American Scientific Research Journal for Engineering, Technology, and Sciences (ASRJETS)
ISSN (Print) 2313-4410, ISSN (Online) 2313-4402

http://asrjetsjournal.org/

Multicriteria Decision Making Problems Using Variable Weights of Criteria Based on Alternative Preferences

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Abstract

In this study, we consider multicriteria decision making problems with respect to variable domination structures. These structures are considered in relation to variable weights of criteria based on alternatives preferences. Such problems have many applications in logistics, strategic management, economics, and many decision making processes. For solving such problems, we define a technique based on simple additive weighting, technique for order of preference by similarity to ideal solution, and preference ranking organization method for enrichment evaluation (PROMETHEE) methods. Then, we utilize the PROMETHEE method for facility location selection.

Keywords: Multicriteria decision making problems; variable ordering structure; variable weights of criteria; SAW; TOPSIS; PROMETHEE.

1. Introduction

Decision making is a significant tool in business and life. Right decisions facilitate the success of activities in business and life. By applying a proper decision theory, productivity and efficiency can be increased at the individual level and in organizations, institutes, and companies. Many approaches to solving multicriteria decision making (MCDM) problems use weights to represent the relative importance of criteria, see [1, 2, 3, 4]. Selecting the appropriate weights of criteria is a significant part of the decision making process because the varied weights of criteria represent the different alternatives rankings. In many MCDM problems, the weights of criteria are used to represent the importance of each criterion and compare the alternatives with respect to them [5]. In the decision making process, knowing the preferences of the decision maker (DM) and determining the weights of criteria are very difficult. Several methods can be used to assign appropriate value to the weights of criteria; for more detail, we refer the reader to [6, 7, 8, 9, 10].

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For MCDM problems, an expert (DM) or experts (in group decision making) must evaluate every alternative to each attribute, determine the attribute weights, and select the most favorable one among the alternatives. The weights can be determined using either the objective or subjective weighting methods. The objective method selects the weights of criteria through mathematical calculation, whereas the subjective method is besed on the judgments of the DMs [9, 11]. Some methods for determining and selecting the weights of criteria are as follows: 1. Entropy 2. Linear programming for multidimensional analysis of preference (Linmap) 3. Least squares 4. Eigenvector. The weights of criteria were used to solve different MCDM problems, and all the alternatives were compared with respect to considered weights. Because, in many real problems, different preferences or characterizations correspond to each alternative, it is ideal to consider the weight of criterion based on each alternative. This model is very useful in location problems, logistics, strategic management, economics, organization decisions, and many decision making processes. In general, we can divide the different weights of criteria into two parts. The first one refers to the MCDM problems wherein the weights of criteria vary based on time or condition (but they are not related to the alternatives), as performed by [12]. Furthermore, some of dynamic decision-making problems are in this type [13, 14]. The second type refers to the MCDM problems with variable weights of criteria based on the preferences for alternatives. For instance, if the aim is selecting the ideal location among different candidate locations, according to some criteria, such as quality, cost, time, customer satisfaction, proximity to market, proximity to the supplier and other objectives, we may determine the variable weights of criteria for each location. In some locations (based on the preference of the DM), minimizing the time is preferable than minimizing the cost; in other locations, the reverse is the case. Therefore, each criterion has different weights in various locations. Considering these variations can facilitate the best decision. In this study, the second type of variable weights is considered, and the weight vector of criteria corresponding to each alternative is used. The consideration of variable preferences of alternatives is based on the vector optimization, with respect to the variable domination structure, for more detail see [15]. This consideration can be incorporated into several methods in this area, such as simple additive weighting method (SAW), technique for order of preference by similarity to ideal solution (TOPSIS), elimination et choice translating reality (ELECTRE), and preference ranking organization method for enrichment evaluation (PROMETHEE), to solve MCDM problems [16, 17, 18, 19]. In this study, we present a method that can be incorporated by the DM into the simple additive weighting (SAW), TOPSIS, and PROMETHEE methods. The consideration of variable preferences of alternatives is based on the vector optimization, with respect to the variable domination structure. Therefore, Section 2 presents a preliminary study of vector optimization problems, with respect to the variable ordering structure and the theorem that gives the method to consider variable weights in the case of variable domination structures. The mathematical method for inserting the weights in the decision matrix is presented in section 3. In Section 4, we examine the use of this technique in the SAW, TOPSIS, and PROMETHEE methods, following which we present the result of applying the technique in the PROMETHEE method.

