

# A Case Study on Outpatient Waiting Time for Treatment with Single Server Queuing Model at Public Eye Hospital in Bangladesh

Obaidul Haque<sup>a\*</sup>, Sharmin Akter<sup>b</sup>, Anwar Hossen<sup>c</sup>, Zillur Rahman<sup>d</sup>

<sup>a,b</sup>*Department of Mathematics, Comilla University, Cumilla, 3506, Bangladesh*

<sup>c,d</sup>*Associate Professor, Department of Mathematics, Comilla University, Cumilla, 3506, Bangladesh*

<sup>a</sup>*Email: obaidul.cou@yahoo.com*

<sup>b</sup>*Email: sharminshimucou@yahoo.com*

<sup>c</sup>*Email: mdahossen@yahoo.com*

<sup>d</sup>*Email: rahman.zillur54@yahoo.com*

## Abstract

In public hospital outdoor patient services all over the world have become an important component of health care. Queuing is a major challenge for healthcare service particularly in developing countries, queuing theory helps decision making to improve waiting problem is not commonly used by managers in developing countries in contrast to their counterparts in the developed world. In this paper we introduce data analysis of observation to reduce outdoor patients waiting time without cost consideration at public hospital.

**Keywords:** Operation research; Queuing Theory; Queue Model; Single Server Model; Public Hospital; BIOEH; Sensitivity Analysis.

## 1. Introduction

Operation research is a rational method of problem-solving and decision making and Queuing theory is a section of operations research in mathematics that models the function of waiting in lines. Queuing theory, also known as the theory of congestion, is a section of operation research that research the familiarity between the want on a service system and the delays bear by the purchaser of the system. The Impetus for the progress of queuing theory was the burgeoning telephone industry in the early 1900s and the history of Queue goes back to primitive man.

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\* Corresponding author.

Below is an earlier queue which is described in the Bible. A. K. (Agner Karup) Erlang published his first paper on Queuing theory in 1909. The study of queues deals with quantifying the phenomenon of waiting in lines using ambassador step of performance, such as average queue length, average waiting time in queue, and average facility utilization. Queue model is structured so that queue lengths and wait times can be forecast. Queuing theory is the mathematical study of waiting lines and it is very useful to define. Modern information technologies require innovations that are based on modelling, analyzing, designing to deal as well as the procedure of traffic control of daily life of human like telecommunications, reservation counter, super market, big bazaar, Picture Cinema hall ticket window and also to determining the sequence of computer operations, computer performance, health services, airport traffic, airline ticket sales [1]. Each user of the queuing model passes through the queue where he may remain for a period of time (positive, possibly zero) and then is processed by a single server because of the parallel arrangement of the servers. Once a purchaser has left the server, after obtaining the service, the purchaser is considered to have left the queuing system as well.

## 2. Single Server Queuing Model

In single server case we introduce two models. First one sets no limit on the maximum number in the system and the second assumes a finite system limit. Both models assume an infinite –capacity source. Arrivals occur at the rate of  $\lambda$  customers per unit time, and the service rate is  $\mu$  customers per unit time [3]. The results of the two models are derived as special cases of the results of the generalized model. The Kendall notation will be used to summarize the characteristics of each situation. Because the derivations of  $p_n$  and of all the measures of performance are totally independent of a specific queue discipline, the symbol GD will be used with the notation. (M/M/1) : (GD/ $\infty$  / $\infty$ ). Using the notation of the generalized model, we have

$$\lambda_n = \lambda$$

$$\mu_n = \mu$$

Also,  $\lambda_{eff} = \lambda$  and  $\lambda_{lost} = 0$ , because all arriving customers can join the system.

