

Quantum Heat Flow Model for Heat Flow in Some Nanotubes

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Abstract

Using Schrodinger equation in a fractional medium a useful expression for heat flow through Nano tubes has been found. Fortunately, this equation resembles that obtained by Moran Wang et al, and Hai- Dong Wang et al. the ordinary thermal conductivity is constant. The effective thermal conductivity temperature dependent resembles that obtained for carbon Nano tubes and Boron Nitride Nano tubes. It is also finite at low temperature which also conforms with experimental data for carbon and Boron. Since Nano materials are described by quantum laws, this new model is thus more suitable for Nano tubes, as for as it is derived using quantum laws.

Keywords: Schrodinger equation; effective thermal conductivity; carbon Nano tubes; Boron Nitride Nano tubes; Alzaim Alazhari.

1. Introduction

Thermal conduction is the transfer of heat by collisions of atomic and subatomic particles within a body. The microscopically colliding particles, that include molecules, atoms and electrons, transfer kinetic and potential energy, jointly known as internal energy. Conduction takes place in solids, liquids, gases [1].

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The rate at which energy is conducted as heat between two bodies is a function of the temperature difference between the two bodies and the properties of the conductive medium through which the heat is transferred. Thermal conduction was originally called diffusion. Conduction: transfer of heat via direct contact. The thermal properties of matter need to be controlled to satisfy industrial requirements and needs. One of the most efficient tools used is nanotechnology [2]. Nanotechnology is defined as the study and use of structures of isolated tiny particles that have sizes in the range 1 nanometer and 100 nanometers in size. To give you an idea of how small that is, it would take eight hundred 100 nanometer particles side by side to match the width of a human hair. This is the most common definition of nanotechnology. Applications of nanotechnology is in solar cells, electronics, energy, environment, medicine and so, on [3,4] one of the properties of Nano materials that can be controlled by the Nano technology are thermal properties [5,6]. All nanotubes are expected to be good conductors of heat along the tube, showing property known as ballistic conductivity, but at the same time they act as good insulators in a normal direction to the its [7]. Experiment have shown that single-walled carbon nanotubes have the ability to deliver room temperature along its axis up to 3500W.m⁻¹. K⁻¹. This is compared with copper, a metal known to be a good conductor of heat, 385W.m⁻¹. K⁻¹. Since single-walled carbon Nanotubes have the ability transfer or deliver room temperature across their axis to approximately 1.52W.m⁻¹.K⁻¹, which is almost thermally conductive as soil [8,9].the temperature stability of carbon nanotubes is estimated at up to 2800°C in vacuum and up to 7500°C in air. This work is concerned with using quantum model for frictional resistive medium. The derivation of this model is exhibited in section 2. sections 3 and 4 are devoted for discussion and conclusion.

2. Quantum Heat Flow Model

Phonon mass density is related phonon mass m and number density by using the relation

$$\rho = mn \tag{1}$$

The rate of mass flow is given by [Moran wang, wang, physical letter]

$$m = \frac{q}{c^2} = \rho u \tag{2}$$

The phonon mass density is related to matter density ρ_m and specific heat c_m and temperature T through the relation

$$\rho c^2 = \rho_m c_m T \tag{3}$$

Using Einstein energy relation for kinetic energy K , one can write

$$mc^2 - m_0c^2 = k \tag{4}$$

$$nm c^2 - nm_0c^2 = nk$$

$$\rho c^2 - \rho_0c^2 = \rho_m c_m T \tag{5}$$

Thus relation (3) is modified.

According to equation (2), the heat flow rate (or heat flux) is defined by

$$q = \rho c^2 u = \rho \frac{m}{m} c^2 u \quad (6)$$

Where one assumes that phonon energy is equal to the thermal energy,

$$mC^2 = kT \quad (7)$$

$$q = \rho \frac{k}{m} Tu \quad (8)$$

Defining

$$k_0 = \frac{k}{m} \quad (9)$$

One gets

$$q = \rho k_0 Tu \quad (10)$$

Let the rate of heat flow be defined by

$$q = k_0 T \rho u \quad (11)$$

The Einstein energy relation

$$E^2 = C^2 P^2 + m^2_0 c^4 \quad (12)$$

The linear Einstein energy- momentum relation is given by

$$E = c_1 pc + c_2 m_0 c^2 \quad (13)$$

Multiplying both sides by q, one gets.

$$Eq = c_1 pcq + c_2 m_0 c^2 q \quad (14)$$

Assuming u to be constant, the time and spatial differentiation of q, gives

$$\frac{\partial q}{\partial t} = k_0 \frac{\partial T}{\partial t} \rho u + k_0 T \frac{\partial \rho}{\partial t} u \quad (15)$$

$$\frac{\partial q}{\partial x} = k_0 \frac{\partial T}{\partial x} \rho u + k_0 T \frac{\partial \rho}{\partial x} u = k_0 \rho u \frac{\partial T}{\partial x} + k_0 T u \frac{\partial \rho}{\partial x} \quad (16)$$

