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# Mathematical Modelling of Dynamic Behavior of Droplets of Saliva as a Vehicles for Respiratory Pathogens Transmission

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**Abstract**

This study performs an analytical description of the dynamic behavior of the droplets and the effect of the environmental air temperature on their drop time and horizontal velocity to highlight some aspects that improve the fight against the spread of the respiratory pathogens, such as the COVID-19, between humans. According to this study, when the droplet exits the human body, its horizontal velocity reduces drastically. The smaller the size of the droplets, the higher is the rate of this decay. The droplets achieve its largest horizontal range instantaneously. Thus, the horizontal displacement for droplets with less than 100 µm in diameter does not exceed 380 mm. However, droplets with larger than 300 µm in diameter have a horizontal range that can reach distances greater than 1600 mm, without significant reduction of their heights to the ground. This can invalid the current rule of social distance (least 1.5 m — 2 m between people). Because of this, if a person is within the horizontal displacement range of a droplet he will be hit in the face which can increase the contagion. The droplets with less than 1 µm in diameter remain airborne for at least 15 hours, which can further increase of contagion by inhalation. The temperature of the environment has a negative influence on droplets drop time. Faced with this, the people in warmer countries can inhale more virus of suspension than those in colder countries. Finally, the higher the environmental air temperature, the lower the horizontal range achieved by the droplets.

***Keywords:*** Droplets; Dynamic Behavior; Respiratory Pathogens; Virus; COVID-19; Coronavirus.

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## 1. Introduction

Drops of saliva can contain a wide variety of pathogens such as influenza virus [1], measles [2] and *Mycobacterium tuberculosis* [3]. For example, high loads of COVID-19 virus were found in the saliva droplets of positive patients, both in symptomatic and asymptomatic [4, 5]. The human body can exhale these droplets not only through coughing or sneezing but also through simple conversations between people. A study carried out in [6] shows that a droplet of saliva may contain over 2 million viruses. According to the authors in [7], the human body can expel approximately 3,000 droplets of saliva at a speed that can vary between 6 and 28 m/s whenever one coughs. In a single sneeze, a person may expel 40,000 droplets of saliva at speeds than can reach 50 m/s, which is a quantity able to contaminate closed spaces such as bedrooms, offices and living rooms, as the study carried out by authors in [8] shows. In a normal conversation, a person can exhale thousands of droplets per seconds, with different diameters, and this indicates a very strong possibility that a normal speaking causes an airborne virus transmission meanly in a close environment [9]. The authors in [10] showed that the mouth can exhale around 112 to 6720 droplets of saliva with 16.0 µm in diameter at 3.9 m/s because of speak, while the coughing liberates between 947 to 2085 droplets with 13.5 µm in diameter at a mean velocity around 11.7 m/s. The widely accepted idea between health professionals that the transmission of the respiratory pathogens like coronavirus occurs essentially by direct hand-to-mouth contact may sometimes deviate from reality. In fact, small droplets of saliva (with diameter less than 30 µm), which potentially remain suspended in the air for significant periods of time, have been marginalized and can undoubtedly be significant sources of contagion by direct inhalation of viruses [11]. The chance of infection spreads through the air depends mainly on how long it remains airborne and this is essentially determined by the droplet sizes. For example, the authors in [12] showed that the droplets with 30 µm in diameter can be airborne for 10 min while for droplets with 1 µm in diameter can remain airborne for 17 hours. The same authors showed yet that droplets larger than about 100 µm in diameter drop to the ground within 1 or 2 s, while droplets that are initially smaller than 100 µm evaporating before dropping to the ground and so form residues or “droplet-nuclei” that may remain airborne for several hours or even some days. In addition, the authors in [13] also showed that the original diameter of droplets reduce between 20% and 34% because of their dehydration and this can slow down their motion. The work realized by the same authors showed that the original diameter of the respiratory drops ranges from 1 µm to 2000 µm and 95% of them present diameter between 2 µm and 100 µm, been the most common droplets exhibit values of diameter from 4 µm to 8 µm. According to the referred authors, the most common droplets-nuclei present diameter between 1 µm to 2 µm and in calm air 90% of the bacteria-carrying droplets-nuclei takes 30 min to 60 min to disappear from the air, while the smaller nuclei (with less 4 µm in diameter) remain airborne for a much longer period and sometimes they take 30 hours at least. A study carried out by the US Centers for Disease Control and Prevention (CDC) shows that droplets of saliva may travel a horizontal distance of between 1 and 2 meters [14]. However, studies recently carried out by MIT researchers show that these droplets can travel distances of over 8 meters [15]. Also, in this context, an experimental study carried out by the authors in [16] shows that the horizontal stagnation of the movement of droplets occurs quickly.