Letting  $p = \frac{\lambda}{\mu}$ , the expression for  $p_n$  in the generalized model then reduces to

$$p_n = p^n p_0, n=0,1,2, \dots$$

To determine the value of  $p_0$ , we use the identity

$$p_0(1+p+p^2+\dots) = 1$$

Assuming  $p < 1$ , the geometric series will have the finite sum  $(\frac{1}{1-p})$ , thus

$$p_0 = 1-p, \text{ provided } p < 1$$

The general formula for  $p_n$  is given by the following geometric distribution

$$p_n = (1-p)p^n, n=1,2,\dots,(p<1)$$

The mathematical derivation of  $p_n$  imposes the condition  $p<1$  or  $\lambda < \mu$

If  $\lambda \geq \mu$ , the geometric series will not converge, and the steady-state probabilities  $p_n$  will not exist. This result makes intuitive sense, because unless the service rate is larger than the arrival rate, queue length will increase indefinitely. The measure of performance  $l_q$  can be derived in the following manner [3]:

$$\begin{aligned} l_s &= \sum_{n=0}^{\infty} np_n = \sum_{n=0}^{\infty} n(1-p)p^n \\ &= (1-p)p \frac{d}{dp} \sum_{n=0}^{\infty} p^n \\ &= (1-p)p \frac{d}{dp} \left( \frac{1}{1-p} \right) = \frac{p}{1-p} \end{aligned}$$

Because  $\lambda_{eff} = \lambda(1-p)$  for the present situation, the remaining measures of performance is computed using the relationship. Thus,

$$w_s = \frac{l_s}{\lambda} = \frac{1}{\mu(1-p)} = \frac{1}{\mu - \lambda}$$

$$w_q = w_s - \frac{1}{\mu} = \frac{p}{\mu(1-p)}$$

$$l_q = \lambda w_q = \frac{p^2}{1-p}$$

$$\bar{c} = L_s - L_q = p$$

### 3. Reduce outdoor patients waiting time without cost consideration at public hospital with the help of single channel two phase system

This is a new approach for solving Queuing problem. In this method we apply single channel Multiple phase system. The objective of outpatient scheduling is to find an appointment system for which a particular measure of performance is optimized in a clinical environment an application of resource scheduling under uncertainty [2].

#### 3.1. Research Methodology

This research based on the hospital survey. Interview with management, doctors and records staff were conducted to validate the secondary data and to gather information required to construct the structural model of the routings in and out of OPD and ED. Questionnaires were also used to gather information on daily arrival rates, patients view on queuing at the hospital, waiting time to consult a doctor etc. In all, 250 patients were

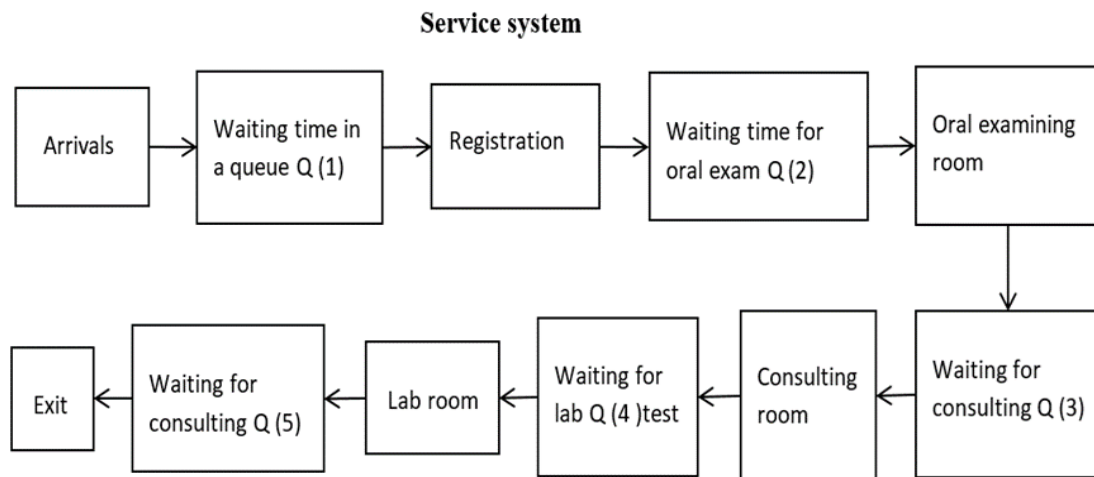
surveyed for this study. The outdoor department was selected. To get information went to the hospital information department, patient waiting room, doctors, nurses, outdoor and emergency department. Then we gathered information about hospital total survey. Observation made during a month. After the observation it was decided for data retrieval done on Sunday and Thursday which is on the busy days of the week. The data used are the number of patient arrival per specialists, per day arrival time of doctor and patient.

