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Mathematical Digital Terrain Model (DTM) Data for Testing DTM Generation Methodologies and Software

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Abstract

Developing a method for interpolation of data points for forming DTM involves thousands of complicated computations. Checking of such computations requires errorless and well-defined input data and end results from a large number of DTM points. Measured, or actual, DTM data points may not be available or invariably have errors which affect the course of the computations and analysis. Most of these problems can be overcome by mathematically generating DTM data points. This paper focuses on developing software, called *MathDTM*, for generating mathematical (simulated) DTM data at desired specifications. An overview of the development and capabilities of *MathDTM* software platform based on Windows system is presented. The developed software was used for testing the performance and accuracy of different interpolation methods of *Surfer* software. The results of these tests were then discussed and evaluated.

Keywords: Digital Terrain Model; Simulated DTM Data; *MathDTM* Software; Accuracy Analysis.

1. Introduction

A DTM is a means or representing the shape of natural surfaces in digital form suitable for storage in a computer [2, 5, 9, 10]. To form a DTM, a detail survey is carried out in the area for which the DTM is required. Since the shape of natural surfaces varies in a random way, the network of points surveyed to represent the shape of the ground will usually form a random pattern consisting of horizontal coordinates with associated heights.In many cases, photogrammetric methods involving aerial surveys are used to provide surface information for DTMs.

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These methods are well suited to obtain three-dimensional information over large areas where ground techniques would become laborious.

A DTM is usually formed from the field data using one of the following techniques:

- A square grid DTM is one in which data points are obtained at the nodes of a square grid. This model is formed by the computer interpolating the height of each grid node from the field data provided.
- A triangular grid DTM is one in which data points are interpolated at the corners of linked triangles which are positioned to give the best representation of the ground surface.

The quality of the DTM can vary greatly depending on the data source and the interpolation technique. Representation of the DTM as a grid is quite common, as this format lends itself well to computer computations. This research is concerned solely with gridded DTMs. Here, the term DTM will refer to elevation represented by a regular gridded matrix. Deriving a method of interpolation of data points for forming DTM involves thousands of computer instructions. Debugging and checking of such instructions requires errorless and well-defined input data and end results from a large number of DTM points. Actual DTM data points invariably have errors which affect the course of the computations and analysis and thereby make it difficult to evaluate which effects are due to the method used for the determination of X, Y, Z ground coordinates of DTM data points, which due to the geometry of the problem, and which due to shortcomings in developed software. The problem of non-availability of error free actual DTM data can, however, be easily solved by mathematically generating DTM data.

The aims of this paper are:

- Development of a software for generating the mathematical DTM data named as *MathDTM* (an acronym of Mathematical DTM) software, and
- Using the developed software for generating the necessary data for testing the *Surfer* Software.

2. Development of *MathDTM* Software

2.1 Specifications for Generating the DTM Data Points

The used specifications for generating the DTM mathematical data are:

- Area size: The software has no restriction of any kind about the generated ground area size. The software is able to generate the DTM data points for the ground area of any size.
- Height Variation: Height variation of ground points is taken as a percentage of the larger ground area dimension. This factor may define the terrain type as depicted in Table 1.
- Origin of X & Y ground coordinates: X and Y ground coordinates of DTM data points are referred to the lower left corner of the ground area.
- Grid size: This factor defines the distances between DTM data points in X- and Y- directions.
- DTM data points distributions: The generated points can be distributed as:

- Regular Distribution: In this case the points are spaced at equal distances in X- and Y- directions forming a regularly spaced grids of data.
- Irregular Distribution: In this case the points are irregularly spaced. To attain this case, the software generates regularly spaced grids of data points and introduces discrepancies to the X and Y ground coordinates of the grid nodes. Special random discrepancies generator [3] is used for this purpose. The values of discrepancies change between –0.4d to +0.4d where d is the grid size in X- and Y- directions.

