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Analysis Over the Accuracy of Dynamic Formulas for Predicting Ultimate Load Capacity in Deep Foundations

Alcineide D. Pessoa[a], Gean Carlos L. Sousa[b], Gérson J.M. Anjos[c], Rodrigo C. Araujo[d]*

[a]Lecturer, State University of the Tocantina Region of Maranhão (UEMASUL), Imperatriz-MA, Brazil

[b]Assistant Professor, Federal University of Maranhão (UFMA), São Luís-MA, Brazil

[c]Associate Professor, Federal University of Pará (UFPA), Belém-PA, Brazil

[d]Associate Professor, Federal University of Maranhão (UFMA), São Luís-MA, Brazil

[a]Email: alcineide.sousa@uemasul.edu.br

[b]Email: gean.sousa@ufma.br

[c]Email: mirandadosanjos@gmail.com

[d]Email: araujo.rodrigo@ufma.br

Abstract

In Civil Engineering, particularly in Geotechnics, several empirical methods, commonly referred to as dynamic equations, have been proposed for prediction of ultimate load capacity of driven piles. However, these formulas are admitted inaccurate and, so, there is a need to evaluate the results obtained by them. In this work a comparative analysis of the values obtained by five dynamic equations (Janbu, Danish, Gates, FHWA-Gates and WSDOT) with actual ultimate load capacities (obtained through pile load tests at site) is made. Errors are measured using the root mean squared error and the correlation between the equation´s results and the measured values is verified. The results showed important differences between the ultimate capacities obtained from the analyzed models and the real values verified in field tests. It was also verified a superiority, in terms of lower error and greater correlation, of the WSDOT and Danish formulas. Attempts were made to improve the methods. For this, coefficients were determined that, when multiplied by the results of the formulas, promoted a reduction in error. Once again, WSDOT presented best perform in terms of correlation and error.

Keywords: Ultimate Capacity; Dynamic Equations; Deep Foundations; Root Means Squared Error.

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* Corresponding author.

1. Introduction

In engineering, several models, empirical or not, have been proposed to solve many types of problems. These models go mainly through physical assumptions and construction of approximations by mathematical models, which generate a series of uncertainties and approximation errors. An important problem in geotechnics is the prediction of the ultimate load capacity of deep foundations [1]. It is common to find in the literature several formulas that try to perform this task. Author in [2] explains that several equations have been developed to calculate the ultimate capacity of a pile during driving which are widely used in the field to determine whether a pile has reached a satisfactory bearing value at the predetermined depth. However, the presence of uncertainties in these estimates is a constant [3]. Because the dynamic formulas originate from different hypotheses, the results may differ widely, depending on the formula used [4]. Classical methods such as Janbu´s and Danish formulas are still used as models for calculating ultimate load capacity. On the other hand, more recent models such as that proposed by the Washington Department of Transportation (WSDOT) have been incorporated into the literature [4]. However, it is noticeable that comparative analyzes of the efficiency of each formula are necessary, once these studies are scarce in the literature.Thus, the objective of this work is to evaluate the accuracy of the ultimate capacity estimated through five dynamic equations presented in the literature. To this end, the capacity forecasts resulting from such proposals are evaluated by comparing them with actual results obtained through pile load tests at site. To achieve this goal the work calculates the ultimate capacity using five pile-driving formulas (Janbu, Danish, Gates, FHWA-Gates and WSDOT) and then compares by means of the Root Mean Squared Error (RMSE) and correlation the results obtained by the formulas with the capacities measured in loco.