2. Multicriteria decision-making (MCDM) with respect to the variable ordering structure

Depending on the type of problem, we can select the method of weight consideration. In the case where the preferences of the criteria are variable with respect to the alternatives, the following theorem can be useful. Because this study is based on the assumption of a variable weight vector, the following theorem shows how the weights can be considered as variable preferences. In the remainder of this section, we present the concept of minimal and non-dominated elements in vector optimization problems with respect to the variable domination structure for use in the following theorem.

2.1. Vector optimization with respect to variable domination structure

Consider the objective function $\mathrm{f}: \mathbb{R}^{\mathrm{n}} \longrightarrow \mathbb{R}^{\mathrm{p}}$, $\mathrm{i} = \overline{1, \mathrm{p}}$,

$$f(x) := \begin{pmatrix} f_1(x) \\ \cdots \\ f_p(x) \end{pmatrix}, \qquad x \in X,$$

where, $X \subseteq \mathbb{R}^{\mathrm{n}}$. Let $C: \mathbb{R}^{\mathrm{n}} \rightrightarrows \mathbb{R}^{\mathrm{p}}$ be the ordering map. The corresponding vector optimization problem with the variable domination structure is given by

$$Min\big(f(X), C(\cdot)\big),$$

in which $f(X) := \bigcup_{x \in X} f(x)$.To study this problem, we use the following concepts [20]:

- Suppose $x, x^0 \in X$. $f(x^0)$ is a nondominated element of $f(X)$ with respect to the ordering map $C: \mathbb{R}^{\mathrm{n}} \rightrightarrows \mathbb{R}^{\mathrm{p}}$ if

  $$\forall f(x) \in f(X): \ \ (f(x^0) - C(x) \setminus \{0\}) \cap \{f(x)\} = \emptyset.$$

  The set of all the nondominated elements is denoted by $N\big(f(X), C(\cdot)\big)$.

- Suppose $x^0 \in X$. $f(x^0)$ is a minimal element of $f(X)$ with respect to the ordering map $C: \mathbb{R}^{\mathrm{n}} \rightrightarrows \mathbb{R}^{\mathrm{p}}$ if

  $$(f(x^0) - C(x^0) \setminus \{0\}) \cap f(X) = \emptyset.$$

  The set of all the minimal elements is denoted by $M\big(f(X), C(\cdot)\big)$.

In the following, we recall Theorem 2.1 [15], to show how variable weights can considered in MCDM to achieve a minimal and non-dominated solutions.

Assumption 1. Assuming that $\mathrm{f}: \mathbb{R}^{\mathrm{n}} \longrightarrow \mathbb{R}^{\mathrm{p}}$ is a vector function, m is a positive integer number, $X \subseteq \mathbb{R}^{\mathrm{n}}$, and $X := \bigcup_{k=1}^{m} X_k$,where $X_k \subset \mathbb{R}^{\mathrm{n}}$ for $k = \overline{1.m}$, and $X_s \cap X_r = \emptyset$whenever $s \neq r$. Let $\alpha^k := \big(\alpha_1^k, \dots, \alpha_p^k\big)^T \in \mathbb{R}^{\mathrm{p}}$ for $k = \overline{1.m}$, $C: \mathbb{R}^{\mathrm{n}} \rightrightarrows \mathbb{R}^{\mathrm{p}}$ is the definition of the pointed convex domination map, as follows:

$$C(x) := \begin{cases} \left\{(y_1, y_2, \dots, y_p)^T \in \mathbb{R}^{\mathrm{p}} \middle| \alpha_1^1 y_1 + \alpha_2^1 y_2 + \cdots + \alpha_p^1 y_p \geq 0\right\} & \text{for} \quad x \in X_1 \\ \cdots & \\ \left\{(y_1, y_2, \dots, y_p)^T \in \mathbb{R}^{\mathrm{p}} \middle| \alpha_1^m y_1 + \alpha_2^m y_2 + \cdots + \alpha_p^m y_p \geq 0\right\} & \text{for} \quad x \in X_m. \end{cases}$$

Theorem 1. Let Assumption 1. be fulfilled, $x^0 \in X$, and $k = \overline{1.m}$ satisfies the following inequality:

$$\sum_{i=1}^{p} \alpha_i^k f_i(x^0) \leq \sum_{i=1}^{p} \alpha_i^k f_i(x^k);$$

for all $x^k \in X$. Then, $f(x^0) \in N\big(f(X), C(\cdot)\big)$.

Furthermore, for $x^0 \in X$, $k = \overline{1.m}$, if the following inequality holds,

$$\sum_{i=1}^{p} \alpha_i^0 f_i(x^0) \leq \sum_{i=1}^{p} \alpha_i^0 f_i(x^k);$$

for all $x^k \in X$, $f(x^0) \in M\big(f(X), C(\cdot)\big)$.

Remark 1. In MCDM, $\alpha^k := \big(\alpha_1^k, \dots, \alpha_p^k\big)^T \in \mathbb{R}^p$ represents the weight vector corresponding to the k^{th} alternative.

The nondominated and minimal solution is used, depending on the type of problem and applications. This study is based on the concept of nondominated solution and its application in MCDM problems.

3. Mathematical model to insert weight vectors in decision matrix

Previous models consider the fixed weights of criteria for all the alternatives. In many problems, there are different weights of criteria corresponding to different alternatives that are regarded a function of weights. The general model for inserting these weights in the decision matrix is illustrated below; it can be useful for all decision making models such as: TOPSIS, ELECTRE, and PROMETHEE. Consider a set of alternatives $x^k \in X, k = \overline{1, m}$. The objective function $f_i: \mathbb{R}^n \longrightarrow \mathbb{R}^p$, $i = \overline{1, p}$, is defined by

$$f(x) := \begin{pmatrix} f_1(x) \\ \cdots \\ f_p(x) \end{pmatrix},$$

To study such problems, we introduce the following decision matrix/table

Table 1: Decision matrix

	$f_1(x)$	$f_2(x)$	...	$f_p(x)$
x^1	$f_1(x^1)$	$f_2(x^1)$	...	$f_p(x^1)$
$\vdots$				
x^m	$f_1(x^m)$	$f_2(x^m)$	...	$f_p(x^m)$

Where $f_i(x^k)$ represents the performance of an action, x^k, with respect to the i^{th} objective function. Furthermore, we introduce the weight function of the criteria with respect to the alternatives $w: \mathbb{R}^2 \longrightarrow \mathbb{R}$ is given by:

$$w\left(x^k,\ f_i(x^k)\right) := \alpha_i^k, \qquad \alpha_i^k \in \mathbb{R},$$

α_i^k represents the weight of alternative, x^k, with respect to the i^{th} criteria $f_i(x)$. The matrix of all the weights is represented as

$$\begin{pmatrix} \alpha_1^1 & \cdots & \alpha_p^1 \\ \vdots & \ddots & \vdots \\ \alpha_1^m & \cdots & \alpha_p^m \end{pmatrix}.$$

The decision matrix can be weighted by the corresponding elements α_i^k, as follows

$$\begin{pmatrix} \alpha_1^1 f_1(x^1) & \cdots & \alpha_p^1 f_p(x^1) \\ \vdots & \ddots & \vdots \\ \alpha_1^m f_1(x^m) & \cdots & \alpha_p^m f_p(x^m) \end{pmatrix}.$$

