

# Mathematical Model for Determining the Coffee Leaf Area

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## Abstract

The present study aimed to establish a mathematical model to estimate, in a simple and precise way, the area of the coffee leaves. What has been observed, in other works, already carried out, are many methods and instruments with the purpose of facilitating the measurement of leaf area and most of them are destructive, laborious and costly methods. For this study, 160 leaves of different dimensions were used to test linear and non-linear mathematical models. The linear model, which uses a correction factor ( $ACF = 0.644 \cdot LF \cdot CF$ ) presented results with high precision ( $R^2 = 0.9898$ ), with variations of -1.28% for larger leaves and 0.32% for smaller leaves, validating the method. Therefore, this model can be safely used to estimate the area of Arabica Catuaí 144 Red coffee leaves or similar.

**Keywords:** Coffee leaves; Leaf area; Area estimate; Mathematical models; Leaf sizing.

## 1. Introduction

Knowledge and obtaining the leaf area are crucial in studies, in agronomic, pharmaceutical and food segments, which aim to evaluate plant growth.

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They are related to the physiological parameters, the intensity of sweating, the net assimilation rate and its diffusivity [3, 16]. According to [1], net assimilation rate is the efficiency of the assimilation system, in the vegetable, and they are involved in the production of dry mass, estimating liquid photosynthesis. To quantify or estimate the leaf area of plants, in his work, Reference [13] mentions several methodologies that can be divided into non-destructive and destructive methods. Non-destructive methods estimate the leaf area using regression models [12], and biometric variables [7]. These methods use the basic dimensions of the leaves, such as length and width, or their calculation using digital image analysis software. The methods of quantifying the destructive leaf area are invasive and require the removal of the leaf, or other parts of the plant, which makes this method, in some cases, unfeasible or even costly, due to the limitation of individuals, the difficulty of transport to the laboratory and storage [2, 18]. The main limitation to using this method is that, once tested, the leaves are discarded [11]. In their study on sunflower leaves, Reference [15] describes the importance of using an equation that estimates, in a non-destructive way, the leaf area for physiological and agronomic studies of the vegetative growth phase. Reference [3] propose, in their study, on the determination of the area of coffee leaves (*coffee arabica* L. cv “Bourbon Amarelo”), an abacus to facilitate the calculation of the area of the leaves [14]. performed an area measurement of the sapodilla leaves, comparing six methods, namely measurement of length x width, measurement of leaf discs, measurement of the portable area integrator, measurement of squares, measurement of fresh mass and measurement of the scanner. They concluded that the integrator and squares methods were more efficient. Many studies on leaf area, of different cultures, use digital images and software [19], among them the open access software used is ImageJ. The present work aims to obtain a simplified mathematical model, which can be used, in the field or in scientific works. to determine the leaf area of the individual under study, aiming to estimate the leaf area through the basic dimensions of width and length.

## 2. Material and Methods

The study was carried out with coffee leaves of the type “Arabica Catuaí 144 Vermelho”, from the producing farms in the city of Araxá-MG (Alto Paranaíba region), Brazil [5]. 160 leaves were used, collected randomly from different healthy individuals and different sizes. The collection followed the adapted leaf sampling methodology by [17], in which the entire branches were harvested to keep the leaf moisture in transport at 15 °C to the laboratory. The leaves were identified, with sequential numbers; then, the photographic record of each leaf was performed, in the state they were in, under a clear background and with a graduated scale as a dimensional parameter, Figure 1 [6].

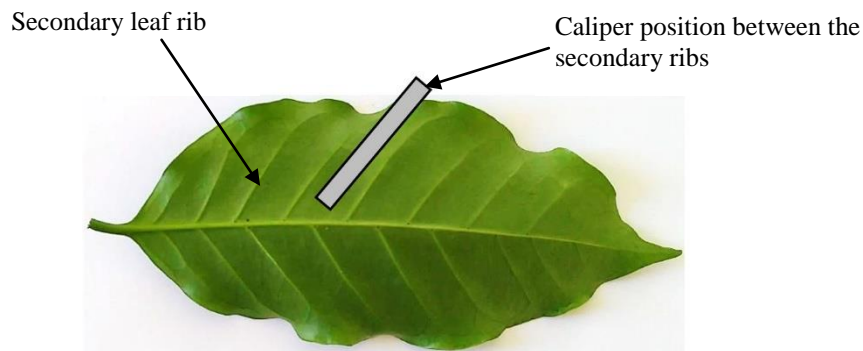


**Figure 1:** Coffee leaf identified and scale in millimeters.

After the photographic record, the measurement of the leaf area and the dimensions of the length and width were taken, using the ImageJ<sup>®</sup> software, version 1.52A, and the data were recorded in an electronic spreadsheet.

### 2.1 Determination of the average thickness of the coffee leaves

The measurement of the thickness of the leaves was between its secondary ribs and performed with the aid of a pachymeter of 0.02 mm resolution, always in the central region of each leaf, with the instrument positioned, as shown in Figure 2, following the inclination of the leaves secondary ribs.

