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Alternative Multiplying Triangular Fuzzy Number and Applied in Fully Fuzzy Linear System

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Abstract

In this paper, a new concept of arithmetic fuzzy number will be introduced using the broad area concept triangular fuzzy number so that we will get the form of multiplying fuzzy number in some cases. New arithmetic concept fuzzy number will be applied to solve the fully fuzzy linear system using Gauss Seidel method and the solution obtained is a single solution.

Keywords: Arithmetic Fuzzy Number, Fully Fuzzy Linear System , Triangular Fuzzy Number.

2010 Mathematics Subject Classification: 94D05, 08A72, 15B15

1. Introduction

System linear is one part of linear algebra which is studied in mathematics. The form of the matrix equation the system linear is $\boldsymbol{Ax} = \boldsymbol{b}$ with all entries real number. Along with the development of mathematics, the system linear is not only used in real number, but variable and constant in system linear can be fuzzy number. Fuzzy in introduced by Lothfi A.Zadeh (1965). The system linear using fuzzy number consists of the fuzzy linear system of equation, the fully fuzzy linear system of equation [1, 2 and 3], and the dual fully fuzzy linear system of equation [4, 5, 6, and 7].

Many methods are to solve the fully fuzzy linear system of equation that has been discussed, using the decomposition [8, 9, and 10], using the direct method and iteration method [11, 12, and 13], Huang's method [14], using LU factorization of the coefficient matrix [4 and 6], and using cramers rules [15]. In this paper, the system linear to be discussed is the fully fuzzy linear system of equation. Dehghan and Hashemi (2006) [8] the fully fuzzy linear system of equation is a system linear with elements from the matrix and the vector is in the

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form fuzzy number. The fully fuzzy linear system of equation $\widetilde{\boldsymbol{A}} \otimes \widetilde{\boldsymbol{x}} = \widetilde{\boldsymbol{b}}$ where $\widetilde{\boldsymbol{A}}$ is matrix with fuzzy number, $\widetilde{\boldsymbol{x}}$ and $\widetilde{\boldsymbol{b}}$ are vector with fuzzy number. In this paper, new definition of positive fuzzy number and negative fuzzy number will be given in some cases using broad comparison, then by defining new fuzzy number, algebra will be constructed from fuzzy number in fully fuzzy linear system of equation. Fuzzy number to be used in this paper is triangular fuzzy number and to solve the fully fuzzy linear system of equation using the Gauss Seidel method.

2. The Basic Definition

Some of the basic definitions of fuzzy number have been in Ming Ma (2000) [16].

Definition 2.1 A fuzzy number is a fuzzy set $\tilde{a}: R \to [0,1]$ which satisfies:

1. $\tilde{a}$ is upper semicontinuous ;
2. $\tilde{a}(x) = 0$ outside some interval $[0,1]$;
3. There are real numbers a, b in $[c, d]$ for which,
   (i) $\tilde{a}(x)$ is monotonic increasing on $[c, a]$;
   (ii) $\tilde{a}(x)$ is monotonic decreasing on $[b, d]$;
   (iii) $\tilde{a}(x) = 1$, for $a \le x \le b$.

An equivalent parametric definition is given in Friedman (1998) [17].

Definition 2.2 A fuzzy number $\tilde{a}$ is a pair $(\underline{a}(r), \overline{a}(r))$ of functions $\underline{a}(r), \overline{a}(r); 0 \le r \le 1$ which satisfy the following requirements:

1. $\underline{a}(r)$ is a bounded left continuous nondecreasing functions over [0,1] ;
2. $\overline{a}(r)$ is a bounded left continuous nonincreasing functions over [0,1];
3. $\underline{a}(r) \le \overline{a}(r), 0 \le r \le 1$.

3. Triangular Fuzzy Number

In this section, we discuss the concept triangular fuzzy number, new definitions for positive triangular fuzzy number and negative triangular fuzzy number, and arithmetic algebraic operations consisting of addition, subtraction, scalar product, multiplication of two fuzzy number and inverse fuzzy number.