2. Capacity in Deep Foundations

Foundation can be defined as a system formed by structural foundation elements (EEF) and the various layers of soil that surround them [5]. Author in [6] explains that all engineered construction resting on the earth must be carried by some kind of interfacing element called a foundation. Reference [7] states that foundations are structural elements whose function is to transmit the loads of the structure to the ground on which it rests. In a similar way, one can say that foundation is the lowest part of a structure generally and its function is to transfer the load of the structure to the soil on which it is resting [8]. The way this transfer takes place is the parameter used to classify the foundations as shallow or deep. Shallow foundations are those whose transmission is made through the base of the structural element of the foundation, considering only the support of the piece on the ground layer, being neglected any other form of load transfer, while deep foundations are those that transfer the loads by either skin friction along the shaft or point resistance [5]. A deep foundation is one whose base rupture mechanism does not appear on the surface of the ground [9, 10]. On the other hand, reference [11] defines that deep foundation is the foundation element that transmits a load either by the base (tip resistance) or by its lateral surface (shaft resistance) or by a combination of both, being its tip or base at a depth greater than twice its smallest dimension in the plant and at least 3.0m [11]. Focusing on deep foundation, it can be said that from the point of view of soil mechanics, there are two general types: the first type may be represented by a foundation installed by some process of excavation or drilling which does not induce significant changes in density or structure of the bearing soil and the second type may be represented by a deep foundation forced into the ground by driving or a similar operation, that induces significant changes in adjacent soil [12]. Considering only pile foundation cases, imagine any pile of length L installed in the ground. If a compression force P is applied to its head and progressively increased, resistant tensions will be mobilized. The transfer of superficial load to the ground may be partially by vertical distribution of the load along the pile shaft and partially by a direct application of load to a lower stratum through the pile point [6, 12]. This way, the ultimate load capacity is usually expected to be obtained by load-carrying capacity of the pile point plus the frictional resistance (skin friction) derived from the soil–pile interface [1, 2]. To determine the ultimate capacity of an isolated pile, three verification mechanisms can be used: static formulas (theoretical or empirical), dynamic equations, or load tests. In the last decades, several studies have applied artificial intelligence to improve the accuracy of bearing capacity predictions specially referring to driven piles [13, 14, 15, 16]. Referring to pile driving formulas its historical popularity among practicing engineers may be attributed to the fact that it reduces the design of pile foundations to a quite simple procedure. However, their obvious deficiencies and unreliability makes that the price one pays for this artificial simplification is remarkably high [17].

2.1. Dynamic equations

Within the context of dynamic equations, these models are based on the permanent penetration of a pile, caused by the application of a hammer or pylon blow, always related to the driving energy. Given its small size, it is usually based on the average value obtained from the last few driving blows. Author in [4] highlights that the fact that these models are based on that permanent penetration corroborates with the requirements in [11], which requires its verification. Five dynamic equations will be presented below. The choice of these formulas was due to the variables necessary in each of them and its availability in the database. The list below is a summary of the notations used in the pile driven formulas:

Qt: Ultimate Load Capacity (kN)

η: Efficiency

W: Hammer´s Weight (kN)

West: Pile´s Weight (kN)

h: Hammer Drop Height (m)

L: Pile Length (m)

s: permanent penetration of a pile, caused by the application of a hammer or pylon blow (m/blow)

A: Pile Cross Section (m²)

Ep: Modulus of Elasticity (kN/m²)

2.1.1. Gates´Formula

The Gates proposal (equation 1) is a strictly empirical relationship between hammer energy and final penetration, and the results of pile load tests. It was developed by a statistical adjustment (based on approximately 100 load proof tests) [18].

$$Q_t = 4{,}44822.\left[\frac{6}{7}\sqrt{\eta\frac{h}{0{,}3048}.\frac{W}{0{,}00444822}}.\log(\frac{0{,}25}{s})\right] \quad (1)$$

2.1.2. WSDOT Formula

The Washington Department of Transportation (WSDOT) has used several methods for predicting mobilized pile resistance, including The Engineering News formula (ENR) and Gates. It has also proposed a dynamic formula (WSDOT) which is presented in equation 2 [18].

$$Q_t = 4{,}44822.\left[6{,}6.\eta.\frac{h}{0{,}3048}.\frac{W}{4{,}44822}.\ln(\frac{0{,}25}{s})\right] \quad (2)$$

2.1.3. Gates Modified (FHWA) Formula

For small projects where a dynamic formula is used, the Gates modified (FHWA) formula is preferable as it correlates best with the static load test results. The FHWA-Gates formula consists of the expression below (equation 3) [18].

$$Q_t = 4{,}44822.\left[1{,}75.\sqrt{\eta.\frac{h}{0{,}3048}.\frac{W}{0{,}00444822}}.\log\left(\frac{0{,}25}{s}\right) - 100\right] \quad (3)$$

2.1.4. Janbu´s Formula

Janbu's formula, proposed in 1953, considers energy losses in the hammer, elastic pile compression and impact. In Janbu's formula the dynamic resistance of the soil is considered assuming a load increment proportional to the static resistance. Expressions 4, 5 and 6 summarize this formula [3].