This matrix can be useful in different decision making methods. In a simple case, the best alternative can be chosen based on the minimum/maximum amount of the sum of each row as follows

$$\begin{pmatrix} \alpha_1^1 f_1(x^1) + \cdots + \alpha_p^1 f_p(x^1) \\ \cdots \\ \alpha_1^m f_1(x^m) + \cdots + \alpha_p^m f_p(x^m) \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{p} \alpha_i^1 f_i(x^1) \\ \cdots \\ \sum_{i=1}^{p} \alpha_i^m f_i(x^m) \end{pmatrix}.$$

In the case of selecting the alternatives by using the minimum amount of the sum of rows, the weighted sum method, with respect to the various weights of criteria and the preferences of alternatives, is deployed as follows:

$$Min \sum_{i=1}^{p} \alpha_i^k f_i(x^k)$$

Moreover, according to the problem, other decision making methods can also be considered.

4. MCDM with variable weight vector with respect to the alternative preferences

We assume the variable weight vector of the criteria corresponding to the alternatives, and presented the method to be used in the SAW, TOPSIS and PROMETHEE methods. Let $\alpha^k = \left(\alpha_1^k, \cdots, \alpha_p^k\right) \in \mathbb{R}^p, k = \overline{1, m}$ represents the corresponding weight vector for the k^{th}alternative. $\alpha_i^k \in \mathbb{R}$ represents the weight of the i^{th}criteria with respect to the k^{th} alternative. Because there are different weights of criteria with respect to the alternative, in the

TOPSIS and PROMETHEE methods, it is important to insert the weights into the right point in the decision making process. For example, in the PROMETHEE method, if the weights are inserted similarly to the previous models, the comparison of the pair of alternatives is not optimal. Below, we show how the SAW, TOPSIS, and PROMETHEE methods can consider variable weight vectors.

4.1. SAW method with respect to weights of criteria according to preference of alternatives

The (SAW) method, which based on the weighted average as the weighted sum scalarization in vector optimization and the Weber problem in location problems, is a simple multiattribute decision making technique. In this technique, the best alternative can be chosen based on the score that is calculated for each alternative by multiplying the weights of the criteria by the scaled value of the alternative. Let the multicriteria decision problem be represented by

$$Min\left\{f_1(x^k), f_2(x^k), \cdots, f_p(x^k) \middle| x^k \in X, k = \overline{1,m}\right\},$$

where $X = \{x^1, x^2, \dots, x^m\}$ is a set of alternatives, locations or possible actions, and $f_i(x), i = \overline{1,p}$ is a set of considered criteria or objective functions; then, $f_i(x^k)$ represents the performance of the k^{th} alternative with respect to the i^{th} criterion. The SAW method selects the best alternative by minimizing the following value

$$Min \sum_{i=1}^{p} w_i f_i(x),$$

where w_i represents the relative weight of the i^{th} criteria. Then, the problems with the variable weight vector of criteria, with respect to the alternatives, can be solved using

$$Min \sum_{i=1}^{p} \alpha_i^k f_i(x^k),$$

where α_i^k represents the weight of the i^{th} criteria with respect to the k^{th} alternative.

4.2. TOPSIS method with respect to weights of criteria according to preference of alternatives

TOPSIS was introduced by Hwang and Yoon (1981) [21] to determine the best among the considered alternatives. The solution is based on minimizing the Euclidean distance from the ideal solution and maximizing the Euclidean distance from the negative ideal solution. We will consider the above MCDM problem. Given a set of alternatives, $x^k \in X, k = \overline{1,m}$, and the criteria or objective functions $f_i: \mathbb{R}^n \longrightarrow \mathbb{R}$, $f_i\ (x), i = \overline{1,p}$, let $f_i(x^k)$ represent the performance of the action, x^k, with respect to the i^{th} objective function and $x_{ik} := f_i(x^k)$.