In this paper, we write a fuzzy number in the form of $\tilde{a} = (a, \alpha, \beta)$, where $\boldsymbol{a}$ is the center, $\boldsymbol{\alpha}$ is the left width, and $\boldsymbol{\beta}$ is the right width. For arbitrary fuzzy number $\tilde{a} = (a, \alpha, \beta)$, the membership function is of the form:

$$\mu_{\tilde{a}}(x) = \begin{cases} 1 - \dfrac{a - x}{\alpha}, & a - \alpha \le x \le a, \\ 1 - \dfrac{x - a}{\beta}, & a \le x \le a + \beta, \\ 0, & otherwise. \end{cases}$$

On the other hand, a parametric fuzzy number $\tilde{a} = [\underline{a}(r), \overline{a}(r)]$ can be represented as:

$$\underline{a}(r) = a - (1 - r)\alpha \text{ and } \overline{a}(r) = a + (1 - r)\beta$$

3.1. Triangular Fuzzy Number Positive and Negative

Triangular fuzzy number $\tilde{a} = (a, \alpha, \beta)$ is said to be positive or negative:

1. If $a - \alpha \geq 0$, then $\tilde{a}$ is said to be positive, and if $a + \beta \leq 0$, then $\tilde{a}$ is said to be negative. Seen in Figure 1:

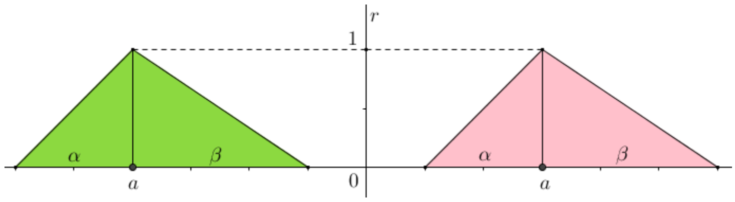

Figure 1: Triangular Fuzzy Number Potitif and Negative

2. If $a > 0$ and $a - \alpha < 0$. Seen in Figure 2:

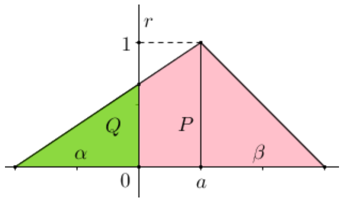

Figure 2: Triangular Fuzzy Number $a > 0$

$\tilde{a} = (a, \alpha, \beta)$ is said to be positive if $P > Q$, and said to be negative if $P < Q$ or can be expressed in form $P = \frac{\beta}{2} + a + \frac{a^2}{2\beta} > \frac{\alpha}{2} - a - \frac{a^2}{2\beta} = Q$ positive, and $P = \frac{\beta}{2} + a + \frac{a^2}{2\beta} < \frac{\alpha}{2} - a - \frac{a^2}{2\beta} = Q$ negative.

3. If $a < 0$ and $a + \beta > 0$. Seen in Figure 3:

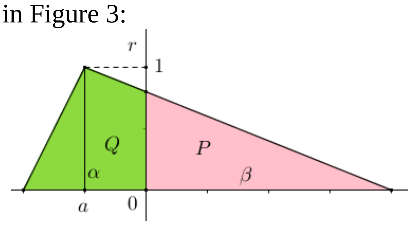

Figure 3: Triangular Fuzzy Number $a < 0$

$\tilde{a} = (a, \alpha, \beta)$ is said to be positive if $P > Q$, and said to be negative if $P < Q$ or can be expressed in form $P = \frac{\beta}{2} + a - \frac{a^2}{2\alpha} > \frac{\alpha}{2} - a + \frac{a^2}{2\alpha} = Q$ positive, and $P = \frac{\beta}{2} + a - \frac{a^2}{2\alpha} < \frac{\alpha}{2} - a + \frac{a^2}{2\alpha} = Q$ negative.

4. If $a = 0$. Seen in Figure 4:

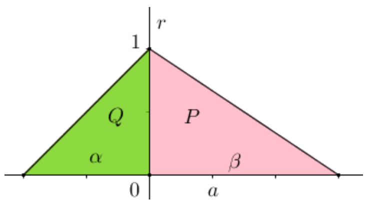

Figure 4: Triangular Fuzzy Number $a = 0$

$\tilde{a} = (a, \alpha, \beta)$ is said to be positive if $P > Q$, and is said to be negative if $P < Q$ or can be expressed in form $P = \beta > \alpha = Q$ Positive, and $P = \beta < \alpha = Q$ negative.