$$Q_t = \frac{\eta.W.h}{s\left[C'.\left(1+\sqrt{1+\frac{\lambda}{C'}}\right)\right]} \quad (4)$$

$$C' = 0{,}75 + 0{,}15.\left(\frac{W_{est}}{W}\right) \quad (5)$$

$$\lambda = \frac{\eta . W . h . L}{A . E_P . s^2} \tag{6}$$

2.1.5. Danish Formula

The Danish formula (equation 7) was developed by authors in [19] who proposed an equation obtained through a dimensional analysis. The proposal takes into consideration the efficiency of the hammer and the energy loss due to the elastic deformation of the pile [3].

$$Q_t = \frac{\eta . W . h}{s + \sqrt{\frac{\eta . W . h . L}{2 . A . E_P}}} \tag{7}$$

3. Materials and Methods

As previously stated, to evaluate the quality of ultimate capacity estimates obtained through dynamic equations, the work compared the capacity predictions resulting from five of these proposals with load capacities obtained through load tests. The database used to carry out the project was compiled from authors in [20, 21, 22] and consisted of 233 load tests, carried out in diverse cities and different countries, for which load capacity, hammer weight, hammer drop height, pile length and pile weight values were available. These values were used to calculate ultimate capacity using the Gates, WSDOT and FHWA-Gates formulas. Of the 233 tests available, 153 also had information on the modulus of elasticity of the pile. These data allowed the estimates by Janbu´s and Danish formulas.

Table 1: presents the efficiencies values adopted for each equation, which have been chosen according to the authors' practical experience.

Equation	(Statics load tests – VDV_9%D)	(Dynamics load tests – PDA)
Janbu	75%	30%
Danish	40%	20%
Gates (1957)	80%	80%
WSDOT	35%	35%
FHWA-Gates (2006)	55%	65%

Table 1: Efficiency Adopted (η)

To measure and thus compare the equations proposed in the literature with the results of load tests Pearson Correlation Coefficient and Root Mean Squared Error metrics were used. Correlation identifies two groups of data with some relationship to each other, that is, if high (low) values of one of the variables implicated in high (or low) values of another variable. A correlation analysis provides a number that summarizes the degree of linear relationship between the two variables, which is called the correlation coefficient. Then, the choice of the correlation coefficient (Equation 8) was due to this metric be widely used to evaluate this type of methodology

[23].

$$\rho = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}} \tag{8}$$

where X and Y are the variables analyzed.

However, the linear correlation coefficient may lead to false conclusions when interpreted as an accuracy index of predictions or simulations. For example, the analysis of a variable whose simulation and observation are highly correlated, it can hide the tendency of simulations to overestimate or underestimate the observed. Thus, a measure often used to verify the accuracy of numerical models is the Root Mean Squared Error (RSME), which is defined as the mean of the difference between the estimator value and the squared parameter. The root mean squared error is obtained by the expression presented in Equation 9 [24].

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y-\hat{y})^2}{n}} \tag{9}$$

Where y is the measured value, $\hat{y}$ is the value obtained in the analyzed model and n is the number of samples.

In other words, the choice of correlation as an evaluation metric was because it is a metric widely used to evaluate comparisons of this type, while the use of the root mean squared error as an evaluation metric is extremely important because its evaluation is a better way to verify the accuracy of the methods used to solve the proposed problem. For a visual evaluation, scatter plots are presented that relate real values and calculated values. Finally, statistical analysis was also performed to determine adjustment coefficients capable of improving each method. After obtaining the adjusted results for each method, by multiplying original formulas by the respective coefficients above mentioned, correlations and RSME were calculated again.

4. Results and Discussion

Using the formulas described in equations 1 to 7, the ultimate load capacities for all samples were calculated. Comparisons between the calculated values and the values obtained by load tests at site are shown in figures 1 to 5. Two groups of simulations were performed. In the first group, the Janbu´s and Danish formulas were used because these models need the values of the modulus of elasticity of the pile. For these simulations 153 samples were used. In Figure 1 the results obtained with Janbu's formula are compared graphically with the load capacity values obtained in loco.

Graphically, it is possible to verify interesting discrepancies between the results obtained by the Janbu´s equation and the actual values. It can be noticed an important difference between the actual values and those obtained byjanbu's formula, highlighting a sample in which the value obtained by the formula (1,708KN) was extremely lower than the actual value (14,740KN). Figure 2 shows the comparison between Danish Formula´s results and actual load tests values.