The first step in applying the TOPSIS method is to normalize the decision matrix as

$$r_{ik} = \frac{x_{ik}}{\sqrt{\sum_{k=1}^{m} x_{ik}^2}}, \quad k = \overline{1,m}, \quad i = \overline{1,p};$$

this is the vector normalization technique. Other normalization techniques can also be used. r_{ik} represents the normalized value of the element, x_{ik}. Then, the weights of criteria, $w = w_i, i = \overline{1,p},$ are inserted into the normalized elements $v_{ik} = w_i x_{ik}$. The next step is to determine the positive and negative ideal points as in the case of the maximization problems; the highest value of each column is the corresponding positive ideal point, $A^+,$ and the minimum value of each column represents the negative ideal point A^- of the specific column as follows:

$$A^+ = \{Max_k(v_{ik})|i = \overline{1,p}, \quad k = \overline{1,m}\} = \{v_1^+, \dots, v_p^+\},$$

$$A^- = \{Min_k(v_{ik})|i = \overline{1,p}, \quad k = \overline{1,m}\} = \{v_1^-, \dots, v_p^-\},$$

The TOPSIS method calculates the distance of each element from the positive and negative ideal points. The separation value is obtained by using the Euclidean distance as follows:

$$D_k^+ = \sqrt{\sum_{i=1}^{p} [v_{ik} - v_i^+]^2} \ , \quad k = \overline{1,m},$$

and

$$D_k^- = \sqrt{\sum_{i=1}^{p} [v_{ik} - v_i^-]^2} \ , \quad k = \overline{1,m}.$$

The similarities to the positive ideal point are derived using

$$C_k^+ = \frac{D_k^+}{D_k^+ + D_k^-} \quad k = \overline{1,m}.$$

The best alternative has the minimum amount of C_k^+. In the case of the variable weights in the TOPSIS method, to obtain the different weights of criteria corresponding to each alternative, we apply a similar condition as that of the SAW method. It is very important to consider the weights before calculating the positive and the negative ideal points. Because it is clear that, in the case of different weights of criteria, inserting the weights after calculating A^+ and A^- will change the value of $A^+, A^-$and the next sequences; however it can be applied to previous problems(with one weight vector). Let α_i^k be a weight vector of the k^{th} alternative with respect to the i^{th} criteria; then, we introduce $\bar{v}_{ik}$ instead of v_{ik} and an element obtained by $\bar{v}_{ik} = \alpha_i^k x_{ik}$. A similar calculation is applied to minimize the distance from the positive ideal solution and maximize the distance from negative ideal solution, as follows:

$$\bar{A}^+ = \{Max_k(\bar{v}_{ik})|i = \overline{1,p}, \quad k = \overline{1,m}\} = \{\bar{v}_1^+, \dots, \bar{v}_p^+\},$$

$$\bar{A}^- = \{Max_k(\bar{v}_{ik})|i = \overline{1,p}, \quad k = \overline{1,m}\} = \{\bar{v}_1^-, \dots, \bar{v}_p^-\},$$

$\overline{D}_k^+$ and $\overline{D}_k^-$ obtained by

$$\overline{D}_k^+ = \sqrt{\sum_{i=1}^{p}[\overline{v}_{ik} - \overline{v}_i^+]^2}\ , \qquad k = \overline{1,m},$$

$$\overline{D}_k^- = \sqrt{\sum_{i=1}^{p}[\overline{v}_{ik} - \overline{v}_i^-]^2}\ , \qquad k = \overline{1,m}.$$

The best alternative has the minimum value of $\overline{C}_k^+$, where

$$\overline{C}_k^+ = \frac{\overline{D}_k^+}{\overline{D}_k^+ + \overline{D}_k^-} \quad k = \overline{1,m}.$$