Table 1: Mathematical Terrain Types

No.	**Terrain Type**	**Height Variation as Percentage of Largest Area Side**
1	Nearly Flat	2%
2	Modestly Undulating	5%
3	Hilly	10%
4	Mountainous	25%

- Computation of the ground heights of DTM data points: The heights of DTM data points are computed, by default, using cubic polynomial distribution as follows [7]:

$$Z(X,Y) = A_{00} + A_{01}Y + A_{02}Y^2 + A_{03}Y^3 + A_{10}X + A_{11}XY + A_{12}XY^2$$

$$+ A_{20}X^2 + A_{21}X^2Y + A_{30}X^3 \qquad (1)$$

Where:

Z	is the height of point
X and Y	are the ground coordinates of point
$A_{00},,,,A_{30}$	are coefficients

Furthermore, the user of the software can specify, if necessary, the polynomial order.

Figures 1 and 2 illustrate contour and DTM maps for mountainous terrain type.

2.2 MathDTM Software Output

The software generates three ASCII files as follows:

- The first file has X, Y, Z ground coordinates of all DTM data points
- The second file contains X, Y, Z ground coordinates of 50% of DTM data points. These points can be used for the generation of the DTM
- The third file includes X, Y, Z ground coordinates of 50% of DTM data points. These points may be used as check points after generating the DTM.

2.3 MathDTM Software Portability and Hardwares

The software is menu based software. It has been implemented using Visual C++ Compiler V6.0 [4,6,8] and designed to be flexible and portable to 32-bit Windows platforms (Windows 95, 98, XP, and Windows 8, 10).