### 3.2. Data collection and Analysis in BIOEH

We considered the Outdoor department of BIOEH in Cumilla. This department is one of the modern units in Bangladesh and consists of several major areas: triage, resuscitation room, immediate care unit, space for minor emergencies, room for minor surgeries, critical observer. The data used in the case study includes detailed information's over the April (1 –30),2019.

### 3.3. Patient flow

Patient flow management requires addressing three aspects of an outpatient unit: Arrival of patients, service process and queuing process working on the patient's arrival includes controlling its patient panel size, balancing patient volume across available sessions and achieving desirable patient arrival pattern within a session. Patient flow can be described by one of two complementary approaches: clinic or operational. Regardless of approach, all patient flows share four common characteristics: an entrance, an exit and the random nature of health care system. In table1 The average number of patients per day and the figure 1 shows the service system.



**Figure 1:** The service system [5]

**Table 1:** The average number of patients per day [4]

Department	Sun	Mon	Tue	Wed	Thus
Outdoor	287	213	254	248	395
Emergency	53	19	48	51	73

In the table 2 shows the time of patient arrivals. A large number of patients come between 7.30 am to 10 am. This condition through to lead a long patient waiting times. So, this research will focus on the busy hours is 20 people per hour.

**Table 2:** The average number of patient’s arrival per hours [4]

Time	Number of patient / day				
	1(Sun)	2(Mon)	3(Tue)	4(Wed)	5(Thu)
<7.29	87	45	74	72	122
7.30-8.29	59	38	61	49	108
8.30-9.29	52	45	65	48	77
9.30-9.59	63	47	47	73	84
10-11	79	58	55	57	73
11.00 > Registration close	0	0	0	0	0

This hospital has 5 specialists. One is senior doctor. The number of waiting rooms are 3. There is a huge space in front of this waiting room. This hospital survey system is multi server. Each patient waiting time is 0.5 hours. After waiting 0.5 hour they are in waiting line. Each patient minimum total survey is 3-5 minute. Outdoor is open from 7.30 am till 2.00 pm giving a total 6.5 hours per day. It is important to note that after 2.00 pm there is still an emergency out-patient clinic. Using the information above the average patient arrival rate is given by: 279 patients per day divided by 6.5 hours per day resulting 43 patients per hour. From interviews with the doctors, one doctor averagely uses 2.68 minutes to complete the care of one patient. This implies that a doctor can see 22.38 patients per hour.

**3.4. Service Capacity**

A total of 4 four doctors and one senior attend to patients at the out patient’s department. Hence, the number of doctors is considered to be 5. Out of 200 questionnaires administered, 150 responded to this question and the data generated is shown in table 3

**Table 3:** Average time taken before to seeing a doctor

Average time taken before seeing a doctor (hours)	Number of Respondents	Percentage
0 - 0.5	6	4
0.5 – 1.0	25	16.67
1.0 – 2.0	47	31.33
2.0 – 3.0	62	41.13
>3.0	10	6.67
Total number of respondents	150	100

From the above data, the model average time it takes to see a doctor on arriving for care at the hospital is between 2 to 3 hours. Therefore, more people will spend an average 2.25 hours before seeing a doctor. Outpatient waiting time :Single channel system queuing formula used was few terms : Arrival rate  $\lambda$ ; service rate  $\mu$ ; Average server utilization  $p = \frac{\lambda}{\mu}$ ; Average number of customers in the queue  $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$ ; Average number of customers in the system  $L_s = \frac{\lambda}{\mu-\lambda}$ ; Average waiting time in the queue  $W_q = \frac{\lambda}{\mu(\mu-\lambda)}$ ; Average time in the system  $W_s = \frac{1}{\mu-\lambda}$ ; based on calculation for first server using one month daily data, the table 4 below is the result.