4.3. PROMETHEE method with respect to weights of criteria according to preference of alternatives

The PROMETHEE method is a MCDM method developed by Brans [22, 23] and his colleagues (1986). It is a considerably simple ranking method in conception and application, compared with other methods for multicriteria analysis. It is well suited for problems where a finite number of alternatives are to be ranked based on several criteria (sometimes conflicting criteria). The evaluation table is the starting point of this method. In this table, the alternatives are evaluated based on the different criteria (decision table/matrix). The implementation of PROMETHEE requires two additional types of information, namely:

- Information on the relative importance of the criteria (i.e. the weights) considered.
- Information on the DM's preference function (Table 2), which is used during the comparison of the contribution of the alternatives based on each distinct criterion.

The implementation of the method is shown below. Let a multicriteria decision problem be represented as follows:

$$Min\{f_1(x^k), f_2(x^k), \cdots, f_p(x^k) | x^k \in X, k = \overline{1,m}\},$$

where $X = \{x^1, x^2, \ldots, x^m\}$ is a finite set of alternatives, locations, or possible actions, $f_i(x), i = \overline{1,p}$ is a set of considered criteria or objective functions, and $f_i(x^k)$ represents the performance of action x^k with respect to the i^{th} criterion. If for a given pair of alternatives, a and b have $f_i(a) \geq f_i(b)$ for $i = \overline{1,p}$, and, at least, one inequality is strict, then, a dominates b. Let X be a finite set of alternatives for MCDM problems, and assuming a preference function, g_i defined for each f_i for each pair of alternatives $a, b \in X$, when $a > b$ in the i criterion $g_i(a, b) = g_i(d_{ab|i})$ indicates that the degree to which Alternative a is preferable to Alternative b with distance of performance $d_{ab|i} = f_i(a) - f_i(b)$ in the i^{th} criterion. $\pi(a, b)$ is a preference index for all the criteria defined by:

$$\pi(a,b) = \sum_{i=1}^{p} w_i \, g_i(a,b),$$

where w_i represents the weight of i^{th} criteria. The PROMETHEE method defines different preference functions [24], as presented in Table 2. This table could help the DM to choose the preference function corresponding to his preferences. Moreover, another preference function can also be considered by the DM [25].

Table 2: PROMETHEE preference function

	Types of criteria	Analytical Definition	Parameter
1	Usual criterion	$H(d) = \begin{cases} 0, d = 0; \\ 1, \lvert d \rvert > 0. \end{cases}$	$-$
2	Quasi criterion	$H(d) = \begin{cases} 0, & \lvert d \rvert < q; \\ 1, & otherwise. \end{cases}$	q
3	$V - sharp\ criterion$	$H(d) = \begin{cases} \frac{\lvert d \rvert}{p}, & \lvert d \rvert < p; \\ 1, & \lvert d \rvert > 0 \end{cases}$	p
4	Level criterion	$H(d) = \begin{cases} 0, & \lvert d \rvert \le q \\ \frac{1}{2} q < \lvert d \rvert \le p \\ 1, & otherwise. \end{cases}$	q, p
5	Linear criterion	$H(d) = \begin{cases} 0, \lvert d \rvert & < q \\ \frac{\lvert d \rvert - q}{p - q} q < \lvert d \rvert \le p \\ 1, & otherwise. \end{cases}$	q, p
6	Gaussian criterion	$H(d) = 1 - e^{-\frac{d^2}{2\sigma^2}}$	σ

Alternatives can be ranked according to the following factors:

- The sum of indices $\pi(a,i)$ indicating the preference of Alternative " a " over all the others. It is termed 'leaving flow' $\phi^+(a)$, and shows how 'good' Alternative " a " is.

$$\phi^+(a) = \frac{1}{1-p} \sum_{b \in X} \pi(a,b).$$

- The sum of indices $\pi(i,a)$ indicating the preference of all the other alternatives, compared to " a ". It is termed the 'entering flow' $\phi^-(a)$, and shows how 'inferior' Alternative " a " is.