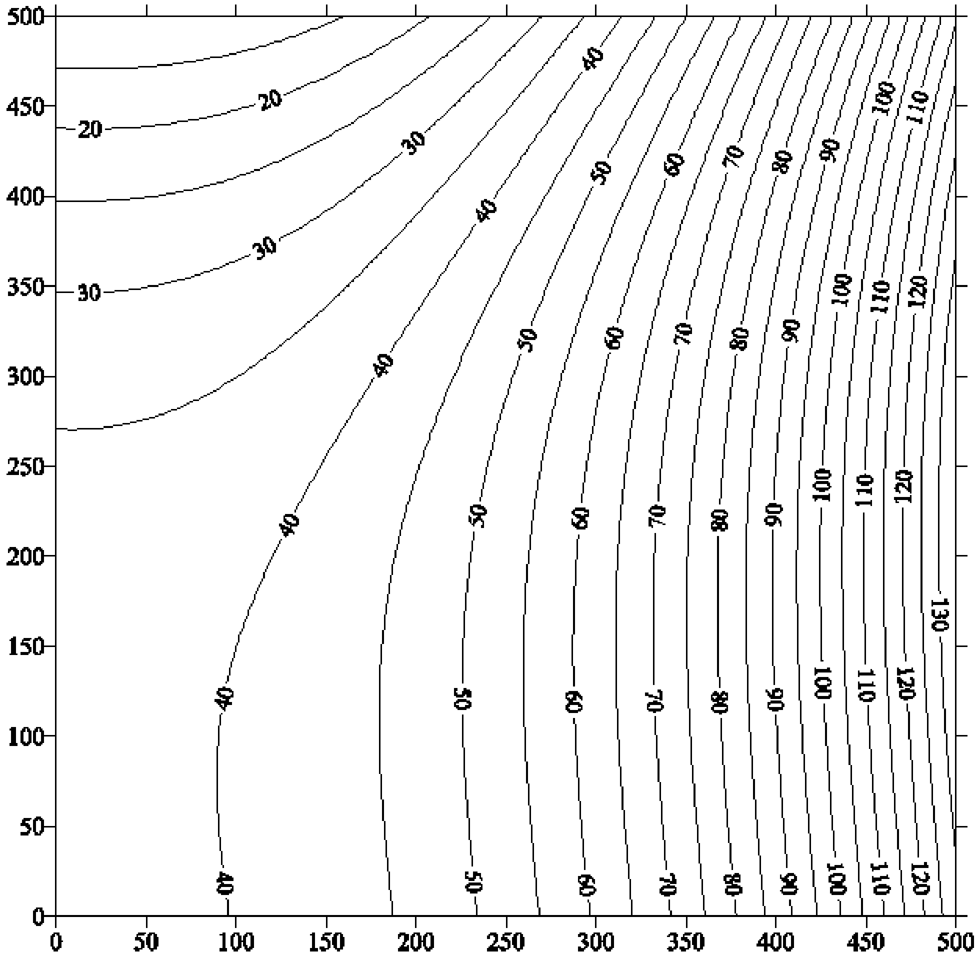

Figure 1: Contour Map for Mathematical Mountainous Terrain Type

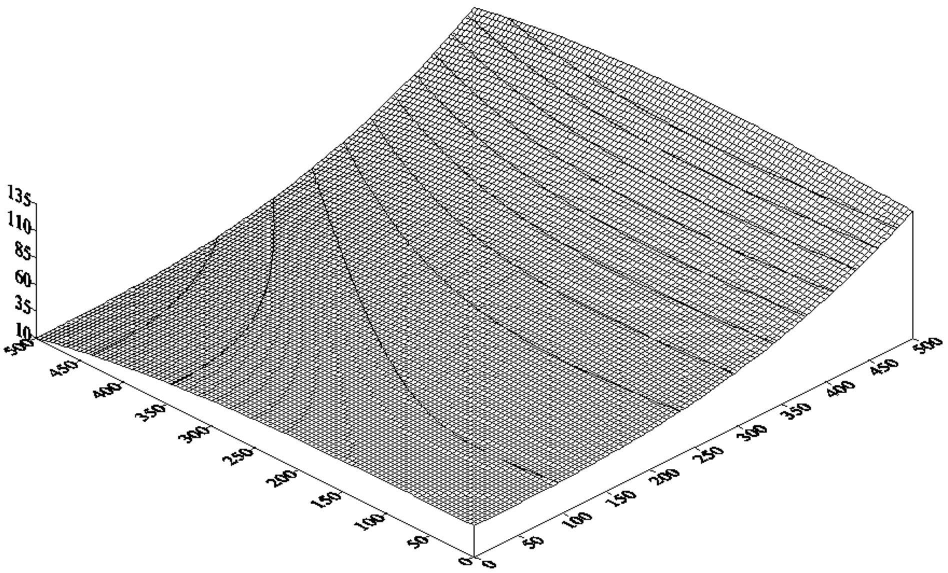

Figure 2: DTM Map for Mathematical Mountainous Terrain Type

The software requires a minimum of 512 megabytes of memory (i.e. RAM) and approximately 100 megabytes of disk space on the hard disk. The software is released in a compressed format on CD-ROM with a reference manual.

3. Surfer Software Overview

Table 2: Grid Interpolation Methods of *Surfer*

Number	**Grid Interpolation Method**
1	Inverse Distance to a power
2	Kriging
3	Minimum Curvature
4	Modified Shepard's Method
5	Natural Neighbor
6	Nearest Neighbor
7	Polynomial Regression
8	Radial Basis Function
9	Triangulation with Linear Interpolation

Surfer is one of the well-known software packages used for contour and DTM generations. It is a product of Golden, Inc., Colorado, U.S.A. The software reads the input data in the form of X, Y, Z ground coordinates of DTM data points and creates a regularly spaced grid of data from irregularly spaced data. Grid density may be specified by either the number of grid lines along each side of the grid (X or Y) or the distance between grid lines in X or Y data units along each side of the grid. Furthermore, the software allows the user to specify the interpolation method (griding method) to be used. There are nine different interpolation methods available as shown in Table 2. The full description of these interpolation methods is explained in [7].

4. Testing *Surfer* Software

The testing phase of *Surfer* software consisted of:

- Testing the system error, and
- Testing the accuracy of each method of grid interpolation.

