure.
3. Diffraction forces on the surface of structures that in the horizontal directions are not small as compared to the wavelength.

The following paragraph will focus on wave and current forces imposed on the floating hull with the cross-sectional dimension D of the hull being much smaller than the wave length L . Typically, severe marine conditions, such as for the British North Sea waters, a wave length of about (450-560m) [7] and for $D = 12.5\text{m}$

(typical diameter of the supporting hull). Therefore, D is much less than $0.2L$ and given this condition these forces can with sufficient confidence be determined by Morison's formula [5].

9.1 The Morison force

For a circular cylinder, Figure (2) of diameter D and cross-sectional area $A = \pi D^2/4$ placed in a flow of water with mass density ρ with particle velocity v and particle acceleration a , the force per unit length of the cylinder is given by equation:

$$q = \frac{1}{2} \rho C_D D V_c |V_c| + \rho C_M A a_c \tag{1}$$

Where V_c m/sec and a_c m/sec² are velocity and acceleration of the water particles orthogonal to the cylinder axis respectively. The mass density ρ of the seawater is typically about 1025 kg/m³. The values of the coefficients C_D (drag coefficient) and C_M (inertia coefficient) are partly empirical. They reflect the size of the drag force and the inertial force respectively.

This approach is conservative as it assumes hydrodynamic force on fixed installation. In this case the forces on the floating body are reduced due to the ability of the structure to move. However due to the size of the inertia (dominant force in this case) involved and the fact that the mooring will provide restraint at the stage of loading, hence the relative speed of the floating body and water particles is large enough for this assumption to provide an acceptable representation. This approach was widely mentioned to be accurate enough throughout the literature [5].

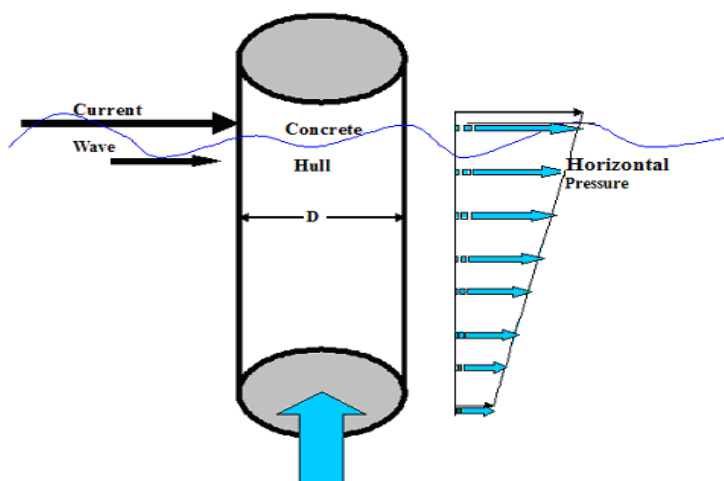


Figure 2: Environmental forces on hull vertical side

Buoyancy pressure

Detailed discussions are given for the inertia and drag coefficients throughout the fluid and hydrodynamics literature [5,8,9,10]. For the scalar drag force and mass force coefficient case the mass force coefficient C_M is typically about 2 while the drag force coefficient varies between 0.7 and 1.2. Det Norske (DS449 1983) [6] gives the following values for circular tubes in connection with Stokes' 5th order wave theory:

$$C_M = 2.0, C_D = \begin{cases} 1.2 \text{ for } R \leq 2.10^5 \\ .7 \text{ for } R \geq 4.10^5 \end{cases}$$

$R = \frac{D|V_C|}{\nu}$ Is Reynolds' number the kinematics viscosity ν is $1.11 \times 10^{-6} \text{ m}^2/\text{sec}$. For the seawater giving

$$R \approx 0.9 * 10^6 D|V_C| \quad \text{with } D \text{ in meters and } V_C \text{ in m/sec (values in between may be linearly interpolated).}$$

The Department of Energy [8], [9], [10], recommends Morison's equation to be used for wave loading with appropriate values for the coefficients. For Stokes 5th order theory $C_D = 0.8$ in the splash zone and = 0.6 elsewhere and $C_M = 2$ unless found lower by diffraction analysis are used, if Airy wave theory is used $C_D = 1$