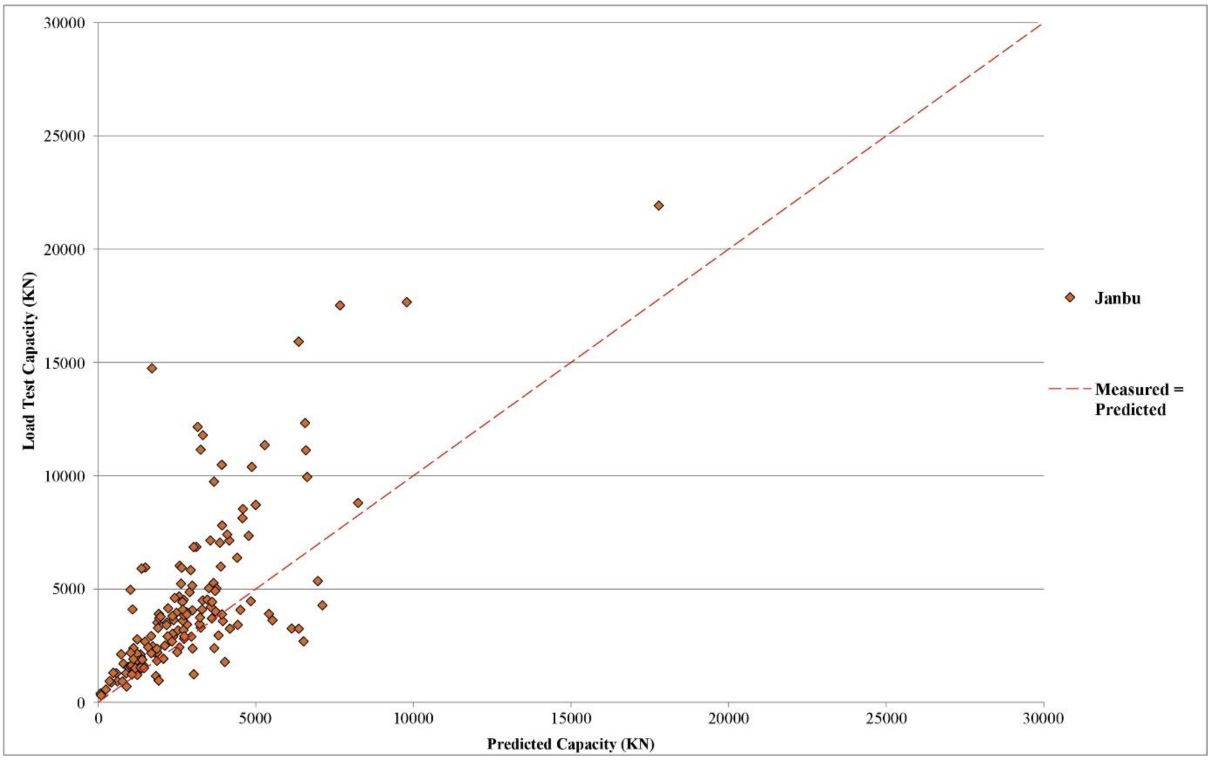

Figure 1: Comparison between Janbu´s results and load tests values

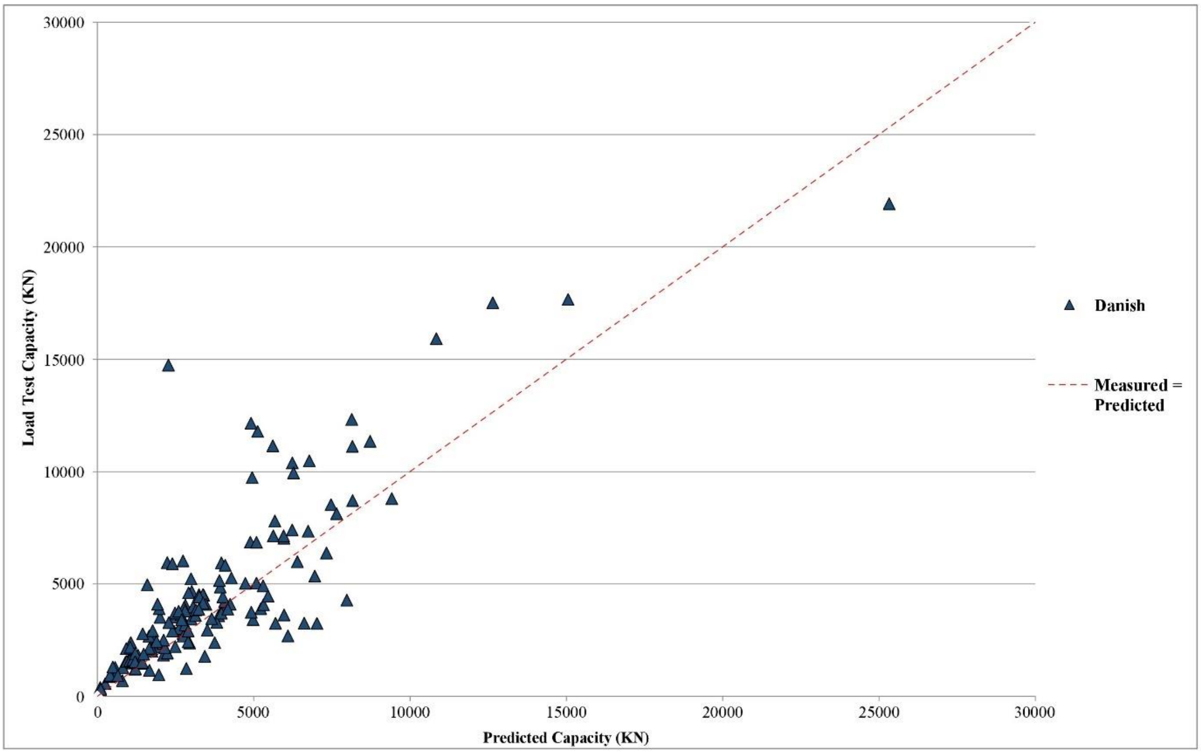

Figure 2: Comparison between Danish results and and load tests values

When the Danish formula was used an improvement in the results was noticed, but only through the evaluation metrics will a more appropriate quantitative analysis be possible. However, the discrepancy in the same pile remains. This fact can be the object of future investigation, that is, to analyze specifically what characteristics in the pile load capacity provoke the values obtained by the formula to be so discrepant of the originals. The second group of simulations consisted of the application of those formulas that do not use the modulus of elasticity of the pile: Gates, WSDOT and FHWA-Gates. A comparison between the results obtained by the FHWA-Gates formula and those tested by load proof is presented in Figure 3.