As has already been mentioned, mathematical DTM data can be advantageously used for testing of grid interpolation methods and systems since in this case error free input data and end results are both known. Testing of the *Surfer* software, therefore, was carried out by using the mathematically generated DTM data points for which, as already explained, *MathDTM* software was developed.

4.1 Testing the System Error of Surfer

System error consists of two parts. The first part of this error is due to rounding off of values during intermediate computations. This part of error may be minimised by using double precision computations as far as possible. The second part of the system error occurs due to truncation of higher order terms while forming the linearised equations from the non linear equations.

The specifications used for generating the DTM data points were:

- Area size : 100 * 100 m.
- Height Variation : 25% i. e. 25 m.
- Grid size : 1m
- DTM data points distributions : Regular Distribution

For this test, the total number of points is 10201 points. For each grid interpolation method, the test was performed as following:

- Generating the DTM using all points (10201 points).
- Using the X and Y coordinates of all points (as check points) to predicate their levels
- Computing the Maximum Absolute Error (MAE) using:

$$AbsoluteError = \left| known\ level - predicated\ level \right| \qquad (2)$$

- Computing the Root Mean Square Error (RMSE) of the levels of check points using:

$$RMSE = \sqrt{\sum_{i=1}^{n} (known\ \text{level} - predicated\ \text{level})_i^2 / n} \tag{3}$$

Where n is the number of check points.

The results of each grid interpolation method showed that zero values for errors were obtained and it can be concluded that *Surfer* software is free from system error.

4.2 Testing the Accuracy of Each Method of Grid Interpolation

Studying the accuracy of each method of grid interpolation (Table 2) was done by generating mathematical DTM data for ground area of 500m * 500m size. The specifications for generating the mathematical DTMs took into consideration the effects of three factors. These factors are:

- Terrain type: Terrain type is defined as a percentage of the height variation. Four terrain type were defined as shown in Table 1.
- Grid size: It consisted of generating DTMs of 20m, 10m and 5m grid size.
- Point distribution: It contained both methods of point distribution (regular and irregular).

The number of generated DTM points varies according to the grid size. 50% of the generated points were used to generate the DTM using the specified grid interpolation method while the other 50% (in most tests) of the generated points were used as check points. In all tests, the values of MAE and RMSE of the levels of check points were computed and tabulated in Tables 3, 4, 5 and 6 for nearly flat, undulating, hilly and mountainous terrain types respectively.

From these tables, it can be seen that:

- Polynomial Regression method gave approximately zero values for MAE and RMSE because both this method [7] and *MathDTM* software utilize the same mathematical model (Equation 1) for computing the DTM data. For this reason, the results of this method will not be discussed through this comparative study.
- Terrain type has significant effect on the accuracy of the DTM. For example, using Modified Shepard’s Method produces 0.01٠m and 0.00١m maximum absolute error and RMSE values respectively for generating DTM of 20m grid size for nearly flat terrain while it produces 0.1٢١m and 0.012m maximum absolute error and RMSE respectively for generating DTM of 20m grid size for mountainous terrain. Generally, the accuracy of DTM decreases with increasing the height variation.
- In terms of absolute error and RMSE, there is a slight improvement in the RMSE values using irregular points distribution. Furthermore, using regular distribution of points may decrease the values of maximum absolute error.

- Increasing the grid size has significant effect on decreasing the accuracy of the generated DTM. For example, in Table 6, using Kriging method, the values of RMSE are 0.625m, 0.084m and 0.034m for DTM of 20m, 10m and 5m grid size respectively with irregular distribution of points while the max absolute error values are 2.628m, 0.704m and 0.527m for the same generated DTMs.
- The best three methods for grid interpolation which give the smallest values of RMSE and hence the highest accuracies, for all terrain types using the data generated by *MathDTM* software are:

– Modified Shepard's method

– Natural Neighbor method

– Triangulation with Linear Interpolation method

Table 3: Maximum Absolute Error and RMSE of Check Points Levels for Nearly Flat Terrain