$$\phi^-(a) = \frac{1}{1-p} \sum_{b \in X} \pi(b,a).$$

According to PROMETHEE Ⅰ, Alternative " a " is superior to Alternative " b ", if the leaving flow $\phi^+(a)$ of

" a " is greater than that of " b ", and the entering flow $\phi^-(a)$ of " a " is smaller than that of " b ". In other words:

$$\begin{cases} a\ P^+\ b \ \text{ iff } \ \phi^+(a) > \phi^+(b); \\ a\ I^+\ b \ \text{ iff } \ \phi^+(a) = \phi^+(b); \end{cases}$$

$$\begin{cases} a\ P^-\ b \ \text{ iff } \ \phi^-(a) < \phi^-(b); \\ a\ I^-\ b \ \text{ iff } \ \phi^-(a) = \phi^-(b); \end{cases}$$

Where P and I represent preferences and indifference, respectively. The equality in ϕ^+ and ϕ^-indicates the indifference to the two compared alternatives. In the case where the leaving flows indicate that a is better than b, while the entering flows indicate the reverse, a and b are considered incomparable. Therefore, the PROMETHEE I provides a partial ranking of the alternatives. In PROMETHEE II, the net flow ϕ (the difference of the leaving flow and entering flows) is used, which enable a complete ranking of all the alternatives. The alternative with the highest net flow is superior.

$$\phi(a) = \phi^+(a) - \phi^-(a).$$

We now return to the main topic of this section. If the weights of criteria are dependent on the alternatives, the previously described PROMETHEE can not be effective for this. With a slight change in the definition of $d_{ab|i}$, we obtain a new method to solve the problem and because the weights are positive (for a zero weight, one could delete the related criteria), one may multiple every $d_{ab|i}$ by the corresponding weight for the criterion, f_i. If α_i^k represents the weight of the i^{th} criterion, with respect to the k^{th}alternative:

$$\bar{d}_{x^j x^s|i} := \alpha_i^k \left(f_i(x^j) - f_i(x^s) \right),$$

for $j, s = \overline{1, m}$, and $i = \overline{1, p}$ and

$$g_i(a, b) = g_i(d_{ab|i}).$$

Define

$$\bar{\pi}(x^j, x^s) = \sum_{i=1}^{p} \bar{g}_i(x^j, x^s).$$

Using $\bar{\pi}$, one may define ϕ^+, ϕ^-, and ϕ, and use both PROMETHEE I and PROMETHEE II, similarly. Furthermore, we consider an example by Athawale [26], and investigate the term of the weight of criteria, with respect to the alternative preferences in the PROMETHEE method.

Example. The goal is to select the best facility location for a given industry. This example considers eight criteria, $i = \overline{1,8}$, and three candidate locations, $k = \overline{1,3}$, These eight criteria are the closeness of the market (CM), closeness to raw material (CR), land transportation (LT), air transportation (AT), labor cost (CLR), availability of labor (AL), community education (E), and business condition(BC). The normalized decision matrix is shown in Table 3.

Table 3: Normalized decision matrix

Location	*CM*	*CR*	*LT*	*AT*	*CLR*	*AL*	*E*	*BC*
$\boldsymbol{x^1}$	0.6735	1	0	0	1	1	0	1
$\boldsymbol{x^2}$	1	0	0	1	0	0	0.4839	1
$\boldsymbol{x^3}$	0	0	1	0	0.6667	0	1	0

It is important to insert the weights in this step. Because there are different weights of criteria with respect to the alternatives, and these weights could vary based on the results of the preference functions. The criteria weights are as follows: α_1^1=0.1267, α_2^1=0.1267, α_3^1=0.0883, α_4^1=0.0517, α_5^1=0.0929, α_6^1=0.0706, α_7^1=0.1668, α_8^1=0.2764, α_1^2=0.2033, α_2^2=0.0325, α_3^2=0.0726, α_4^2=0.3777, α_5^2=0.0926, α_6^2=0.0552, α_7^2=0.1227, α_8^2=0.0439, α_1^3=0.1732, α_2^3=0.1553, α_3^3=0.2102, α_4^3=0.0322, α_5^3=0.0322, α_6^3=0.1240, α_7^3=0.1057, α_8^3=1672. Table 4 is obtained by taking all of them into account.

