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Dynamics of Gully-Formation by Considering the Wave Motion of Flow

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Abstract

The dynamics of the formation of ravines (gullies) by taking the wave motion of flow into account. The article considers the question of impact of the wave motion of flow on the erosion intensity in the ravines. The technique to predict the occurrence of waves on a free surface of alluvium-carrying flows is proposed. Wave motion stimulates the intensity of the soil and ground washout. The influence of wave formation on the process of the erosion intensity is assessed by using the correction factor for the average flow velocity in the clear section.

Keywords: stimulation of the intensity of erosion processes; wave formation.

1. Introduction

The main role in the formation of ravines is played by surface flows. At the beginning of the flow, there develops deep intense erosion. The profile of the ravine is a V-shaped one at the beginning. Deep erosion slows down and there occurs a lateral erosion of the wandering flow [1]. The transverse profile takes the form of a trough and the depth of flow decreases. The reduced depth loses its original steady shape, and undulation starts.

With the wave motion, the motion of flows starts to intensely capture and transport different-size solid particles. Below are the proposed methods to predict the occurrence of waves on a free stream surface both, in the water and in the sedimentary flows, which support an intense washout of soil and ground in the ravines.

The studies have confirmed that the impact of the wave-formation on the intensity of the erosion processes in ravines should be taken into account in the available calculated dependencies using correction factor $V_в = 1{,}5V$ for the average flow velocity in the clear section [2, 3].

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2. Materials and methods

A number of theoretical and experimental studies are devoted to the problems of brook (gully) soil erosion. Most of the papers deal with the issues of uniform motion of clear shallow water, i.e. the flow not containing sediment suspension and not allowing the occurrence of the waves on the free surface of a uniformly moving stream. In shallow streams, creeping waves are often observed during heavy rains along the inclined sections of streets, even with a slight surface roughness.

The equations of the dynamics of a drift-carrying flow for unsteady motion are as follows [2, 3, 4]:

$$\frac{1}{g}\frac{\partial v}{\partial t}+\frac{\alpha v}{g}\frac{\partial v}{\partial x}-\frac{(\alpha-1)}{g}\frac{v}{\omega}\cdot\frac{\partial \omega}{\partial t}+T\frac{\partial H}{\partial t}-i_0+I_{mp}=0 \qquad (1)$$

Continuity equation:

$$\frac{\partial Q}{\partial x}+\frac{\partial \omega}{\partial t}=0 \quad , \qquad (2)$$

where: V is the average mixture velocity in the clear section; ω is the area of the clear section of the mixture; g is the acceleration of gravity; H is the maximum depth of mixture; α is a complete correction of the amount of movement, which considers the uneven distribution of the averaged velocities and pulsating head over the flow section; i_0 is the gradient of the brook (ravine) bottom; Q is the consumption of the mixture; I_{mp} is the sum of all dissipative members representing the gradient of the hydraulic resistance of the mixture;

$$T=\frac{w\omega+\sigma K_2 B v\left(S_{cp}-S_0\right)}{w\omega\left(1+\sigma S_{cp}\right)} \qquad (3)$$

so-called dimensionless parameter of the hydraulics of the suspension flows; $\sigma=\frac{\rho^x-\rho}{\rho}$ is the relative density; ρ^x, ρ are the density of the suspended matter and water, respectively; B is the typical width of the mixture; w is the average hydraulic particle size of the suspension in the mixture; v is kinematic coefficient of viscosity of the mixture; $K_2=0{,}6$ is the experimental coefficient considering the difference between the coefficients of the turbulent exchange of the carrying and carried phases.

It is easy to notice that if we consider the steady state of motion when $S_{cp}=S_0=0$ and neglect the pulsating component of the complete correction factor, dependence (1) coincides with the equation of Prof. G.A. Petrov for water flow, while when $\alpha=1$, the equation of Prof. I.I. Konovalov is gained [see 4].

We will try to solve the problem using the method of "small" perturbations.