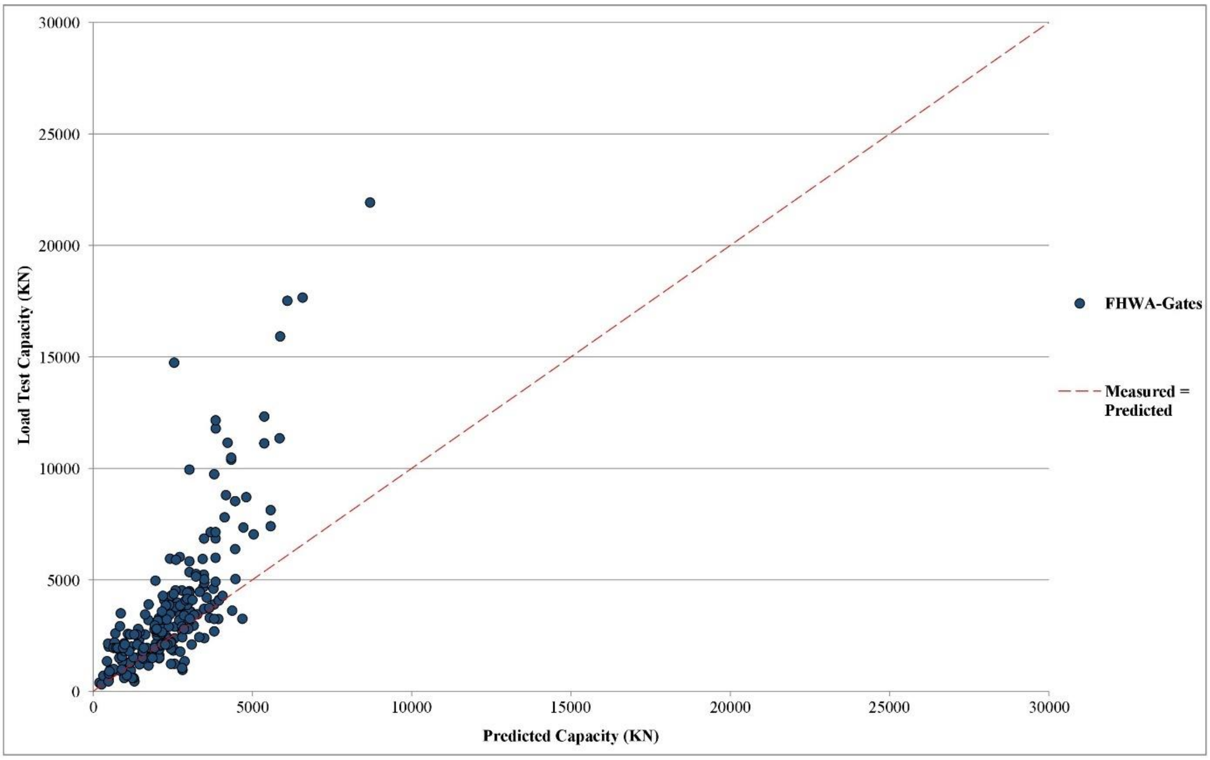

Figure 3: Comparison between FHWA-GATES results and load tests values

Differently from what was observed in the application of the Janbu and Danish formulas, the FHWA-GATES formula generally provided underestimated values. Most of the values obtained by the formula were lower than those measured by load test. In some cases, it is possible to observe in the chart, the difference between the measured value and that obtained by the formula is remarkably high. These observations show that this model is composed of important weaknesses and its use seems to require special attention. The results obtained through Gates´ proposal were also compared to load tests values (Figure 4).