Interpolation Method	**Regular Distribution of Points**						**Irregular Distribution of Points**					
	Grid Size 20m Data Points 338 Check Points 338		Grid Size 10m Data Points 1301 Check Points 1300		Grid Size 5m Data Points 5101 Check Points 5100		Grid Size 20m Data Points 5101 Check Points 5100		Grid Size 10m Data Points 1301 Check Points 1300		Grid Size 5m Data Points 5101 Check Points 5100	
	MAE (m)	*RMSE* (m)	*MAE* (m)	*RMSE* (m)	*MAE* (m)	*RMSE* (m)	*MAE* (m)	*RMSE* (m)	*MAE* (m)	*RMSE* (m)	*MAE* (m)	*RMSE* (m)
Inverse Distance to a power	0.540	0.066	0.150	0.021	0.074	0.008	0.591	0.090	0.300	0.052	0.170	0.031
Kriging	0.096	0.037	0.041	0.006	0.019	0.002	0.210	0.047	0.056	0.007	0.042	0.003
Minimum Curvature	0.248	0.055	0.049	0.007	0.020	0.002	0.358	0.076	0.052	0.010	0.062	0.009
Modified Shepard's Method	0.010	0.001	0.000	0.000	0.000	0.000	0.012	0.001	0.000	0.000	0.000	0.000
Natural Neighbor*	0.010	0.004	0.004	0.002	0.002	0.000	0.011	0.004	0.005	0.002	0.003	0.001
Nearest Neighbor	0.228	0.099	0.400	0.097	0.204	0.046	0.509	0.102	0.444	0.097	0.221	0.083
Polynomial Regression	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Radial Basis Function	0.196	0.048	0.048	0.007	0.023	0.002	0.210	0.051	0.064	0.008	0.048	0.003
Triangulation with Linear Interpolation	0.011	0.004	0.008	0.003	0.002	0.001	0.011	0.006	0.008	0.004	0.002	0.002

For example, the above mentioned methods give RMSE of 0.028m, 0.048m and 0.073m respectively for generating DTM of 20m grid size for mountainous terrain type with irregular distribution of points while the smallest RMSE value obtained by the other methods is 0.625m using Kriging method for generating the DTM with the same specifications.

Table 4: Maximum Absolute Error and RMSE of Check Points Levels for Modestly Undulating Terrain

Interpolation Method	**Regular Distribution of Points**						**Irregular Distribution of Points**					
	Grid Size 20m Data Points 338 Check Points 338		Grid Size 10m Data Points 1301 Check Points 1300		Grid Size 5m Data Points 5101 Check Points 5100		Grid Size 20m Data Points 5101 Check Points 5100		Grid Size 10m Data Points 1301 Check Points 1300		Grid Size 5m Data Points 5101 Check Points 5100	
	MAE (m)	*RMSE* (m)	*MAE* (m)	*RMSE* (m)	*MAE* (m)	*RMSE* (m)	*MAE* (m)	*RMSE* (m)	*MAE* (m)	*RMSE* (m)	*MAE* (m)	*RMSE* (m)
Inverse Distance to a power	1.350	0.164	0.375	0.052	0.186	0.019	1.479	0.224	0.748	0.129	0.424	0.077
Kriging	0.490	0.117	0.102	0.016	0.048	0.005	0.526	0.128	0.141	0.017	0.105	0.007
Minimum Curvature	0.370	0.035	0.122	0.018	0.050	0.006	0.894	0.190	0.131	0.026	0.154	0.023
Modified Shepard's Method	0.029	0.002	0.000	0.000	0.000	0.000	0.024	0.006	0.001	0.000	0.001	0.000
Natural Neighbor*	0.026	0.009	0.010	0.004	0.006	0.001	0.027	0.010	0.012	0.005	0.008	0.002
Nearest Neighbor	1.110	0.226	0.569	0.202	0.453	0.108	1.274	0.256	0.993	0.244	0.509	0.116
Polynomial Regression	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.001	0.000	0.001	0.000
Radial Basis Function	0.491	0.119	0.120	0.018	0.057	0.006	0.526	0.128	0.159	0.020	0.119	0.008
Triangulation with Linear Interpolation	0.026	0.010	0.019	0.007	0.005	0.002	0.026	0.015	0.020	0.009	0.010	0.004