If Chez formula is suitable for the stable unperturbed state of a uniform turbulent sediment-carrying flow:

$$i = I_{ap} = \frac{V^2}{C^2 R} \quad , \tag{4}$$

then the perturbed state of such a flow will be described by equations (1) and (2), while supposing that $V = V_0 + u$; $H = H_0 + h$; $\omega = \omega_0 + B_0 h$, where the index "0" means the attributing the marked values to a uniform motion, and « u » and « h »" are the velocity and height of the disturbance wave, respectively, which are so small that their derivatives and squares can be neglected and, following [2, 3, 4] , we obtain the basic differential equation of the perturbed state of a sediment-carrying turbulent flow as follows:

$$\frac{\partial^2 h}{\partial t^2} + 2\alpha V \frac{\partial^2 h}{\partial x \partial t}\left[\alpha v^2 - T_0 \frac{g\omega_0}{\mathrm{B}_0}\right]\frac{\partial^2 h}{\partial x^2} + \frac{2i_0 g}{v_0} \cdot \frac{\partial h}{\partial t} + \frac{i_0 \chi g \omega_0}{H_0 B_0} \frac{\partial h}{\partial x} = 0 \quad , \tag{5}$$

where χ is the hydraulic indicator of the riverbed according to B.A. Bakhterev [5] for flow module: $\frac{K}{K_0} = \left(\frac{H}{H_0}\right)^{\frac{\chi}{2}}$

If accepting that $\alpha = 1$, $S_{cp} = S_0 = 0$, then equation (5) becomes an equation of the theory of small amplitude for water flow without suspension.

In order to establish the criterion of the loss of the stability of the initial uniform motion and appearance of waves on a free surface, when the sediment-carrying flow moves through the streams and ravines, we will obtain a particular solution of a linear differential equation of a disturbed state of a sediment-carrying flow as a simple oscillation with K_1 frequency corresponding to the distribution of (wave) disturbance along positive value x (i.e. in the direction of translational flow):

$$h = f(x)\cos K_1 t \quad , \tag{6}$$

where: $f(x)$ is some function depending only on x .

By using Euler formula, this equation can be presented in a complex form convenient for further transformations:

$$h = f(x)e^{ik_1 t} \quad , \tag{7}$$

where: $f(x)$ is some function with real and imaginary parts, depending on x .

By differentiating (7) and accepting that $i=\sqrt{-1}$; $i^2=-1$,we gain:

$$\frac{\partial^2 h}{\partial t^2}=-f_1(x)i^2K_1^2e^{ik_1t} \quad , \tag{8}$$

By substituting the corresponding values in the differential equation of the perturbed state of the sediment-carrying flow (5), after reducing by e^{ik_1t} and grouping the terms, we will obtain an ordinary second-order differential equation with constant coefficients:

$$\left[\alpha V_0^2+T_0\frac{g\omega_0}{B_0}\right]f_1{}''(x)+\left[2\alpha V_0^2ik_1+\frac{i_0\chi g\omega_0}{H_0B_0}\right]f_1{}'(x_0)+\left[\frac{2i_0g}{V_0}iK_1-K_1^2\right]f_1(x)=0 \tag{9}$$

If making the following designations:

$$T_1^2=\left[\alpha V_0^2-T_0\frac{g\omega_0}{B_0}\right] \tag{10}$$

$$T_2=\frac{i_0\chi g\omega_0}{B_0H_2} \tag{11}$$

$$T_3=\frac{2i_0g}{V_0} \tag{12}$$

By considering (10), (11) and (12), dependence (9) is as follows:

$$T_1^2y^2+\left(2\alpha V_0^2iK_1+T_2\right]y+\left[T_3iK_1-K_1^2\right]=0 \tag{13}$$

The solution of (13) will yield:

$$y=\frac{-\left(2\alpha V_0iK_1+T_2\right)\pm\sqrt{\left(2\alpha V_0iK_1+T_2\right)^2-4T_1^2\left(T_0iK_1-K_1^2\right)}}{2T_1^2} \tag{14}$$