3.2. New Arithmetic Triangular Fuzzy Number

Arithmetic algebraic operations will be given for triangular fuzzy number. For $\tilde{a} = (a, \alpha, \beta)$ and $\tilde{b} = (b, \gamma, \delta)$, then the parametric forms are as follows:

$\tilde{a} = [\underline{a}(r), \overline{a}(r)] = [a - (1-r)\alpha, a + (1-r)\beta]$

$\tilde{b} = [\underline{b}(r), \overline{b}(r)] = [b - (1-r)\gamma, b + (1-r)\delta]$

Then for the arithmetic algebraic process the two triangular fuzzy numbers are as follows:

a. Addition

$$\begin{aligned} \tilde{a} \oplus \tilde{b} &= [\underline{a}(r) + \underline{b}(r), \overline{a}(r) + \overline{b}(r)] \\ &= [(a - (1-r)\alpha) + (b - (1-r)\gamma), (a + (1-r)\beta) + (b + (1-r)\delta)] \\ &= [(a+b) - (1-r)(\alpha + \gamma), (a+b) + (1-r)(\beta + \delta)] \end{aligned}$$

Transforming back into the triangular form, we have:

$$\tilde{a} \oplus \tilde{b} = (a + b, \alpha + \gamma, \beta + \delta)$$

b. Subtraction

$$\begin{aligned} \tilde{a} \ominus \tilde{b} &= [\underline{a}(r) - \overline{b}(r), \overline{a}(r) - \underline{b}(r)] \\ &= [(a - (1-r)\alpha) - (b + (1-r)\delta), (a + (1-r)\beta) - (b - (1-r)\gamma)] \\ &= [(a-b) - (1-r)(\alpha + \delta), (a-b) + (1-r)(\beta + \gamma)] \end{aligned}$$

Transforming back into the triangular form, we have:

$$\tilde{a} \ominus \tilde{b} = (a - b, \alpha + \delta, \beta + \gamma)$$

c. Scalar product as $\lambda \otimes \tilde{a} = \begin{cases} (\lambda a, \lambda\alpha, \lambda\beta) & \lambda \geq 0, \\ (\lambda a, -\lambda\beta, -\lambda\alpha) & \lambda < 0. \end{cases}$

d. Multiplication

If $\tilde{a} = [\underline{a}(r), \overline{a}(r)]$ and $\tilde{b} = [\underline{b}(r), \overline{b}(r)]$ are two positive fuzzy numbers, then $\tilde{c} = \tilde{a} \otimes \tilde{b} = [\underline{c}(r), \overline{c}(r)]$ for every $r \in [0,1]$. The following is given some cases for triangular fuzzy number to multiplication operations.

(i) If $\tilde{a}$ is positive and $\tilde{b}$ is positive, then:

$$\begin{cases} \underline{c}(r) = \underline{a}(r)\underline{b}(1) + \underline{a}(1)\underline{b}(r) - \underline{a}(1)\underline{b}(1) \\ \overline{c}(r) = \overline{a}(r)\overline{b}(1) + \overline{a}(1)\overline{b}(r) - \overline{a}(1)\overline{b}(1) \end{cases} \tag{1}$$

From equation (1) :

$$[\underline{c}(r), \overline{c}(r)] = [(a - (1-r)\alpha)b + a(b - (1-r)\gamma) - ab, (a + (1-r)\beta)b + a(b + (1-r)\delta) - ab]$$

we have $\tilde{c} = [\underline{c}(r), \overline{c}(r)] = [ab - (1-r)(a\gamma + b\alpha), ab + (1-r)(a\delta + b\beta)]$ *(2)*

If we let $\tilde{c} = (c, \xi, \psi)$, then the parametric fuzzy number is of the form:

$$\tilde{c} = [c - (1-r)\xi, c + (1-r)\psi] \tag{3}$$

From equation (2) and (3) we have: $\xi = a\gamma + b\alpha \quad \psi = a\delta + b\beta$

So multiplication $\tilde{a}$ positive and $\tilde{b}$ positive can be written as $\tilde{c} = \tilde{a} \otimes \tilde{b} = (ab, a\gamma + b\alpha, a\delta + b\beta)$

(ii) If $\tilde{a}$ is positive and $\tilde{b}$ is negative, then:

$$\begin{cases} \underline{c}(r) = \overline{a}(r)\underline{b}(1) + \overline{a}(1)\underline{b}(r) - \overline{a}(1)\underline{b}(1) \\ \overline{c}(r) = \underline{a}(r)\overline{b}(1) + \underline{a}(1)\overline{b}(r) - \underline{a}(1)\overline{b}(1) \end{cases} \tag{4}$$