The underestimation verified by the FHWA-GATES formula is the accentuated when the model used is Gates. For this case specifically it is noticeable in the chart the existence of a considerable error, probably the largest among the models. For the verification or refutation of this perception the mean quadratic errors of each model will be calculated. The approximation between the results obtained by the WSDOT model and the values verified in loco is graphically presented in Figure 5.

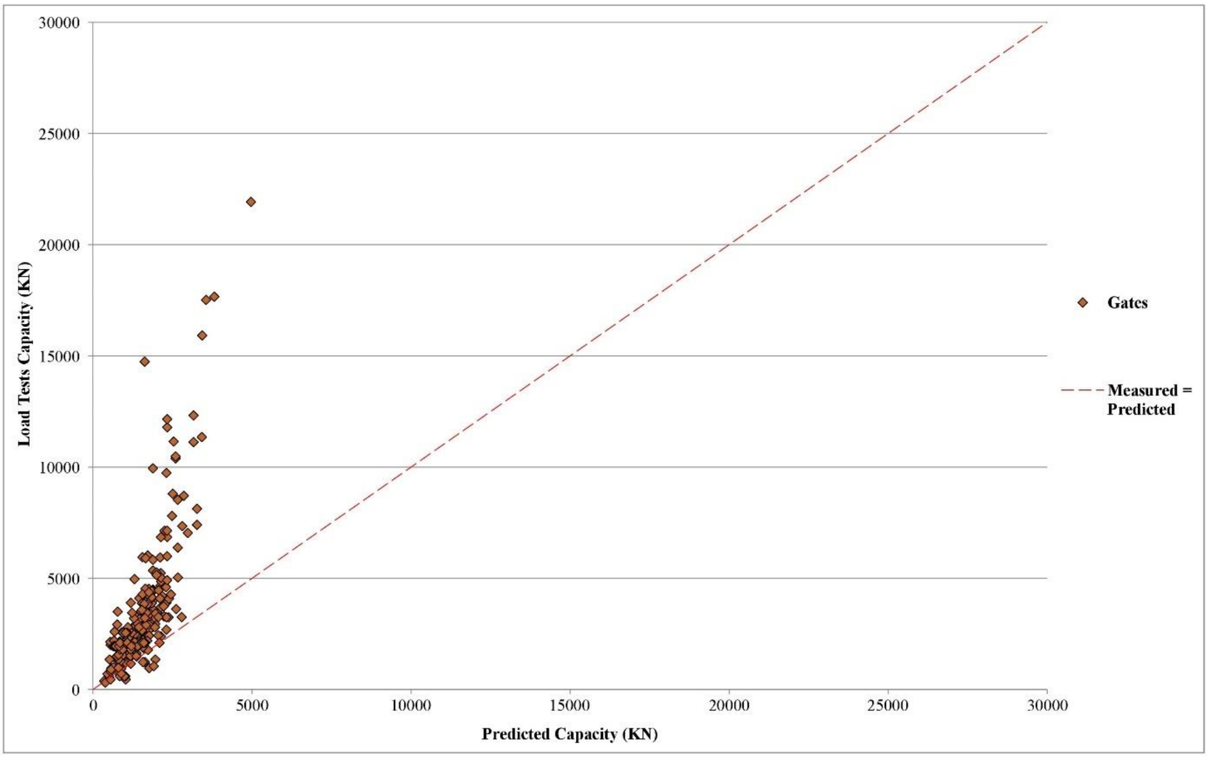

Figure 4: Comparison between Gates results and load tests values.