In order to separate the real and imaginary parts in dependencies (13) (14), let us accept the following notations:

$$(a+ib)^2=\left(2\alpha V_0^2iK_1+T_2\right)^2-4T_1^2\left(T_3iK_1-K_1^2\right) \tag{15}$$

or

$$a^2 + 2aib + b^2 = T_2^2 + 4\alpha V_0 iK_1T_2 - 4\alpha V_0^2 iK_1^2 - 4T_1^{\,2}T_3 iK_1 + 4T_1^{\,2}K_1^2 \ ,$$

i.e.:

$$a^2 - b^2 = T_1^{\ 2} - 4a^2V_0^{\ 2}K_1^{\ 2} + 4T_1^{\ 2}K_1^{\ 2} \qquad (16)$$

$$2aib = 2i\left(2\alpha V_0^{\ 2}T_2K_1 - 2T_1^{\ 2}T_3K_1\right)$$

or

$$ab = 2\alpha V_0T_2K_1 - 2T_1^{\ 2}T_3K_1$$

As a result, we will gain:

$$b = \frac{K_1}{a}\left(2\alpha V_0T_2 - 2T_1^{\ 2}T_3\right) \qquad (17)$$

Let us denote the right side of (16) by »Π_0»,

$$\text{i.e.: } \Pi_0 \approx T_1^{\ 2} - 4\alpha^2V_0K_1^{\ 2} + 4T_1^{\ 2}K_1^{\ 2} \qquad (18)$$

By substituting (17) and (18) in (16), following simple transformations, we will gain:

$$a = \left[\frac{\Pi_0}{2} \mp \sqrt{\frac{\Pi_0^{\ 2}}{4} + K_1^{\ 2}\left[2\alpha V_0T_2 - 2T_1^{\ 2}T_3\right]^2}\right]^{1/2} \qquad (19)$$

By knowing "a" and "b" as (19) and (17), the solution (14) by considering (16) can be written down as follows:

$$Y_{1,2} = \frac{1}{2T_1^{\ 2}}\left\{-\left(2\alpha V_0 iK_1 + T_2\right) \pm \left(a + ib\right)\right\} \qquad (20)$$

Or, by introducing the following notations:

$$Б_1{}' = \frac{\alpha - T_2}{2T_1} \qquad (21)$$

$$Б_2{}' = \frac{b - 2\alpha V_0K_1}{2T_1} \qquad (22)$$

$$Б_1'' = -\frac{a + T_2}{2T_1} \quad (23)$$

$$Б_2'' = \frac{b + 2\alpha V_0 K_1}{2T_1^2} \quad (24)$$

We will obtain:

$$\Upsilon_1 = Б_1^1 + iБ_2^2 \ ; \qquad У_2 = Б_1'' + iБ_2'' \quad (25)$$

Thus, particular solution (5) corresponding to the propagation of waves of disturbance along the motion, will be:

$$h = \Pi\left[f_1(x)e^{ik_1t}\right] = \Pi\left[Me^{yx+ik_1t}\right] \quad (26)$$

where: *П* is the symbol of the real part; *M* is an arbitrary constant; *Y* is one of the values of a characteristic equation.

By introducing the following notation:

$$M = A_0 e^{i\psi}, \quad (27)$$

where: A_0 is a new constant.

Following the separation of the real part, expression (26) will be as follows:

$$h = A_0 e^{\sigma_1 x} \cos(Б_2 x + K_1 t + \psi), \quad (28)$$

At the same time, $Б_1$ and $Б_2$ in this expression, depending on which of the roots of (13) is adopted in expression (26), in accordance with formulas (25), are identified by dependencies $(21 \div 23)$.

It is easy to notice that the adoption of $y = y_2$ in expression (25) and $Б_1 = Б_1''$, and $Б_2 = Б_2''$ in expression (26) leads to the damping condition of the agitation along the movement, but at the same time, it is hard to determine the conditions of a uniform flow under which the disturbance will attenuate, i.e. movement will be stable.