Table 5: Maximum Absolute Error and RMSE of Check Points Levels for Hilly Terrain

Interpolation Method	**Regular Distribution of Points**						**Irregular Distribution of Points**					
	Data Points 338	Check Points 338	Data Points 1301	Check Points 1200	Data Points 5101	Check Points 5100	Data Points 5101	Check Points 5100	Data Points 1301	Check Points 1300	Data Points 5101	Check Points 5100
	MAE (m)	*RMSE* (m)	*MAE* (m)	*RMSE* (m)	*MAE* (m)	*RMSE* (m)	*MAE* (m)	*RMSE* (m)	*MAE* (m)	*RMSE* (m)	*MAE* (m)	*RMSE* (m)
Inverse Distance to a power	2.700	0.328	0.751	0.105	0.372	0.038	2.957	0.448	1.496	0.258	0.848	0.154
Kriging	0.980	0.250	0.203	0.031	0.096	0.011	1.051	0.264	0.281	0.034	0.205	0.013
Minimum Curvature	1.789	0.379	0.243	0.036	0.099	0.011	2.741	0.475	0.262	0.052	0.308	0.045
Modified Shepard's Method	0.048	0.005	0.001	0.000	0.000	0.000	0.059	0.011	0.002	0.001	0.002	0.001
Natural Neighbor*	0.050	0.018	0.022	0.008	0.010	0.002	0.053	0.019	0.025	0.009	0.015	0.004
Nearest Neighbor	2.139	0.495	1.987	0.487	1.018	0.232	2.547	0.511	2.200	0.852	1.107	0.416
Polynomial Regression	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.001	0.002	0.001	0.002	0.001
Radial Basis Function	0.983	0.239	0.240	0.036	0.114	0.012	1.052	0.253	0.318	0.039	0.238	0.016
Triangulation with Linear Interpolation	0.051	0.020	0.039	0.013	0.010	0.004	0.052	0.020	0.039	0.018	0.021	0.009

Table 6: Maximum Absolute Error and RMSE of Check Points Levels for Mountainous Terrain

Interpolation Method	**Regular Distribution of Points**						**Irregular Distribution of Points**					
	Data Points 338 Check Points 338		Data Points 1301 Check Points 1200		Data Points 5101 Check Points 5100		Data Points 5101 Check Points 5100		Data Points 1301 Check Points 1300		Data Points 5101 Check Points 5100	
	MAE (m)	*RMSE* (m)	*MAE* (m)	*RMSE* (m)	*MAE* (m)	*RMSE* (m)	*MAE* (m)	*RMSE* (m)	*MAE* (m)	*RMSE* (m)	*MAE* (m)	*RMSE* (m)
Inverse Distance to a power	6.750	0.821	1.877	0.262	0.930	0.094	7.393	1.119	3.741	0.644	2.120	0.384
Kriging	2.451	0.584	0.508	0.078	0.240	0.027	2.628	0.625	0.704	0.084	0.527	0.034
Minimum Curvature	4.471	0.949	0.608	0.089	0.249	0.028	6.852	1.189	0.655	0.129	0.769	0.113
Modified Shepard's Method	0.121	0.012	0.001	0.000	0.000	0.000	0.147	0.028	0.006	0.002	0.006	0.002
Natural Neighbor[*]	0.114	0.046	0.049	0.021	0.027	0.006	0.133	0.048	0.063	0.022	0.035	0.009
Nearest Neighbor	2.847	1.239	4.966	1.218	2.544	0.580	6.368	1.279	5.549	2.129	2.766	1.040
Polynomial Regression	0.000	0.000	0.000	0.000	0.000	0.000	0.006	0.002	0.006	0.002	0.006	0.002
Radial Basis Function	2.457	0.597	0.600	0.091	0.285	0.031	2.630	0.633	0.794	0.099	0.600	0.040
Triangulation with Linear Interpolation	0.130	0.051	0.097	0.034	0.025	0.011	0.131	0.073	0.098	0.046	0.093	0.035

Moreover, the obtained RMSE values (Tables 3, 4, 5 and 6) were compared with the permissible limits according to the specifications of ASPRS (American Society for Photogrammetry and Remote Sensing) [1] as tabulated in Table 7.