## 2. Methodology

The method of this study deals with the calculations of a set of physical quantities described in the following.

### *2.1. The Vertical and horizontal velocities of the droplets*

Assuming a droplet as a perfect sphere in motion in the air medium, with no rotation or deformation, from the free-body diagram, the forces acting on it are the drag force ($D_x$) in the $X$ (horizontal) direction and the weight ($W$), the buoyancy force ($I$) and the drag force ($D_y$), in the $Y$ (vertical) direction [17]. The drag force $D_x$ acts as a resistance slowing the horizontal motion of the droplets. The drag force $D_y$ and the buoyancy force $I$ are opposite of the downward motion of the droplets. Thus, applying Newton Second Law in the two directions of the motion the following equations we get.

$$\begin{cases} -D_x = m_d \frac{dV_x}{dt} & \leftarrow X \\ I + D_y - W = m_d \frac{dV_y}{dt} & \uparrow Y \end{cases} \tag{1}$$

Where, $V_x$ and $V_y$ are, respectively, the velocity components in the $X$ and $Y$ directions and $m_d$ is the mass of the droplets. In the above equations, $m_d = \rho_{H_2O} \frac{4\pi}{3} R_d^3$ , $W = gm_d = \gamma_{H_2O} \frac{4\pi}{3} R_d^3$ , $I = \gamma_{air} \frac{4\pi}{3} R_d^3$ in which $g = 9.81 \, {}^{m}/_{s^2}$, $R_d$ , $\rho$, $\gamma$,are, respectively, the specific gravity, the radius of the droplets, the specific mass and the specific weight. We will analyze the two drag forces, separately as we do it in the following subsection because of their nature.

### *2.2. The drag forces $D_x$and $D_y$*

The drag force appears whenever the body moving in a fluid medium and it is a function of Reynolds Number ($Re = \frac{2UR_d}{\vartheta}$, $\vartheta$ is the kinematic viscosity of the fluid and $U$ is the flow velocity ) that represents the ratio between Inertial and Viscous forces in a fluid motion [18]. Here, the droplets move in the air medium that causes the appearance of the two drag forces mentioned before. If the velocity of the body is very low that conduit to $Re \leq 1$, the mathematical expression for drag force ($D$) acting on droplets is represented by [19]:

$$D = 0.5\rho_{air} A_d U^2 C_D, with\ C_D = \frac{24}{Re} \tag{2}$$

In the above equations,$A_d$, $C_D$ are, respectively, the projected area of the droplets $A_d = \pi R_d^2$ and the drag coefficient (the dimensionless drag force). For $0.2 \leq Re \leq 2 \times 10^3$ the drag coefficient can be calculated by the following expression [20]:

$$C_D = \frac{a}{Re} + \frac{b}{\sqrt{Re}} + 0.25 \tag{3}$$

Where $a = 21.12$ and $b = 6.3$. Thus, we get the new mathematical formula of drag force from the eq. (2), replacing the $C_D$ parameter by it expression given by the eq. (3). Finally, replacing the expressions of $D_x$ and $D_y$ in the eq. (1) we get the following set of differentials equations that produce the velocity field of the droplets.

$$\begin{cases} \frac{dV_x}{dt} = K_0 V_x \\ \frac{dV_y}{dt} = K_1 + K_0 V_y \end{cases} \quad \text{for } Re \leq 1 \tag{4}$$

Where, $K_0 = -\frac{6\rho_{air} A_d \vartheta}{m_d R_d}$ and $K_1 = \frac{w-I}{m_d}$.