From equation (4) :

$$\left[\underline{c}(r),\overline{c}(r)\right] = [(a+(1-r)\beta)b + a(b-(1-r)\gamma) - ab, (a-(1-r)\alpha)b + a(b+(1-r)\delta) - ab]$$

we have $\tilde{c} = \left[\underline{c}(r),\overline{c}(r)\right] = [ab-(1-r)(a\gamma - b\beta), ab+(1-r)(a\delta - b\alpha)]$ *(5)*

From equation (5) and (3) we have: $\xi = a\gamma - b\beta \quad \psi = a\delta - b\alpha$

So multiplication $\tilde{a}$ positive and $\tilde{b}$ negative can be written as $\tilde{c} = \tilde{a} \otimes \tilde{b} = (ab, a\gamma - b\beta, a\delta - b\alpha)$

(iii) If $\tilde{a}$ is negative and $\tilde{b}$ is positive,then:

$$\begin{cases} \underline{c}(r) = \underline{a}(r)\overline{b}(1) + \underline{a}(1)\overline{b}(r) - \underline{a}(1)\overline{b}(1) \\ \overline{c}(r) = \overline{a}(r)\underline{b}(1) + \overline{a}(1)\underline{b}(r) - \overline{a}(1)\underline{b}(1) \end{cases} \tag{6}$$

From equation (6) :

$$\left[\underline{c}(r),\overline{c}(r)\right] = [(a-(1-r)\alpha)b + a(b+(1-r)\delta) - ab, (a+(1-r)\beta)b + a(b-(1-r)\gamma) - ab]$$

we have $\tilde{c} = \left[\underline{c}(r),\overline{c}(r)\right] = [ab-(1-r)(-a\delta + b\alpha), ab+(1-r)(-a\gamma + b\beta)]$ *(7)*

From equation (7) and (3) we have: $\xi = -a\delta + b\alpha \quad \psi = -a\gamma + b\beta$

So multiplication $\tilde{a}$ negative and $\tilde{b}$ positive can be written as $\tilde{c} = \tilde{a} \otimes \tilde{b} = (ab, -a\delta + b\alpha, -a\gamma + b\beta)$

(iv) If $\tilde{a}$ is negative and $\tilde{b}$ is negative, then:

$$\begin{cases} \underline{c}(r) = \overline{a}(r)\overline{b}(1) + \overline{a}(1)\overline{b}(r) - \overline{a}(1)\overline{b}(1) \\ \overline{c}(r) = \underline{a}(r)\underline{b}(1) + \underline{a}(1)\underline{b}(r) - \underline{a}(1)\underline{b}(1) \end{cases} \tag{8}$$

From equation (8) :

$$\left[\underline{c}(r),\overline{c}(r)\right] = [(a+(1-r)\beta)b + a(b+(1-r)\delta) - ab, (a-(1-r)\alpha)b + a(b-(1-r)\gamma) - ab]$$

We have $\tilde{c} = \left[\underline{c}(r),\overline{c}(r)\right] = [ab-(1-r)(-a\delta - b\beta), ab+(1-r)(-a\gamma - b\alpha)]$ *(9)*

From equation (9) and (3) we have: $\xi = -a\delta - b\beta \quad \psi = -a\gamma - b\alpha$

So multiplication $\tilde{a}$ negative and $\tilde{b}$ negative can be written as $\tilde{c} = \tilde{a} \otimes \tilde{b} = (ab, -(a\delta + b\beta), -(a\gamma + b\alpha)$

e. Inverse

The identity element for triangular fuzzy number is:

$$\tilde{I} = (1,0,0)$$

where $\tilde{I} = (1,0,0)$ is positive. Let two fuzzy number $\tilde{a} = (a,\alpha,\beta)$ and $\tilde{b} = (b,\gamma,\delta)$ have inverse:

$$\tilde{b} = \frac{1}{\tilde{a}}$$

will be indicated $\tilde{a} \otimes \tilde{b} = (1,0,0)$. Fuzzy number $\tilde{a} = (a,\alpha,\beta)$ the condition for having inverse is $a \neq 0$. Therefore, inverse for triangular fuzzy number consist of two cases, as follows:

(i) If $\tilde{a} > 0$ and $\tilde{b} > 0$, $\tilde{a} \otimes \tilde{b} = (a,\alpha,\beta) \otimes (b,\gamma,\delta) = (ab, a\gamma + b\alpha, a\delta + b\beta) = (1,0,0)$

(ii) If $\tilde{a} < 0$ and $\tilde{b} < 0$, $\tilde{a} \otimes \tilde{b} = (a,\alpha,\beta) \otimes (b,\gamma,\delta) = (ab, -(a\delta + b\beta), -(a\gamma + b\alpha) = (1,0,0)$

So the inverse for triangular fuzzy number is:

$$\tilde{b} = \frac{1}{\tilde{a}} = \left(\frac{1}{a}, \frac{\beta}{a^2}, \frac{\alpha}{a^2}\right)$$

4. Solving Fully Fuzzy Linear System of Equation

Let the fully fuzzy linear system be as follows:

$$(\tilde{a}_{11}\otimes \tilde{x}_1) \oplus (\tilde{a}_{12}\otimes \tilde{x}_2) \oplus \dots \oplus (\tilde{a}_{1n}\otimes \tilde{x}_n) = \tilde{b}_1$$

$$(\tilde{a}_{21}\otimes \tilde{x}_1) \oplus (\tilde{a}_{22}\otimes \tilde{x}_2) \oplus \dots \oplus (\tilde{a}_{2n}\otimes \tilde{x}_n) = \tilde{b}_2$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$(\tilde{a}_{n1}\otimes \tilde{x}_1) \oplus (\tilde{a}_{n2}\otimes \tilde{x}_2) \oplus \dots \oplus (\tilde{a}_{nn}\otimes \tilde{x}_n) = \tilde{b}_n$$

The matrix form the fully fuzzy linear system of equation is $\widetilde{\boldsymbol{A}} \otimes \widetilde{\boldsymbol{x}} = \widetilde{\boldsymbol{b}}$, where $\widetilde{\boldsymbol{A}} = (\widetilde{\boldsymbol{a}}_{\boldsymbol{ij}}) = (a_{ij}, \alpha_{ij}, \beta_{ij})$ is a fuzzy matrix $n \times n$, $\widetilde{\boldsymbol{x}} = (\tilde{x}_1, \dots, \tilde{x}_n)$ and $\widetilde{\boldsymbol{b}} = (\tilde{b}_1, \dots, \tilde{b}_n)$ are fuzzy vectors $n \times 1$. A matrix $\widetilde{\boldsymbol{A}} = (\widetilde{\boldsymbol{a}}_{\boldsymbol{ij}})$ is called a fuzzy matrix, if each element of $\widetilde{\boldsymbol{A}}$ is a fuzzy number. We may represent fuzzy matrix $\widetilde{\boldsymbol{A}} = \left(\widetilde{\boldsymbol{a}}_{\boldsymbol{ij}}\right)_{n\times n}$ that $\left(\widetilde{\boldsymbol{a}}_{\boldsymbol{ij}}\right) = \left(a_{ij}, \alpha_{ij}, \beta_{ij}\right)$ with new notation $\widetilde{\boldsymbol{A}} = (A, M, N)$, where $A = \left(a_{ij}\right)$, $M = \left(\alpha_{ij}\right)$, and $N = \left(\beta_{ij}\right)$ are three $n\ x\ n$ crips matrices.

Next, to get a solution the fully fuzzy linear system of equation $\widetilde{\boldsymbol{A}} \otimes \widetilde{\boldsymbol{x}} = \widetilde{\boldsymbol{b}}$, where $\widetilde{\boldsymbol{A}} = (A, M, N)$, $\widetilde{\boldsymbol{x}} = (x, y, z)$, and $\widetilde{\boldsymbol{b}} = (b, g, h)$ so that it is obtained:

$$(A, M, N) \otimes (x, y, z) = (b, g, h)$$

by using algebra multiplication of two fuzzy number for $\widetilde{\boldsymbol{A}} > \boldsymbol{0}$, $\widetilde{\boldsymbol{b}} > \boldsymbol{0}$, and $\widetilde{\boldsymbol{x}} > \boldsymbol{0}$ the formula that applied is $(Ax, Ay + Mx, Az + Nx) = (b, g, h)$. Therefore, it can be concluded that:

$$Ax = b$$

$$Ay + Mx = g \tag{11}$$

$$Az + Nx = h$$

Furthermore, proving equation (11) satisfies strictly diagonal dominant with the following formula:

$$|a_{ii}| > \sum_{j=i, j\neq i}^{n} \left|a_{ij}\right| \quad i = 1,2, \dots, n$$