Table 7: ASPRS (American Society for Photogrammetry and Remote Sensing) Topographic Elevation Accuracy Requirement for Well-Defined Points

Contour Interval in Meters	***ASPRS* Limiting *RMSE* in Meters Spot or Digital Terrain Model Elevation Points**		
	Class I*	**Class II**	**Class III**
0.5	0.08	0.16	0.25
1.0	0.17	0.33	0.5
2.0	0.33	0.67	1.0
4.0	0.67	1.33	2.0
5.0	0.83	1.67	2.5

* The maps are divided into three classes:

Class I holds the highest accuracies. Site plans for construction fit this category.

Class II has half the overall accuracy of Class I. Typical projects may include excavation, road grading, or disposal operations.

Class III has one third the accuracy or three times the allowable error of Class I maps. Large area cadastral, city planning, or land classification maps are typically in this category.

From tables 3, 4, 5 and 6, and taking into account only the specifications for the highest accuracy (Class I of Table 7) it can be seen that:

- All grid interpolation methods are suitable for generating contour maps of 4m or 5m contour interval for any terrain type with regular or irregular distribution of points using grid size of 20m or less.
- For generating contour map of 2m contour interval:
  - All methods are suitable for Nearly Flat and Undulating terrain types using grid size of 20m or less.
  - Minimum Curvature and Nearest Neighbor methods are suitable for Hilly terrain type when using grid size of 10m or less which is not economic compared with the other methods which are suitable using 20m or less grid size.
  - Modified Shepard's, Natural Neighbor and Triangulation with Linear Interpolation methods are suitable for Mountainous Terrain type using 20m or less grid size. The other methods are suitable when using grid size of 10m or less (such as Kriging method) or 5m or less (such as Inverse Distance to a power method) which is not economical compared with the other methods.
- For generating contour map of 1m contour interval:
  - All methods are suitable for Nearly Flat using grid size of 20m or less.
  - Modified Shepard's, Natural Neighbor and Triangulation with Linear Interpolation methods are

suitable for any terrain type using 20m or less grid size. The other methods are applicable when decreasing the grid size which is not economically accepted.

- For generating contour map of 0.5m contour interval:

– Nearest Neighbor method is suitable for Nearly Flat terrain type when using 10m or less grid size and the other methods are applicable using 20m or less grid size.

– Modified Shepard's, Natural Neighbor and Triangulation with Linear Interpolation methods are suitable for any terrain type without any restriction for the grid size. The other methods are applicable when using grid size less than 20m which is not cost effective.

From above, it can be concluded that, using the data generated by *MathDTM* software, the applicable grid interpolation methods for generating contour maps of 0.5m or larger contour interval using 20m or less grid size for any distribution of points and terrain type pertaining ASPRS specifications are:

- Modified Shepard's method
- Natural Neighbor method
- Triangulation with Linear Interpolation method

5. Conclusions

- The developed software, *MathDTM*, provides a real time solution for testing the commercial DTM generation software and eliminates the need of real data which are costly, unavailable in all times, and subjected to unknown sources of errors.
- *MathDTM* is quite versatile and provide affordable tool to the students and researchers in the universities and academic centres for:

    - Testing the accuracy of the available DTM software before using it
    - Providing the researchers with the necessary DTM data for testing their programs for DTM generation during/after their development stages.

- Out of the different methods of grid interpolation of *Surfer* software, the following methods are recommended:

    - Modified Shepard's method
    - Natural Neighbor method
    - Triangulation with Linear Interpolation method

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