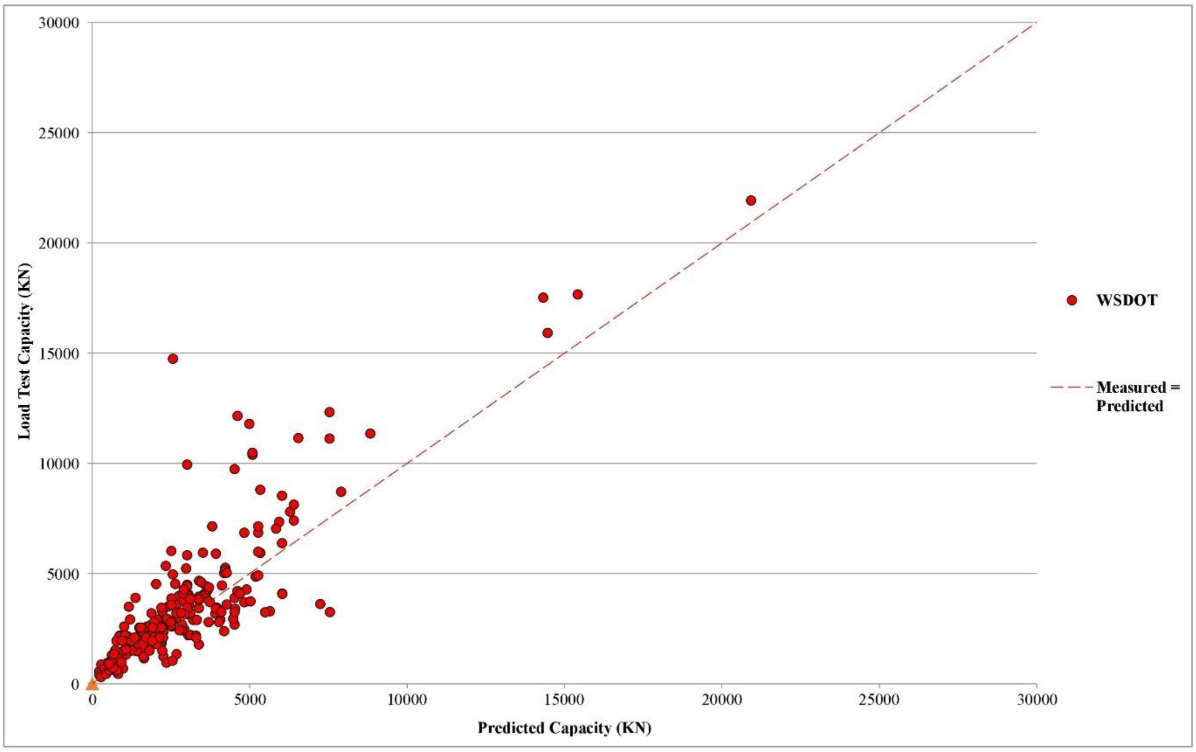

Figure 5: Comparison between WSDOT results and load tests values.

It can be noticed in Figure 5 that the WSDOT model apparently presented similar performance to that obtained with the Danish Formula and performed better than Janbu´s (Figure 1), Gates (Figure 2) and FHWA-Gates (Figure 3). However, to measure these differences an appropriate metric is required. The metric used for this is root-mean-square error. To quantitatively compare the results of the studied models, the root mean squared error between the results of each model and values measured in loco, as well as their respective correlations, were determined. These metrics are presented in table 2.

Table 2: Statistical metrics of comparison between the Equations results and Load Tests Values

	Janbu	Danish	Gates	WSDOT	FHWA
RMSE	2,879.9	2,094.6	3,328.4	1,792.6	2,540.6
Correlation	0.735	0.837	0.791	0.848	0.791

The RMSE values presented in Table 2 numerically confirm what can be observed in the graphs presented, that is, the WSDOT formula was the model that presented results closest to the actual values. On the other hand, the biggest discrepancy was presented by Gates' formula. This fact also confirms the observations made when analyzing the graphs presented. Considering the results, attempts were made to improve the methods. For this, coefficients were determined that, when multiplied by the results of the formulas, promoted a reduction in error, maintaining the correlation. Table 3 presents the new results obtained, also indicating the value of the coefficient applied to each method.