In brief the application of Morison's equation depending on the values of the water particle speed V_c and water particle acceleration a_c in the flow direction, in turn the values of both V_c and a_c are dependent on the wave theory used to determine them. Wave theories describe the kinematics of waves of water on the basis of potential theory. In particular they serve to calculate the particle velocities and accelerations and the dynamic pressure as function of the service elevation of the waves. The waves are assumed to be long-crested i.e. they can be described by a two-dimensional flow field and are characterised by the parameters: wave height (H), period (T), wave length (L), relative height (z) and water depth (d). Different wave theories of varying complexity developed on the basis of simplifying assumptions are appropriate for a different range of wave parameters. Among the most common theories are: the linear Airy wave theory and the Stokes' fifth order theory. It is obvious that both wave and current velocity and accelerations are decaying with depth; therefore, their effect is decaying as well.

To quantify the linear distributed pressure along the hull vertical axis, a certain wave theory finding of V_c and a_c need be followed; meanwhile the values of drag and inertia coefficients are usually recommended by guidelines and mainly decided by Reynolds' number.

Based on Airy linear wave theory, the maximum horizontal particle velocity can be calculated according to Department of Energy, [8], through, [10]:

$$V_C = \frac{\pi \times H}{T} \times \frac{\cosh(k \times (z + d))}{\sinh(k \times d)} \times \cos \phi$$

Where H is the wave height in metres, T is wave period in seconds, d is the water depth in metres, z is the depth in metres at which water particle horizontal velocity in m/sec is calculated and k is empirical wave number

($k = \frac{2\pi}{L}$). The cosine term is the wave phase which is set to 1 to calculate the maximum horizontal particle velocity in m/sec.

The second part of Morison's force is the inertia which is dominant in this case. The inertia force is much dependent on water particle acceleration as well as the inertia coefficient as discussed earlier. Following Airy linear theory and previous source:

$$a_c = \frac{\pi \times H \times g}{L} \times \frac{\cosh(k \times (z + d))}{\cosh(k \times d)} \times \sin \phi$$

g is acceleration due to gravity 9.81 m/sec² other variables as defined before and the sine term is set to 1 to calculate the maximum particle acceleration.

Having calculated the parameters involved in Morison's force either at different depths or at the water surface and bottom, a pressure profile in the shape of Figure (2) is established representing the vertical distributed force on the hull orthogonal to hull vertical axis. If the above dimensions were followed the resulting force will be in N/m or force per unit length of the vertical cylinder wall.

Morison's force therefore is:

$$F \text{ (N/m)} = F_{\text{drag}} + F_{\text{inertia}} \text{ acting in the discussed sense.}$$

Quantifying this formula to create loads to apply them to the model is left to the user and fully developed in Mohamed, [5]. For moderate depths typically between 100m and 500m this distributed pressure profile could be assumed constant on the hull for the purpose of analysis [5], the error involved is insignificant. When quantifying the hydrodynamic loads reference will be made to [7,11] for the used parameters.

10. Conclusions

Through introduction of explicit dynamic finite element analysis code and deep investigation of the most important forces exerted on a floating heavy structure is introduced. Environmental forces namely, wind forces on rotor blades, wave and current forces as applied to floating wind turbine structure is mathematically calculated and ready to be included in the finite element analysis explicit LS-DYNA3D commercial code for dynamic structural analysis as a load vector.

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Appendix

$$\tan \phi = \frac{(1 - a) U_{\infty}}{(1 + a') \omega r} \tag{A1}$$

$$\alpha = \phi - \theta \tag{A2}$$

$$C_N = C_L \cos \phi + C_D \sin \phi \tag{A3}$$

$$C_T = C_L \sin \phi - C_D \cos \phi \tag{A4}$$

$$\sigma(r) = \frac{c_r B}{2\pi r} \tag{A5}$$

$$F = \frac{2}{\pi} \arccos\left(\exp\left(\frac{B}{2} \frac{R - r}{r \sin \phi}\right)\right) \tag{A6}$$

$$a = \frac{1}{\frac{4F \sin^2 \phi}{\sigma C_N} + 1} \quad \text{A7}$$

$$a' = \frac{1}{\left(\frac{4F \sin \phi \cos \phi}{\sigma Cr} - 1 \right)} \quad \text{A8}$$