Table 4: Weighted decision matrix

Location	*CM*	*CR*	*LT*	*AT*	*CLR*	*AL*	*E*	*BC*
$\boldsymbol{x^1}$	0.0853	0.1267	0	0	0.0929	0.0706	0	0.2764
$\boldsymbol{x^2}$	0.2033	0	0	0.3777	0	0	0.0591	0.0439
$\boldsymbol{x^3}$	0	0	0.2102	0	0.0215	0	0.1057	0

The preference function used here is as follows:

$$\begin{cases} \pi(a, b) = 0 & \text{if} \quad f_i(a) \leq f_i(b); \\ \pi(a, b) = d_{ab|i} & \text{if} \quad f_i(a) > f_i(b); \end{cases}$$

Table 5 shows the preference function for all these pairs of the alternatives.

Table 5: Weighted decision matrix

Location pair	*CM*	*CR*	*LT*	*AT*	*CLR*	*AL*	*E*	*BC*
$(\boldsymbol{x^1}, \boldsymbol{x^3})$	0	0.1267	0	0	0.0929	0.0706	0	0.2325
$(\boldsymbol{x^1}, \boldsymbol{x^3})$	0.0853	0.1267	0	0	0.0714	0.0706	0	0.02764
$(\boldsymbol{x^2}, \boldsymbol{x^1})$	0.118	0	0	0.3777	0	0	0.0591	0
$(\boldsymbol{x^2}, \boldsymbol{x^3})$	0.2033	0	0	0.3777	0	0	0	0.0439
$(\boldsymbol{x^3}, \boldsymbol{x^1})$	0	0	0.2102	0	0	0	0.1057	0
$(\boldsymbol{x^3}, \boldsymbol{x^2})$	0	0	0.2102	0	0.0215	0	0.0466	0

The aggregated preference function is as shown in Table 6.

Table 6: Aggregate preference function

Location	x^1	x^2	x^3
x^1	$-$	0.5227	0.6304
x^2	0.5548	-	0.6249
x^3	0.3159	0.2783	$-$

Table 7 represents the leaving and entering flows, and considers the net outranking flow in ranking the alternative locations.

Table 7: Leaving/entering and net flows

Location	leaving flow	entering flow	net outranking flow	Rank
x^1	0.57655	0.43535	0.1412	2
x^2	0.58985	0.4005	0.18935	1
x^3	0.2971	0.62765	-0.33055	3

The best choice of location for the given alternatives is Location 2.

The best choice of location for the given alternatives is Location 2. This method is useful in many actual applications while, in many practical problems it is necessary to consider different weights corresponding to different alternatives. For instance, if the goal is to select the proper location among candidate locations, it is more useful to consider a variable criteria weight according to several criteria such as time, quality, cost, time, proximity to market, proximity to supplier, and other objectives. Minimizing cost is preferred at some locations and minimizing time is preferred at other locations.

5. Conclusion

In many real-world problems, there are variable preferences of criteria (objective functions) with respect to the alternatives, especially in location problems. Consequently, considering of the variable weights with respect to the preferences of alternatives or facilities enable to take proper decisions and rank the alternatives correctly. This study is focused on the variable weights of criteria related to the alternative preferences. The mathematical model of this variety is presented for use in the decision matrix in different MCDM models. Furthermore, we demonstrate the application of the proposed model in location problems. Finally, we propose a model for use in the SAW, TOPSIS and PROMETHEE methods. The application of the variable weights of criteria with respect to the alternative preferences for the nondominated solution shall be explored in future research work.

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