Then do the iteration process using the intial value. Gauss seidel iteration stop if tolerance has been achieved:

$$\left|\frac{\tilde{x}_i^{(j)} \ominus \tilde{x}_i^{(j-i)}}{\tilde{x}_i^{(j)}}\right| \tag{12}$$

Example: The fully fuzzy system linear of equation with positive and positive multiplication operations as follows:

$$(19,1,1)\otimes(x_1, y_1, z_1)\oplus(12,1.5,1.5)\otimes(x_2, y_2, z_2)\oplus(6,0.5,0.2)\otimes(x_3, y_3, z_3) = (1897,427.7,536.2)$$

$$(2,0.1,0.1)\otimes(x_1, y_1, z_1)\oplus(4,0.1,0.4)\otimes(x_2, y_2, z_2)\oplus(1.5,0.2,0.2)\otimes(x_3, y_3, z_3) = (434.5,76.2,109.3)$$

$$(2,0.1,0.2)\otimes(x_1, y_1, z_1)\oplus(2,0.1,0.3)\otimes(x_2, y_2, z_2)\oplus(4.5,0.1,0.1)\otimes(x_3, y_3, z_3) = (535.5,88.3,131.9)$$

Solution: The steps to solve the fully fuzzy linear system of equation as follow:

Change the form of equations into the matrix $\widetilde{\boldsymbol{A}} = (A, M, N)$ and $\widetilde{\boldsymbol{b}} = (b, g, h)$ are:

$$A = \begin{bmatrix} 19 & 12 & 6 \\ 2 & 4 & 1.5 \\ 2 & 2 & 4.5 \end{bmatrix} \qquad M = \begin{bmatrix} 1 & 1.5 & 0.5 \\ 0.1 & 0.1 & 0.2 \\ 0.1 & 0.1 & 0.1 \end{bmatrix} \qquad N = \begin{bmatrix} 1 & 1.5 & 0.2 \\ 0.1 & 0.4 & 0.2 \\ 0.2 & 0.3 & 0.1 \end{bmatrix}$$

$$b = \begin{bmatrix} 1897 \\ 434.5 \\ 535.5 \end{bmatrix} \qquad g = \begin{bmatrix} 427.7 \\ 76.2 \\ 88.3 \end{bmatrix} \qquad h = \begin{bmatrix} 536.2 \\ 109.3 \\ 131.9 \end{bmatrix}$$

Because $\widetilde{\boldsymbol{A}} > \mathbf{0}$, $\widetilde{\boldsymbol{b}} > \mathbf{0}$, and $\widetilde{\boldsymbol{x}} > \mathbf{0}$, the formula that applied in equation (11). Next, change the matrix into system linear of equation as follows:

$Ax = b$

$$\begin{bmatrix} 19 & 12 & 6 \\ 2 & 4 & 1.5 \\ 2 & 2 & 4.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1897 \\ 434.5 \\ 535.5 \end{bmatrix} \tag{13}$$

$Mx + Ay = g$

$$\begin{bmatrix} 1 & 1.5 & 0.5 \\ 0.1 & 0.1 & 0.2 \\ 0.1 & 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 19 & 12 & 6 \\ 2 & 4 & 1.5 \\ 2 & 2 & 4.5 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 427.7 \\ 76.2 \\ 88.3 \end{bmatrix} \tag{14}$$

$Az + Nx = h$

$$\begin{bmatrix} 19 & 12 & 6 \\ 2 & 4 & 1.5 \\ 2 & 2 & 4.5 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 1 & 1.5 & 0.2 \\ 0.1 & 0.4 & 0.2 \\ 0.2 & 0.3 & 0.1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 536.2 \\ 109.3 \\ 131.9 \end{bmatrix} \tag{15}$$

For equation (13), the equation obtained is as follows:

$19x_1 + 12x_2 + 6x_3 = 1897$

$2x_1 + 4x_2 + 1.5x_3 = 434.5$

$2x_1 + 2x_2 + 4.5x_3 = 535.5$

To get the values x_1, x_2 and x_3 it must first be proven that equation (13) strictly diagonally dominant in the following:

$|a_{11}| \geq |a_{12}| + |a_{13}| \rightarrow |19| \geq |12| + |6| = 19 \geq 18$

$|a_{22}| \geq |a_{21}| + |a_{23}| \rightarrow |4| \geq |2| + |1.5| = 4 \geq 3.5$

$|a_{33}| \geq |a_{31}| + |a_{32}| \rightarrow |4.5| \geq |2| + |2| = 4.5 \geq 4$

All equations (13) are proven to be diagonally dominant, so obtained:

$x_1 = \frac{1}{19}(1897 - 12x_2 - 6x_3)$

$x_2 = \frac{1}{4}(434.5 - 2x_1 - 1.5x_3)$

$x_3 = \frac{1}{4.5}(535.5 - 2x_1 - 2x_2)$

Then the iteration process starts with the intial value $(0, 0, 0)$, which is as follows:

First iteration

$x_1^{(1)} = \frac{1}{19}(1897 - 12(0) - 6(0)) = \frac{1}{19}(1897) = 99.8421$

$x_2^{(1)} = \frac{1}{4}(434.5 - 2(99.8421) - 1.5(0)) = \frac{1}{4}(234.8158) = 58.7039$

$x_3^{(1)} = \frac{1}{4.5}(535.5 - 2(99.8421) - 2(58.7039)) = \frac{1}{4.5}(218.408) = 48.5351$

Second iteration

$x_1^{(2)} = \frac{1}{19}(1897 - 12(58.7039) - 6(48.5351)) = \frac{1}{19}(901.3426) = 47.4390$

$x_2^{(2)} = \frac{1}{4}(434.5 - 2(47.4390) - 1.5(48.5351) = \frac{1}{4}(266.8194) = 66.7048$

$x_3{}^{(2)} = \frac{1}{4.5}$(535.5 – 2(47.4390) – 2(66.7048)) = $\frac{1}{4.5}$(307.2124) = 68.2694

Based on equation (12), the iteration process until 10th iteration is obtained the value x_1 = 37.0021, x_2 = 61.9987, x_3 = 74.9996.

For equation (14), the equation obtained is as follows:

$x_1 + 1.5x_2 + 0.5x_3 + 19y_1 + 12y_2 + 6y_3$ = 427.7

$0.1\,x_1 + 0.1x_2 + 0.2x_3 + 2y_1 + 4y_2 + 1.5y_3$ = 76.2

$0.1\,x_1 + 0.1x_2 + 0.1x_3 + 2y_1 + 2y_2 + 4.5y_3$ = 88.3

Because the value x_1, x_2, x_3 has been obtained, substitution value is put into equation (14) so that the new equation is obtained as follows:

$19y_1 + 12y_2 + 6y_3$ = 260.2

$2y_1 + 4y_2 + 1.5y_3$ = 51.3

$2y_1 + 2y_2 + 4.5y_3$ = 70.9

To get the values y_1, y_2 and y_3, it must first be proven that equation (14) strictly diagonally dominant in the following:

$|a_{11}| \geq |a_{12}| + |a_{13}| \rightarrow |19| \geq |12| + |6|$ = 19 ≥ 18

$|a_{22}| \geq |a_{21}| + |a_{23}| \rightarrow |4| \geq |2| + |1.5|$ = 4 ≥ 3.5

$|a_{33}| \geq |a_{31}| + |a_{32}| \rightarrow |4.5| \geq |2| + |2|$ = 4.5 ≥ 4

All equations (14) are proven to be diagonally dominant, so obtained:

$y_1 = \frac{1}{19}(260.2 - 12y_2 - 6y_3)$

$y_2 = \frac{1}{4}(51.3 - 2y_1 - 1.5y_3)$

$y_3 = \frac{1}{4.5}(70.9 - 2y_1 - 2y_2)$

Then the iteration process starts with the intial value $(0, 0, 0)$, which is as follows:

First iteration

$y_1{}^{(1)} = \frac{1}{19}$(260.2 – 12(0) – 6(0)) = $\frac{1}{19}$(260.2) = 13.6947

$y_2{}^{(1)} = \frac{1}{4}$(51.3 – 2(13.6947) – 1.5(0)) = $\frac{1}{4}$(23.9106) = 5.9776

$y_3{}^{(1)} = \frac{1}{4.5}$(70.9 – 2(13.6947) – 2(5.9776)) = $\frac{1}{4.5}(31.5554) = 7.0123$