And,

$$\begin{cases} \frac{dV_x}{dt} = K_2 V_x + K_3 V_x^{1.5} + K_4 V_x^2 \\ \frac{dV_y}{dt} = K_1 + K_2 V_y + K_3 V_y^{1.5} + K_4 V_y^2 \end{cases} \text{for } 0.2 \leq Re \leq 2 \times 10^3 \tag{5}$$

In the above equations $K_2 = -\frac{0.5a\rho_{air} A_d \vartheta}{2m_d R_d}$, $K_3 = -\frac{0.5b\rho_{air} A_d \sqrt{\vartheta}}{m_d \sqrt{2R_d}}$, $K_4 = -\frac{0.5 \times 0.25\rho_{air} A_d}{m_d}$.

### *2.3. Euler method to solve differential equations*

Some above differential equations are nonlinear and require a numerical method to solve it. To do this, we use the Euler method that produces the following set of equations [21].

$\underline{Re \leq 1}$

$$X_{(i+1)} = X_{(i)} + hV_{x(i)}$$

$$V_{x(i+1)} = V_{x(i)} + hK_0 V_{x(i)}$$

$$Y_{(i+1)} = Y_{(i)} + hY_{x(i)} \tag{6}$$

$$V_{y(i+1)} = V_{y(i)} + h(K_0 + K_1 V_{y(i)})$$

$\underline{0.2 \leq Re \leq 2 \times 10^3}$

$$X_{(i+1)} = X_{(i)} + hV_{x(i)}$$

$$V_{x(i+1)} = V_{x(i)} + h(K_2 V_{x(i)} + K_3 V_{x(i)}^{1.5} + K_4 V_{x(i)}^2)$$

$$Y_{(i+1)} = Y_{(i)} + hY_{x(i)} \tag{7}$$

$$V_{y(i+1)} = V_{y(i)} + h(K_2 V_{y(i)} + K_3 V_{y(i)}^{1.5} + K_4 V_{y(i)}^2)$$

On the above equations $h = \frac{\Delta t}{N}$, where $\Delta t$ is the time interval and $N$ is the number of points for which we calculate the variables of the motion. Thus, the previous equations are solved at each point $= 0,1,2,3, \ldots N$ .

### *2.4. Dynamic and kinematic air viscosity*

The fluid viscosity is one of the most important fluid characteristics and represents its ability to flow. The viscosity of liquids decreases with temperature while for gases (air) this quantity increase with temperature. The dependence of the air viscosity with temperature is expressed by Sutherland equation as follow [22]:

$$\mu = \frac{145 \times 10^{-8} T^{1.5}}{T + 110.4} \tag{8}$$

Where $\mu[Pas]$ and $T[K]$ are, respectively the dynamic air viscosity and the air temperature. Finally, the kinematic viscosity is, by definition, the ratio between the dynamic viscosity and the fluid specific mass as follows.

$$\vartheta = \frac{\mu}{\rho} \qquad (9)$$

## 3. Results and discussion

Figure 1 shows the horizontal velocity of the droplets as a function of time. Yet, this figure shows the curves of velocity for three droplets with different diameters. As it is possible to see, the horizontal velocity decreases with time. But the rate of this decay is higher for smaller droplets and decrease as the size of the droplets increase. This behavior of the horizontal velocity is because the deceleration of the droplets, defined by the ratio $D_x/m_d$,is much higher for smaller droplets as shown in Figure 2. Figure 2 shows the ratio between the deceleration acting on the $1\mu m$ diameter droplet and that acting on a $10\mu m$ diameter. Even though the referred ratio decreases over time, its value is very high at the beginning, getting to be about 50 times higher in droplets of $1\mu m$ than in droplets of $10\mu m$ in diameter.