**Table 4:** One-month daily data [4]

Arrival rate ( $\lambda$ )	43
Service rate ( $\mu$ )	45
Average server utilization (p)	0.95%
Average number of customers in the queue ( $L_q$ )	20.54 patients
Average number of customers in the system( $L_s$ )	21.5 patients
Average waiting time in the queue ( $W_q$ )	0.47 Hour
Average time in the system ( $W_s$ )	1 Hour
Probability (% of time) system is empty ( $p_0$ )	0.05%

**3.5. Patient views on Queuing at the Hospital**

Questionnaires were administered to determine the views of patients in relation to the effects of queuing on them. Out of 150 respondents, 99% experienced positive effects with queuing at hospital but the major problem is long waiting time and waiting line. Out of this number, 0% felt frustration with the service, 80% tiredness from waiting and most of the people desire to use the facility again. Queuing is therefore a problem of concern at the hospital, as is the case in several hospitals in the country. The study accordingly goes on to model 1

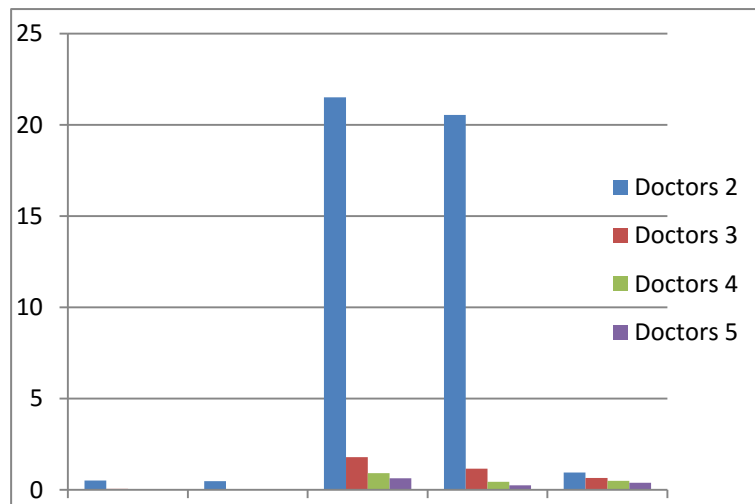
scenarios observer capacity at the hospital applying the queuing theory to establish whether this can inform an optimal scenario setting that will reduce the queuing burden.

### 3.6. Sensitivity Analysis

Four scenarios of server capacity were considered, comparing capacity utilization, average waiting time in the queue and average time in the system, average waiting time in the queue and average time in the system by patients using Software: QM for windows. Based data, sensitivity analysis shown in table 5 and figure 2.

**Table 5:** Sensitivity analysis

Scenarios	Avg. Service Utilization	Avg. patient in queue( $L_q$ )	Avg. patient in system( $L_s$ )	Avg. waiting in queue( $W_q$ )	Avg. waiting in system ( $W_s$ )
2 Doctors	0.95	20.54	21.50	0.47	0.5
3 Doctors	0.6418	1.1499	1.791	0.02674	0.04166
4 Doctors	0.4777	0.4371	0.91489	0.01016	0.02127
5 Doctors	0.3839	0.2392	0.62318	0.00556	0.01449