On the contrary, when $y = y_2$, i.e. when $Б_1 = Б_1'$ и $Б_2 = Б_2'$ in equation (28), $Б_2 \langle 0$ (see equation (20)) only given:

$$T_2 \rangle a \tag{29}$$

Т.к. $a \rangle 0$ и $T_1^2 = \alpha V_0 - T \dfrac{g\omega}{B} \rangle 0$, consequently, the initial flow of the current will be stable under condition (29).

Now, by substituting “a” value from (19) into inequality (29), we will obtain:

$$T_2^{\,2} \rangle \frac{\Pi_0}{2} \pm \sqrt{\frac{\Pi_0^{\,2}}{4} + K_1^{\,2}\left(2\alpha V_0 T_2 - 2T_1^{\,2} T_3\right)^2} \,,$$

Therefore:

$$T_2^{\,2} - \frac{\Pi_0^{\,2}}{2} \rangle \frac{\Pi_0^{\,2}}{4} + K_1^{\,2}\left(2\alpha V_0 T_2 - 2T_1^{\,2} T_3\right)^2,$$

Or

$$T_2^{\,4} - \Pi_0 T^2 \rangle 4K_1^{\,2}\alpha^2 V_0^{\,2} T_2^{\,2} - 8K_1^{\,2}\alpha V_0 T_1^{\,2} T_3 T_2 + 4K_1^2 T_1^4 T_3^2$$

By considering (18) and following the reduction it by $4K_1^2 T_1^{\,2}$, we will obtain:

$$-T_2^{\,2} \rangle T_1^{\,2} T_3^{\,2} - 2\alpha T_2 T_3 V_0$$

By considering (10) (11) (12), we will obtain:

$$T_0 \frac{g\omega_0}{B_0 V_0^{\,2}} \rangle \alpha + \frac{\chi^2 \omega_0^{\,2}}{4H_0^{\,2} B^2} + 2\alpha\left(\frac{\chi\omega}{2B_0 H_0}\right)$$

Hence,

$$\frac{1}{F_{z_0}} \rangle \frac{1}{T_0}\left[m^2 - 2\alpha\mu + \alpha\right] \quad , \tag{30}$$

where:

$$F_{z_0} = \frac{V_0^{\,2} B_0}{g\omega_0} \tag{31}$$

$$\mu = \frac{\omega_0 \chi}{2B_0 H_0} \tag{32}$$

Condition (30) expresses the criteria relation between the stability of the initial uniform motion of a turbulent of the sediment-carrying flow, i.e. under (30), no waves are formed on the free flow surface in the ravines.

In the given dependences, hydraulic indicator of the ravine χ depends on the shape of the cross-section and roughness. A specific value can be calculated under the recommendation of B. A. Bakhmetev or according to the dependence of R. R. Chugayeva.

α value can be determined with formula given by A. S. Obrazovskyi [7].

From dependence (46), it follows that $S = S_0 = 0$, we obtain well-known dependences for the water flow of Vedernikov-Kartvelishvili [8].

The analysis of dependence (30) shows that, depending on the concentration, hydraulic sediment size, suspension density, etc., the sediment-carrying flow, with its degree of stability, may be more, less or equal to the speed of the equivalent water flow.

If there is a possibility of waves occurring on the surface of the stream, the forecast of erosion in the ravine should be calculated not by the uniform flow motion with average section, but by considering the presence of waves, since the wave speed is 1.5 times greater than the speed of an evenly moving flow. In such a case, the flow rate at the height of the roughness protrusion should be assigned as Y_b, where $V_b = 1{,}5V$ what is proved in our works [3,4]. Waves stimulate the intensity of soil and ground washout.