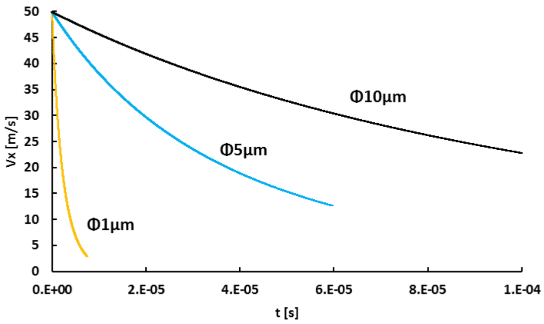


**Figure 1:** Horizontal velocity vs time.

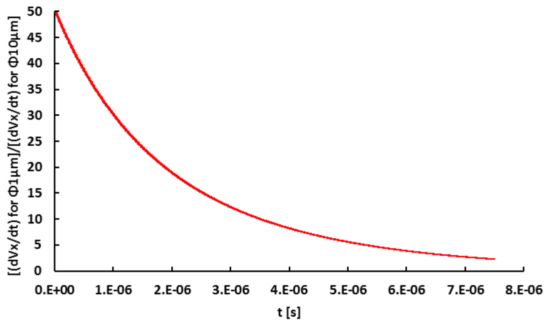


**Figure 2:** Ratio between the decelerations got for droplets of Φ1µm and Φ10 µm vs time.

The vertical velocity of the droplets increases quickly at the beginning of the motion and stabilizes around the terminal speed got from equation (1), assuming null vertical acceleration (Figure 3). This figure also reveals, the bigger the droplet is, the faster it falls. The following figure (Figure 4) shows the trajectory of the droplets, with two different diameters, in the XOY plane. According to this figure, the droplets reach their maximum range almost instantly, given them virtually no time to drop. The droplets reach their maximum range at about the same height from the ground where they were exhaled from the human body. This result is important because it means that if a person is face-to-face with another that sneezing, the droplet will hit him directly in the face. Thus, how it's expected, this situation drastically increases the contagion. Another aspect of the kinematic of the droplets and shown also in Figure 4 has to do with the relationship between their size and the maximum range they can achieve. In fact, the bigger droplet is, the higher is its maximum range. In this context, the maximum range that a droplet with a diameter of less than $100\mu m$ can achieve is 380 mm. However, for droplets with diameter $\varphi \geq 300\mu m$ the maximum range is higher than 1.5 m. Thus, it is important to note that these droplets can put people at risk, even obeying the minimum distance of 1.5 m recommended by WHO.

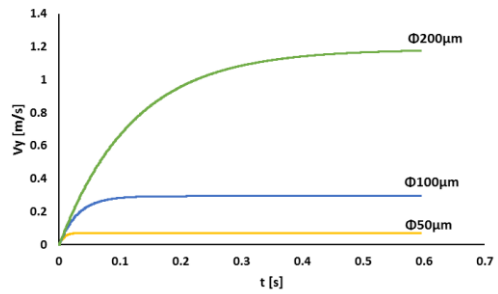


**Figure 3:** vertical velocity vs time.

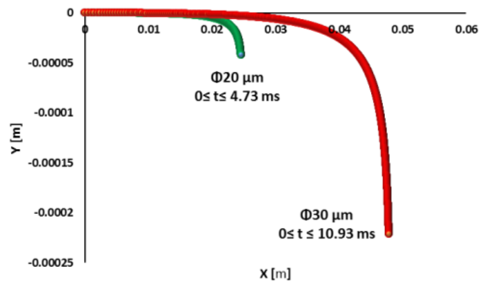


**Figure 4:** Trajectory of droplets in the XOY plane.

As expected, when the environmental air temperature increases it becomes more viscous and, therefore, more resistant to the movement of the droplets decreasing their horizontal range. We present this in the following table that shows the values of the horizontal range of a droplet with 1µm in diameter for different air temperatures and 7.41µs after exhaled from the human body.