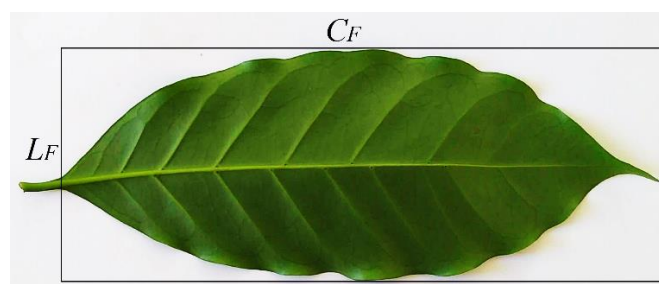


**Figure 2:** Positioning of the caliper for measuring leaf thickness

Each leaf of the experiment was measured and the value found in an electronic spreadsheet was recorded.

### 2.2 Área retangular imaginária da folha

In order to avoid distortions, in the calculation of the area, an imaginary rectangular area was delimited, as shown in Figure 3, which is a circumscribed rectangle. In it, it is possible to register the largest width of the leaf and its length, which extends from the base, discounting the petiole, to its apex, a method used by [10] for mango leaves.



**Figure 3:** Imaginary area demarcated by the circumscribed rectangle.

The mathematical representation of the imaginary area or circumscribed rectangular area  $A_{IF}$  is represented by Equation (1),

$$A_{IF} = L_F \times C_F \tag{1}$$

where  $L_F$  is the largest leaf width and  $C_F$ , the leaf length minus the petiole length.

### 2.3 Leaf area correction factor

The correction factor is a coefficient that relates the real area of the leaf to the area of the rectangle that is circumscribed and can be determined by Equation (2),

$$F_C = \frac{A_{RF}}{A_{IF}} \tag{2}$$

where  $F_C$  is the correction factor, dimensionless.

### 2.4 Correlation coefficients between the variables on the leaf

Pearson's correlation coefficients [9], which measure the degree of correlation between the coffee leaf variables being analyzed, are shown in Table 1.

**Table 1:** Correlation coefficient between leaf variables.

Variável	$L_F$	$C_F$	$A_{RF}$	$A_{IF}$
$L_F$	1	0,76	0,93	0,92
$C_F$		1	0,90	0,94
$A_{RF}$			1	0,98
$A_{IF}$				1

We observed that Pearson's correlations were positive and had different magnitudes. The correlations with the leaf areas show strong values and above 0.90, while the correlations of linear measurements show values below 0.76.

## 3. Results and Discussions

The average values obtained using the ImageJ® software and adjusted in the spreadsheet are shown in Table 2.

**Table 2:** Leaf variables obtained through the software

Variable	Medium dimension	Mean deviation	Minimum dimension	Maximum Dimension
$L_F$ (cm)	5,12	0,61	2,80	7,06
$C_F$ (cm)	13,41	1,63	6,70	17,35
$A_{RF}$ (cm <sup>2</sup> )	42,04	9,50	13,32	76,81
$A_{IF}$ (cm <sup>2</sup> )	65,47	14,75	20,73	117,79

The regressions were calculated considering the actual area of the  $A_{RF}$  leaf, as a dependent variable, the largest  $L_F$  leaf width and the longest  $C_F$  leaf length [8]. The value of  $A_{IF}$  is proportionally greater than that of  $A_{RF}$ . This is possible because the shape of the coffee leaves, when grown and in formation, have an elliptical shape [4], and not a rectangular one, like the imaginary area.

### 3.1 Leaf area correction factor

The correction factor of the leaf area or correction coefficient, after the adjustments, through Equation (2), is shown in Table 3, with the maximum, minimum and average values

**Table 3:** Correction factor of the area calculation

$F_C$ Maximum	$F_C$ Minimum	$F_C$ Medium
0,732	0,560	0,644 ±0,027

In the study by [3], for yellow Bourbon coffee leaves the correction factor value was 0.677.

### 3.2 Mathematical model to determine AF

**Table 4:** Mathematical models for estimating leaf area and their respective  $R^2$ .

Variable	Method	Mathematical model	$R^2$
$L_F$	Exponential	$A_{CF} = 5,8617e^{0,3758L_F}$	0,8754
	Linear	$A_{CF} = 14,193L_F - 30,672$	0,8605
	Logarithmic	$A_{CF} = 66,186 \ln(L_F) - 65,251$	0,8248
	Polynomial	$A_{CF} = 1,4139L_F^2 + 0,2273C_F + 2,9089$	0,8714
	Potentialiation	$A_{CF} = 2,1806L_F^{1,7975}$	0,8830
$C_F$	Exponential	$A_{CF} = 7,2797e^{0,1362C_F}$	0,8218
	Linear	$A_{CF} = 5,1872C_F - 23,002$	0,8207
	Logarithmic	$A_{CF} = 59,986 \ln(C_F) - 108,76$	0,7938
	Polynomial	$A_{CF} = 0,1366C_F^2 + 1,9686C_F - 4,972$	0,8341
	Potentialiation	$A_{CF} = 0,7015L_F^{1,6103}$	0,8302
$A_{IF}$	Exponential	$A_{CF} = 13,953e^{0,0251A_{IF}}$	0,9325
	Linear	$A_{CF} = 0,9732A_{IF} + 1,0086$	0,9667
	Logarithmic	$A_{CF} = 35,764 \ln(A_{IF}) - 90,106$	0,9190
	Polynomial	$A_{CF} = 0,0004A_{IF}^2 + 0,9362A_{IF} + 1,7256$	0,9668
	Potentialiation	$A_{CF} = 1,1343A_{IF}^{0,9655}$	0,9721
$C_F \times L_F$	Correction factor	$A_{CF} = 0,644(L_F \times C_F) = 0,644 \times A_{IF}$	0,9898