3. Results

The above-given dependences can be used for the irrigation of furrows and in stream flows in the right side of the numerator of dependence (3), since $W\omega_0 \rangle\rangle \sigma K_2 B_0 \nu (S_{cp} - S_0)$, then member $\sigma K_2 B_0 \nu (S_{cp} - S_0)$ can be neglected. In such a case, the dimensionless parameter of the hydraulics of the sediment-carrying flow will be as follows:

$$T_0 = \frac{1}{1 + \sigma S_{cp}} \;, \tag{33}$$

what simplifies the calculations without any significant decrease in accuracy.

<u>Example:</u> A uniformly moving sediment-carrying current with a maximum depth of $H = 0{,}1$ m and with an average velocity of $V_0 = 1{,}2\ {}^{m}\!/\!{}_{\sec}$ across the clear section flows through the ravine with a V-shaped section

(i.e. with a triangular cross section with slope embedding factor $m = 1$).

The average diameter of the suspended particles of the soil $d = 1{,}5\ mm$ (hydraulic particle size $W = 0.1256\ {}^{m}\!/_{\sec}$). The density of particles and water $\rho^{*} = 2.65\ {}^{T}\!/_{m^3}$ and $\rho = 1\,{}^{T}\!/_{m^3}$, respectively. Average volume concentration of suspended particles in water $S_{Ave} = 0{,}02$.

The possibility of the appearance of waves on a free flow surface (with the width of $B_0 = 0{,}2\ m$) is to be forecasted, i.e. the loss of stability of the initial uniform motion and its transition to the wave mode of motion is to be established.

<u>Solution:</u> The area of the clear section of flow in the ravine $\omega_0 = \frac{H_0 B_0}{2} = \frac{0{,}2 \times 0{,}1}{2} = 0{,}002\ m^2$. Froude number in case of uniform motion:

$$F_{чо} = \frac{V_0^2 B_0}{g\omega_0} = \frac{1{,}2^2 \cdot 0{,}2}{9{,}81 \cdot 0{,}02} = 1{,}468$$

Let us identify ratio: $\mu = \frac{\omega_0 \chi}{2, B_0 H_0}$

According to B.A. Bakhmetev, the hydraulic indicator of the bed of the ravine with a triangular cross-section, $\chi = 5$, then: $\mu = \frac{0{,}02 \cdot 5}{2 \cdot 0{,}2 \cdot 0{,}1} = 2{,}5$.

The dimensionless parameter of the hydraulics of the sediment-carrying stream (33) is:

$$T_0 = \frac{1}{1 + \sigma\ S_{cp}} = \frac{1}{1 + 1{,}65 \cdot 0{,}02} = 0{,}968$$

Let us use dependence (30):

$$\frac{1}{F_{чо}} > \frac{1}{T_0}\left[\mu^2 - 2\alpha\mu + \alpha\right]$$

To simplify the problem, let us assume that $\alpha = 1{,}2$. Then:

$$\frac{1}{1{,}468} > \frac{1}{0{,}968}\left[2{,}5^2 - 2 \cdot 1{,}2 \cdot 2{,}5 + 1{,}2\right]$$

$$0{,}68 > 1{,}499,$$

It means that the stream loses the original “steady” uniform movement and acquires a wave motion what is to be taken into account when predicting soil erosion. The wave mode stimulates the intensity of the washout of the soils in the upper sections of the ravine.

In such cases, the right choice is to use for example, the ratio of V.G. Goncharov [6], which is generally as follows:

$$U_\Delta = \frac{1{,}25 \cdot 1{,}5V}{\ell g\left(6{,}15\dfrac{H}{\Delta}\right)} = \frac{1{,}875V}{\ell g\left(6{,}15\dfrac{H}{\Delta}\right)}$$

4. **Conclusion**

Thus, in case of the movement of a shallow stream in the upper sections of the ravines (as a brook), rolling waves often occur on the surface of a uniformly moving stream stimulating the intensity of soil and ground washout. The effect of the wave formation on the process of erosion intensity can be taken into account in the existing calculation dependences along the entire length of the ravine, by using correction factor $V_b = 1{,}5V$ for the average flow velocity across the clear section.

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