Table 3: Statistical metrics of comparison between the Improved Equations results and Load Tests Values

	Janbu	Danish	Gates	WSDOT	FHWA
Coefficient	1.4111	1.1197	2.4797	1.1591	1.5661
RMSE	2,463.9	2,016.2	2,088.8	1,682.1	1,983.6
Correlation	0.735	0.837	0.791	0.848	0.791

It should be noted that the WSDOT method continues to produce the lowest errors, combined with a better correlation. It is also noteworthy that such a method, even in its original version, still produces better results than all the other, even in its improved versions. Because it was observed that a pile specifically presented large discrepancies between predicted values and load test value by any of the methods, it was considered that such a point may represent an outlier, or even an incorrect input data. Thus, the determinations of RMSE and correlation were redone, excluding this value from the input data. Results obtained are presented in table 4.

Table 4: Statistical metrics of comparison between the Equations results and Load Tests Values excluding outlier pile

	Janbu	Danish	Gates	WSDOT	FHWA
RMSE	2,689.1	1,842.3	3,222.1	1,607.9	2,416.4
Correlation	0.768	0.871	0.813	0.875	0.812

With the elimination of the outlier pile the results improve considerably, to the point of providing Danish and

WSDOT predictions with better errors and correlations than those found by the improved methods (table 3). Finally, the results obtained with the exclusion of the discrepant pile were also improved by determining coefficients that, when incorporated into the formula, provided reduction of errors (table 5)

Table 5: Statistical metrics of comparison between the Improved Equations results and Load Tests Values excluding outlier pile

	Janbu	Danish	Gates	WSDOT	FHWA
Coefficient	1.4007	1.1117	2.4552	1.1505	1.551
RMSE	2,260.3	1,764.2	1,970.6	1,496.9	1,857,3
Correlation	0.768	0.871	0.813	0.875	0.812

Once again, the adjustment coefficients proved to be useful, providing significant improvements in the results obtained through the reduction of RMSE. Proportionally, the most important ones were in the Gates and FHWA-Gates methods, but in absolute terms, the best results continue to be obtained by WSDOT.

5. Conclusions

After performing the calculations using the analyzed formulae, we realized that, in terms of correlation, all formulas appeared to be potentially viable and with the condition of possible generalization of their use. However, when measuring the errors obtained it is verified that when applying these formulas, one must act cautiously, analyzing conditions imposed by their theoretical elaborations, and thus their generalization is not feasible. The WSDOT Model was the one that presented the highest correlation with the measured values, and this may mean a good adherence of the model to the real situation, which prompts further investigations regarding the generalizability of this mathematical formula. The WSDOT equation was also the one with the lowest root mean squared error among the studied formulas. However, despite being the smallest error is still an expressive value, which means that it cannot be interpreted strictly as a positive aspect of that model. In any case, the combination of the smallest error with the highest correlation guarantees WSTOD the best fit among the studied models. At the other extreme of the results, namely, those with the least satisfactory results are the Janbu´s and Gates formulas. The Janbu´s model presented the lowest correlation (0.73) between the real (load test) values and those obtained by the model. In terms of root mean squared error, Gates presented the highest result. This error value is almost two times the lowest error value obtained (through WSDOT formula) and this demonstrates the discrepancy between model´s results. It is noteworthy, however, that this fact is not sufficient to conclude that these models cannot be used but shows the need to be cautious when adopting them in practice. It is important to note that, although the Danish formula does not stand out as the best result this model presented results remarkably close to the WSDOT formula, that is, the Danish model can be evaluated with the same advantages and disadvantages as WSDOT. It was also found that methods are possible to be obtained in a relatively simple way, only by multiplying each original equation by appropriate coefficients. In this case, improvements are perceived in all methods, and especially in the proposals of Gates and FHWA-Gates, despite WSDOT kept presenting the best results among all methods. Given all that has been exposed and analyzed, it is possible to confirm that the models studied here present significant errors that need to be corrected. On the other hand, it is noticeable the need for models that have the possibility of better generalization than those already existing in the literature. These findings indicate that the problem of estimating ultimate load capacity in a simplified manner based in known pile´s and driving parameters continues to be a major challenge in geotechnics. One limitation faced in this is study is that not all the database presented the modulus of elasticity as a known parameter. Thus, the number of simulations could not be the same for all formulas. This fact represents two problems: first, for analyzes of this type, it is desirable that the number of samples be as large as possible and, second, the difference in the number of samples makes the comparison between formulas a slightly more delicate problem to be analyzed. Suggestions for future studies may include comparisons between the accuracy of such formulas and those of other methods of predicting ultimate bearing capacity of isolated piles, especially static formulas (theoretical or empirical) or the development of improved equations, for example through computational methods such as artificial neural networks.

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