**Table1:** The effect of the air temperature on the horizontal range of the droplets.

| **Air temperature [ºC]** | 0 | 25 | 35 |
|---|---|---|---|
| **X [mm]** | 0.119 | 0.115 | 0.114 |

As it is possible to note from the table above, the horizontal range is almost insensitive to the increase of the environmental air temperature. Figure 5 reveals the influence of the size of the droplets on their drop times. As expected, the drop times decrease as the sizes of the droplets increase. The droplets with diameter $\varphi \leq 1\mu m$ stay in the air (with 25ºC) for a period higher than 15 hours.

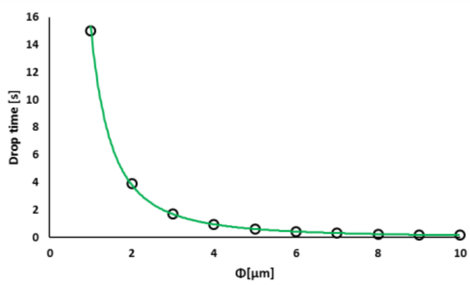


**Figure 5:** Drop time vs diameter of the droplets.

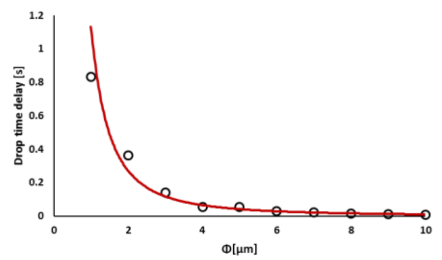


**Figure 6:** Increase of drop time when the air temperature rises from 0ºC to 25ºC, vs size of the droplets.

This last finding is very close to that achieved in [13]. Yet, the same graphic shows that the effect of the size of the droplets on their drop times is less significant for $\varphi \geq 8\mu m$. Figure 6 shows the effect of the air temperature on the droplets drop times. The curve shown in this figure represents the difference between the drop times for 25ºC and 0 ºC air temperature (drop times for 25ºC - drop times for 0ºC). This effect is most significant for small droplets and decreases as the diameter of the droplets increase. Thus, according to this figure, the warmer the country is, the higher is the droplets drop time. This is because the air becomes more viscous and therefore more resistant to the movement of the droplets. Therefore, people living in warmer regions will be subject to a greater likelihood of contagion by inhalation of droplets in suspension.

**4. Conclusion**

As we know, the droplets are perfect vehicles for respiratory pathogens-carriage between humans. So, we can bring to the light important mechanisms of defense against the propagation of the respiratory disease through a correct understanding of the dynamic behavior of droplets and this is the principal aim of this work. The results show that the air act as an important resistance medium reducing quickly the horizontal velocity of the droplets and increasing their drop time. The smaller the droplets are, the higher is the rate of horizontal velocity decay and the higher is their drop time. This behavior of the smaller size droplets is very critical in terms of virus propagation as they stand suspended in the air for a long time. In fact, droplets with a diameter of less than 1μm stay in the air during 15h or higher. These droplets and hence the pathogens they contain are airborne. The droplets achieve their maximum range quickly, for the time interval in which their vertical displacement is insignificant. This means that the droplets attack directly in the face of anyone that standing, at the reach of the drops, face to face with another that sneezing or coughing and this can increase, significantly, the probability of the contagion. Droplets with diameters higher than 300μm can achieve distance higher than 1.58 m that can put in risk even the persons that respect the actual minimum distance rule of 1.5 m to 2 m. The Higher the diameter of the droplets, higher their horizontal range. The air temperature affects negatively the drop time of the droplets. That is, higher air temperature implies higher values of drop time. In the context of virus propagation, this situation becomes the people of the warmer countries more vulnerable than those who live in colder regions.

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