Consider now the expression for the wave function in a resistive medium

$$\psi = \psi_0 e^{-\frac{1}{2}\alpha px - \frac{1}{2}\beta Et}$$

thus the density of particle is given

$$\rho = \rho_0 e^{-\alpha px - \beta Et} \tag{17}$$

$$\text{Thus } \rho = |\psi|^2$$

$$\frac{\partial \rho}{\partial x} = -\alpha p \rho \qquad \frac{\partial \rho}{\partial t} = -\beta E \rho \tag{18}$$

$$\frac{\partial q}{\partial t} = K_0 \rho u \frac{\partial T}{\partial t} - \beta E \rho K_0 T u \tag{19}$$

$$\frac{\partial q}{\partial t} = k_0 \rho u \frac{\partial T}{\partial t} - \beta E q \tag{20}$$

Hence rearranging (20) yields

$$Eq = -\frac{1}{\beta} \frac{\partial q}{\partial t} + \frac{k_0 \rho u}{\beta} \frac{\partial T}{\partial t} \tag{21}$$

$$pq = -\frac{1}{\alpha} \frac{\partial q}{\partial x} + \frac{k_0 \rho u}{\alpha} \frac{\partial T}{\partial x} \tag{23}$$

Sub Eqs (21) and (23) in Eq(14) and setting q_0 is equal to zero ,one gets

$$-\frac{1}{\beta} \frac{\partial q}{\partial t} + \frac{k_0 \rho u}{\beta} \frac{\partial T}{\partial t} = -\frac{c_1 c}{\alpha} \frac{\partial q}{\partial x} - \frac{c_1 c}{\alpha} k_0 \rho u \frac{\partial T}{\partial x} + c_2 m_0 c^2 q \tag{24}$$

$$\frac{1}{\alpha} = E\tau \qquad , \frac{1}{\beta} = E\tau \tag{25}$$

Using (15)

$$-\frac{dq}{dt} + k_0 \rho u \frac{dT}{dt} = -c_1 c \frac{dq}{dx} + c_1 c k_0 \rho u \frac{dT}{dx} + c_3 q$$

$$c_3 = \frac{c_2 m_0 c^2}{Et} \tag{26}$$

Using the fact that $\frac{dT}{dt} = \frac{dT}{dx} \frac{dx}{dt} = u \frac{dT}{dx}$

One gets

$$\frac{dq}{dt} - c_1 c \frac{dq}{dx} + k_0 [c_1 c \rho u - \rho u^2] \frac{dT}{dx} + c_3 q = 0 \tag{27}$$

$$\rho u = \frac{q}{c^2} \quad u = \frac{q}{\rho_m c_m T} \quad (28)$$

Thus

$$\frac{dq}{dt} - c_1 c \frac{dq}{dx} + k_0 \left[\frac{c_1 q}{c} - \frac{q^2}{c^2 \rho_m c_m T} \right] \frac{dT}{dx} + c_3 q = 0 \quad (29)$$

$$\frac{dq}{dt} - c_1 c \frac{dq}{dx} + k_{eff} \frac{dT}{dx} + c_3 q = 0 \quad (30)$$

$$k_{eff} = k_0 q \left[\frac{c_1}{c} - \frac{q}{c^2 \rho_m c_m T} \right] \quad (31)$$

Which can be compared with [Moran, physical letter, equation (11) after] [10]

$$k_{eff} = \frac{qL}{\Delta T} \quad (32)$$

3. Discussion

The use of Schrodinger equation for frictional resistive medium is justifiable as far as heat generated inside any medium is due to the collision of the micro particles with medium particles. According to the laws of electricity the heat generated Q, is given by

$$Q = Ri^2 \quad (33)$$

Where the heat generated vanishes, when no resistive and friction exist. The Schrodinger equation in a resistive medium takes the form

$$\psi = A e^{\frac{\alpha}{2} px - \frac{\beta}{2} Et} \quad (34)$$

Thus the number of particles is given by

$$\rho = |\psi|^2 = A^2 e^{\alpha px - \beta Et} = \rho_0 e^{\alpha px - \beta Et} \quad (35)$$

It is clear that equation (20) resembles that obtained by Moran Wang and Z eng-YuanGuo. [10] It is also important to note that our model is more advance than Moran Wang and Z eng-YuanGuo model since in our model k_0 is constant where its temperature dependent in Moran Wang and Z eng-YuanGuo model

Since

Equations (10) and (31) shows that

$$K_{eff} \propto q \propto T \quad (36)$$

Thus it does not tend to infinity when

$$T \rightarrow 0 \quad (37)$$

While that of Moran tends to ∞ as ($T \rightarrow 0$), since

$$K_{eff} \sim \frac{q^2}{T^3} \sim \infty \quad (38)$$

Which is not physically acceptable as for as physical quantities are finite and since the empirical relations in Fig (1) and (2) shows that K_{eff} is finite as $T \rightarrow 0$.