Second iteration

$y_1{}^{(2)} = \frac{1}{19}$(260.2 – 12(5.9776) – 6(7.0123)) = $\frac{1}{19}$(146.395) = 7.705

$y_2{}^{(2)} = \frac{1}{4}$(51.3 – 2(7.705) – 1.5(7.0123)) = $\frac{1}{4}$(25.3716) = 6.3429

$y_3^{(2)} = \frac{1}{4.5}(70.9 - 2(7.705) - 2(6.3429)) = \frac{1}{4.5}(42.8042) = 9.5120$

Based on equation (12), the iteration process until 10th iteration is obtained the value y_1 = 7.00029, y_2 = 5.49987, y_3 = 10.1998.

For equation (15), the equation obtained is as follows:

$19z_1 + 12z_2 + 6z_3 + x_1 + 1.5x_2 + 0.2x_3 = 536.2$

$2z_1 + 4z_2 + 1.5z_3 + 0.1x_1 + 0.4x_2 + 0.2x_3 = 109.3$

$2z_1 + 2z_2 + 4.5z_3 + 0.2x_1 + 0.3x_2 + 0.1x_3 = 131.9$

Because the value of x_1, x_2, x_3 has been obtained, substitution value is put into equation (15) so that the new equation is obtained as follows:

$19z_1 + 12z_2 + 6z_3 = 391.2$

$2z_1 + 4z_2 + 1.5z_3 = 65.8$

$2z_1 + 2z_2 + 4.5z_3 = 98.4$

To get the values z_1, z_2 and z_3 , it must first be proven that equation (15) strictly diagonally dominant in the following:

$|a_{11}| \geq |a_{12}| + |a_{13}| \rightarrow |19| \geq |12| + |6| = 19 \geq 18$

$|a_{22}| \geq |a_{21}| + |a_{23}| \rightarrow |4| \geq |2| + |1.5| = 4 \geq 3.5$

$|a_{33}| \geq |a_{31}| + |a_{32}| \rightarrow |4.5| \geq |2| + |2| = 4.5 \geq 4$

All equations (15) are proven to be diagonally dominant, so obtained:

$z_1 = \frac{1}{19}(391.2 - 12z_1 - 6z_3)$

$z_2 = \frac{1}{4}(65.8 - 2z_1 - 1.5z_3)$

$z_3 = \frac{1}{4.5}(98.4 - 2z_1 - 2z_2)$

Then the iteration process starts with the intial value $(0, 0, 0)$, which is as follows:

First iteration

$z_1^{(1)} = \frac{1}{19}(391.2 - 12(0) - 6(0)) = \frac{1}{19}(391.2) = 20.5895$

$z_2^{(1)} = \frac{1}{4}(65.8 - 2(20.5895) - 1.5(0)) = \frac{1}{4}(24.621) = 6.1553$

$z_3^{(1)} = \frac{1}{4.5}(98.4 - 2(20.5895) - 2(6.1553)) = \frac{1}{4.5}(44.9104) = 9.9801$

Second iteration

$z_1^{(2)} = \frac{1}{19}(391.2 - 12(6.1553) - 6(9.9801)) = \frac{1}{19}(257.4558) = 13.5503$

$z_2^{(2)} = \frac{1}{4}(65.8 - 2(13.5503) - 1.5(9.9801)) = \frac{1}{4}(23.7292) = 5.9323$

$z_3^{(2)} = \frac{1}{4.5}(98.4 - 2(13.5503) - 2(5.9323)) = \frac{1}{4.5}(59.4348) = 13.2077$

Based on equation (12), the iteration process until 13th iteration is obtained the value z_1 = 13.3016, z_2 = 4.5794, z_3 = 13.9195.

5. Conclusion

In this paper, it can be concluded that the fully fuzzy linear system of equation $\widetilde{\boldsymbol{A}} \otimes \widetilde{\boldsymbol{x}} = \widetilde{\boldsymbol{b}}$ solved by changing it into $(A, M, N) \otimes (x, y, z) = (b, g, h)$, so we will get three system linear of equation. Based on the results from the example, the value obtained with relatively small errors are as follow:

$$\tilde{x} = \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{pmatrix} = \begin{pmatrix} 37,7,13.4 \\ 62,5.5,4.6 \\ 75,10.2,14 \end{pmatrix}$$

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