**Figure 2:** Sensitivity analysis

### 3.7. Data collection and analysis in Comilla Medical College (Ophthalmology Department)

We considered the Outdoor department of Comilla Medical College in Cumilla. This department is one of the modern units in Bangladesh and consists of several major areas: triage, resuscitation room, immediate care unit, space for minor emergencies, room for minor surgeries, critical observer. The data used in the case study includes detailed information's over the May (1– 30),2019. To get information went to the hospital information department, patient waiting room, doctors, nurses, outdoor and emergency department. Then we gathered

information about hospital total survey on eye section. Observation made during a month. After the observation it was decided for data retrieval done on Saturday and Thursday which is on the busy days of the week. The data used are the number of patient arrival per specialists per day arrival time of doctor and patient. Here first seven days' data which are collected in field work given below:

**Table 6:** The average number of patients per day

Department(Eye)	Sat	Sun	Mon	Tue	Wed	Thus
Outdoor	26	22	18	19	17	19
Emergency	2	4	1	3	4	5

**Table 7:** The average number of patient's arrival per hours

Time	Number of patients / day					
	1(Sat)	2(sun)	3(Mon)	4(Tue)	5(wed)	6(Thru)
< 8.29	5	3	1	5	2	1
8.30-9.29	7	6	4	5	4	3
9.30-10.29	4	7	9	1	7	7
10.30-10.59	8	2	1	3	1	5
11.00-12.59	2	3	0	4	2	2
1.00-1.59	0	1	3	0	1	1

This hospital has 31 specialists. 11 is senior doctor. In Ophthalmology department there are 8 doctors. One is senior consultant, one is junior consultant, two's are register doctor, two's are sub register, two's are residential surgeon. This hospital survey system is multi server. Outdoor is open from 8.30 am till 2.00 pm giving a total 5.5 hours per day. It is important to note that after 2.00 pm there is still an emergency out-patient clinic. Using the information above the average patient arrival rate is given by: 20 patients per day divided by 5.5 hours per day resulting arrival rate 4 patients per hour. From interviews with the doctors, one doctor averagely uses enough time completing the care of one patient. So this department of Comilla medical college has no waiting time because the patient flow is very low.

#### 4. Computed Result

In public hospital outdoor patient service all over the world have become an important component of health care. Queuing is a major challenge for healthcare service particularly in developing countries, queuing theory helps decision making to improve waiting problem is not commonly used by managers in developing countries in contrast to their counterparts in the developed world. by hidebound thinking, the medical profession emphasized that a doctor's time is more valuable than a patient's time. In this two analysis we have to show that: In BIOEH, the arrival rate is 43, service rate is 45. Here we see that the arrival rate is greater than the service rate but in average used 2.68 minutes to complete the care of one patients but a patient total waiting time 2.25 hours approximately. Finally, we say that the main problem of this hospital which is given: time loss in



ticket counter, slow work in examine room and the slow movement of total survey system which is unpredictable for the patients. To reduce outdoor patients waiting time, this hospital must be changed the total survey system and the slow movement. In Comilla Medical College Ophthalmology Department, the arrival rate and the service rate are equal. So, there are no waiting time but the patient flow of this hospital is very low because there is a unique eye hospital (BIOEH) which is very popular eye hospital in Cumilla area.

## **5. Conclusion**

This paper introduced that patients are generally dissatisfied with long waiting times and experience a negative effect as a result. It is further established that Queuing theory and modeling is an effective tool that can be used to make decisions on physician and staff needs for optimal performance of the hospital. This research provides suggestions to the hospital to construct the appointment system, take attention to patient flow and make scheduling of the retention to increase the effective and efficient outdoor department performance. This is a primary study that detached each variable separately. This analysis performed to ensure that the waiting time targets not met the minimum service standard of the hospital without consideration of cost. For further research, anyone extends this work with huge data and cost-effective to reduce outdoor patient waiting time.

## **Acknowledgements**

I would like to express my thankfulness to Dr. Mohammad Anwar Hossen, Associate Professor, Department of Mathematics, Comilla University and Zillur Rahman, Assistant Professor, Department of Mathematics for his scholastic help throughout the research work.

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