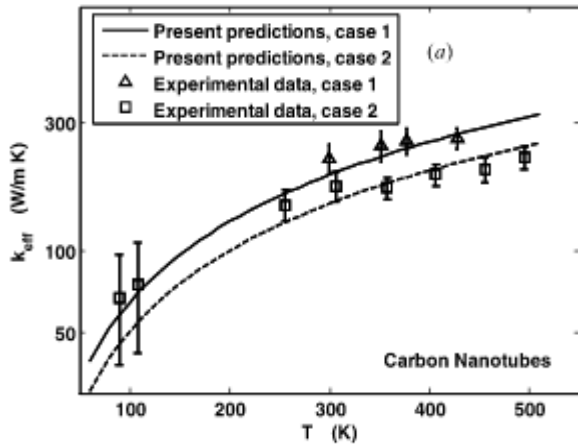


Figure 1

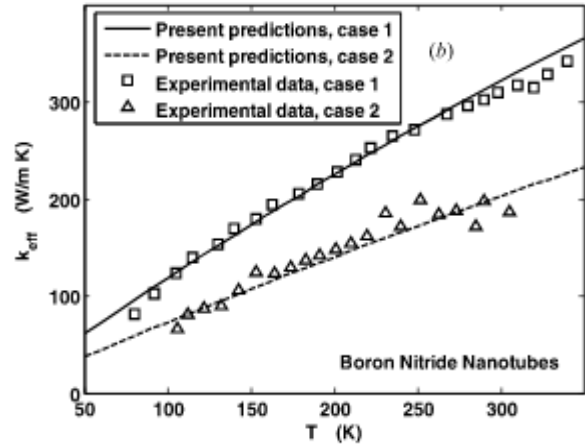


Figure 2

However in the present model $K_{eff} \rightarrow 0$ as $T \rightarrow 0$, which conforms with Figs(1) and (2). This is different from Moran model where K_{eff} tends to ∞ as $T \rightarrow 0$. It is also very interesting to note that the relation between K_{eff} and T in the present model shown in Fig(3) resembles the empirical relation found from the experimental.

It is also very important to note that the present model is more advance than Moran and Hai Dong. This is since the former was derived using quantum laws, which are suitable to describe Nano materials, while the latter was derived from classical laws.

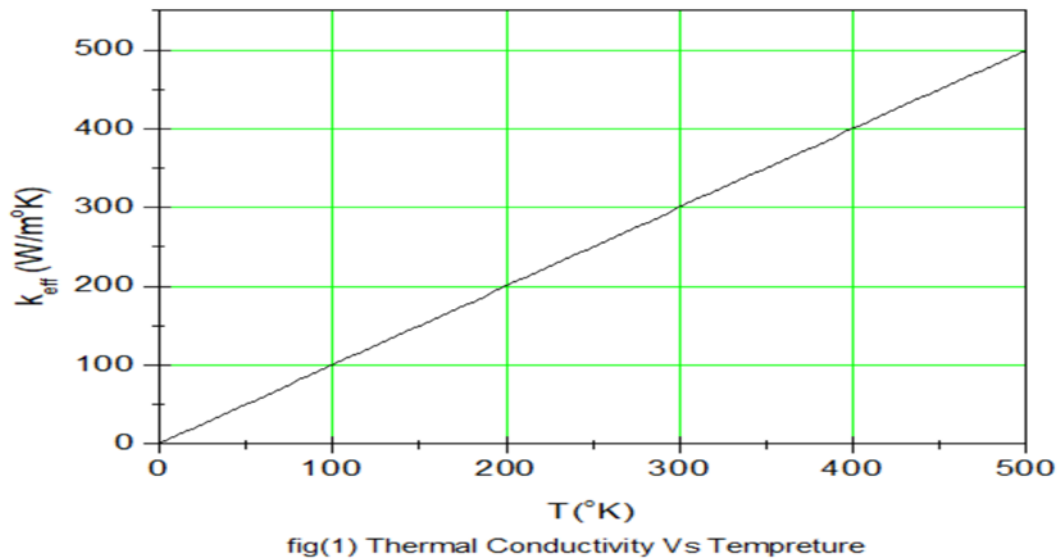


Figure 3: Thermal conductivity Vs Temperature)

4. Conclusion

A quantum model for heat flow was derived using Schrodinger equation in frictional medium. This model is suitable to describe heat flows in Nano materials and tubes as far as they are suitably described by quantum laws. The relation between effective thermal conductivity and temperature is similar to the empirical one. The effective thermal conductivity is finite at low temperatures.

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