The adjustment models were determined in three ways, using the parameters of the actual area of the  $A_{RF}$  leaf, obtained by the software: a) as a function of the width of the  $L_F$  leaf; b) as a function of the length of the  $C_F$  leaf and c) as a function of the imaginary leaf area, obtained through Equation (2). The mathematical models, with their respective adjusted determination coefficients, are noted in Table 4, and were obtained through an electronic spreadsheet. We observed that the adjustment, to estimate the leaf area, with the data obtained by the experiment, was adequate, and made it possible to obtain models with  $R^2 > 0.90$ , which are adequate values [12]. The models generated to estimate the calculated area, from the  $L_F$  width and the  $C_F$  length, were left with  $R^2 < 0.9$ , which are not adequate. The models, using the association of the measurements of the  $A_{IF}$  imaginary area, proved to be more accurate, with values of  $R^2 > 0.90$  (Table 2). Similar results were obtained by [16].

### 3.3 Proof of mathematical models

For better identification and effectiveness of the models, two leaves were separated from those used in the study, with their measurements of length, width and actual area, and the average of all leaves, noted, in Table 5. For better representation, the leaves with values of real areas greater, medium and smaller were adopted.

**Table 5:** Measurements of two of the leaves obtained by the software.

Leaf size	$C_F$ (cm)	$L_F$ (cm)	$A_{RF}$ (cm <sup>2</sup> )	$A_{IF}$ (cm <sup>2</sup> )
Bigger	16,68	7,06	76,81	117,76
Average	12,54	5,12	42,04	64,21
Smaller	7,41	2,80	13,32	20,75

The values presented were the data obtained by the software. The  $A_{IF}$  value is the result of the calculation applied through Equation (1).

We observed that the values obtained by the adjusted equations present distorted leaf area values, in relation to the real area of the  $A_{RF}$  leaf. The values presented by the equation that uses the correction factor are more reliable and present values close to the real area of the leaf, obtained by the software. The columns identified as Deviation, in Table 6, represent the distortion between the real area and the calculated area of the leaves, according to the applied equation.

**Table 6:** Results obtained by applying the values of  $A_{IF}$ ,  $C_F$  and  $L_F$  in mathematical models.

Variable	Method	Bigger leaf	Desviation	Smaller leaf	Desviation
$L_F$	Exponential	83,23	7,71%	16,79	20,66%
	Linear	69,53	-10,47%	9,07	-46,88%
	Logarithmic	64,11	-19,82%	2,90	-360,04%
	Polynomial	74,99	-2,43%	14,63	8,96%
	Potentialiation	73,16	-4,98%	13,88	4,02%
$C_F$	Exponential	70,59	-8,81%	19,97	33,31%
	Linear	63,52	-20,92%	15,44	13,70%
	Logarithmic	60,05	-27,90%	11,38	-17,03%
	Polynomial	65,87	-16,61%	17,12	22,18%
	Potentialiation	65,18	-17,84%	17,65	24,52%
$A_{IF}$	Exponential	268,13	71,35%	23,49	43,29%
	Linear	115,61	33,56%	21,20	37,18%
	Logarithmic	80,44	4,51%	18,35	27,41%
	Polynomial	117,52	34,64%	21,32	37,54%
	Potentialiation	113,31	32,21%	21,20	37,17%
$C_F \times L_F$	Correction factor	75,84	-1,28%	13,36	0,32%

### 3.4 Average leaf thickness

The average thickness obtained from the coffee leaves, after adjustments, was  $0.22 \text{ mm} \pm 0.02$ .

## 4. Conclusions

The linear equation ( $A_{CF} = 0.644 \cdot L_F \cdot C_F$ ), which uses the correction factor, the width and length of the leaf, as parameters, proves to be effective and with more accurate results, when the mathematical and comparative models test is performed with the actual area of the leaves, which were obtained by the software. The variations found were -1.28% for the largest leaf, and 0.32% for the smallest leaf. The other equations (exponential, linear, logarithmic, polynomial and potentialiation) present distorted results, when compared with the real area of the leaf, and should not be used, even if they are for quick calculations, as they present significant differences in area. The method has, as advantages, greater precision of the calculated area, ease of calculation, and simplicity, since it uses only the dimensions of width and length, without the need to remove the leaves.

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