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Confidence Interval Estimation of the Conditional Reliability Function for Time Domain Data

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Abstract

The function of conditional reliability gives the probability of successfully implementing another operation following the successful implementation of a previous operation. The prediction of this function can help software developers in determining optimal release times. In this paper, the Maximum Likelihood Estimation (MLE) method is used to estimate the Non-Homogeneous Poisson Process Log-Logistic (NHPP LL) model's parameters. The upper and the lower bounds of the parameters and conditional reliability function of time domain data are obtained. Real data application is conducted using the coefficient of multiple determination criteria and observed interval length to evaluate the performance of the NHPP LL model and the constructed confidence intervals, respectively. Our results encourage for more assessment of confidence intervals of other measures of reliability of the NHPP models.

Keywords: NHPP log-logistic model; maximum likelihood estimation; confidence interval; conditional reliability function; observed interval length.

1. Introduction

Software reliability is defined as the probability of failure–free operation of a computer program in a specified environment for a specified period of time [1], it received great attention due to its huge impact in our daily life [2,3]. Software reliability models based on Non-Homogeneous Poisson Process (NHPP) of time between failures class have been considered in the literature and validated as an accurate approach for estimating and predicting software reliability [4-7]. Hence, considering the Confidence Intervals (CIs) of software reliability can enhance the precision of the predictions for software testing.

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For software reliability data analysis, Yamada and Osaki [8] examined the Maximum Likelihood (ML) estimates using several SRGMs, they founded CI of the mean value function by the conventional NHPP method. Yin and Trivedi [9] obtained the confidence limits for the model parameters using the Bayesian method via implementing the estimation approach of Yamada and Osaki [8]. Huang [10] also followed the approach of Yamada and Osaki [13] to illustrate the CI of the mean value function graphically.

This paper presents CI of the conditional reliability function of a NHPP model that assumes the time between two successive failures follow a Log-Logistic (LL) distribution. The LL distribution was first considered by Fisk [11], it is like the log-normal distribution, but with a little narrower peak and heavier tails. The LL distribution is among the class of survival time parametric models where the hazard function firstly increases and then decreases and at times can be hump-shaped, its mathematical simplicity and practicality has attracted many researchers in the field of survival analysis [12-14].

The paper layout of is as follows: Section 2 describes the conditional reliability function of the NHPP LL model. Section 3 discusses the parameter estimation and reliability prediction with confidence Intervals for the parameters and conditional reliability function of the NHPP LL model based on the times between failures data. Section 4 presents the analysis of three failure data sets, and Section 5 concludes the paper.

2. Conditional Reliability Function of a NHPP Model

A NHPP model aim to estimate the expected number of faults experienced up to a certain point of time. If N(t) be the cumulative number of faults detected by the time t, F(t) is the distribution function and denote the expected number of faults that would be detected in a given infinite testing time, then the mean value function of a NHPP model is given by [15]:

$$\mu(\mathrm{t_i}; \mathrm{N_0}, \Theta) = \mathrm{N_0} F(\mathrm{t_i}; \Theta), \tag{1}$$

where, $\mathrm{N_0} > 0$ is the expected number of errors, $\mathrm{F}(\mathrm{t_i}; \Theta)$ is the cumulative distribution of $\mathrm{t_i}$, $\mathrm{i} = (1, 2, \dots, \mathrm{n})$, Θ is its unknown parameters. Accordingly, the mean value function of the NHPP Log Logistic Model (NHPP LL model) is given below:

$$\mu(\mathrm{t_i}; \mathrm{N_0}, \boldsymbol{\gamma}, \beta) = \frac{\mathrm{N_0} \gamma x^{\beta}}{1+\gamma x^{\beta}}, \tag{2}$$

where $\beta > 0$ is the shape parameter. and $\gamma > 0$ is positive scale parameter. The corresponding failure intensity function can be found by differentiating Eq. (2) as follows:

$$\eta(\mathrm{t_i}; \mathrm{N_0}, \boldsymbol{\gamma}, \beta) = \frac{\mathrm{N} \gamma \beta \mathrm{t_i}^{\beta-1}}{\left(1+\gamma \mathrm{t_i}^{\beta}\right)^2}, \tag{3}$$

The conditional reliability function at time t of a NHPP model is exponential, given by:

$$\mathrm{R}(\mathrm{t_i}; \mathrm{N_0}, \Theta | \mathrm{x_n}) = \exp\left\{-\left(\mu(\mathrm{t_i} + \mathrm{x_n}; \mathrm{N_0}, \Theta) - \mu(\mathrm{t_i}; \mathrm{N_0}, \Theta)\right)\right\}, \text{ where} \tag{4}$$

$R(t_i; N_0, \Theta|x_n)$ is a monotone non-increasing function of t_i; $R(0; N_0, \Theta|x_n) = 0$ and $R(\infty; N_0, \Theta|x_n) = 1$.

Consequently, the conditional reliability function of the NHPP LL model is given by:

$$R(t_i; N_0, \boldsymbol{\gamma}, \beta|x_n) = \exp\left\{-N_0\gamma\left(\frac{(t_i+x_n)^\beta - x_n^\beta}{\left(1+\gamma x_n^\beta\right)\left(1+\gamma(t+x_n)^\beta\right)}\right)\right\}. \quad (5)$$

More details about the NHPP LL model can be found in Al turk [16].

3. Confidence Interval Estimation of the NHPP LL Model

In this paper the MLE method will be applied to the time-interval between failures class of non- homogeneous Poisson process (NHPP) models.

3.1. Confidence interval estimation of the parameters

Suppose that we have n observations represents the cumulative time to failures denoted by $s_1, s_2, \ldots, s_n$, then by considering Eqs. (2) and (3) the mean value and intensity functions of the NHPP LL model the log-likelihood function of N_0, γ, and β can be written as:

$$L\left(N_0, \boldsymbol{\gamma}, \beta|\underline{S}\right) = e^{-\mu(t_i;N_0,\boldsymbol{\gamma},\beta)} \prod_{i=1}^{n} \eta(t_i; N_0, \boldsymbol{\gamma}, \beta). \quad (6)$$

Taking the natural logarithm of Eq. (6) we obtain:

$$\begin{aligned} \ln L\left(N_0, \boldsymbol{\gamma}, \beta|\underline{S}\right) &= -\mu(t_i; N_0, \boldsymbol{\gamma}, \beta) + \sum_{i=1}^{n} \ln \eta(t_i; N_0, \boldsymbol{\gamma}, \beta) \\ &= -\frac{N_0\gamma S_n^\beta}{1+\gamma S_n^\beta} + \sum_{i=1}^{n} \ln\left(\frac{N_0\gamma\beta S_i^{\beta-1}}{\left(1+\gamma S_i^\beta\right)^2}\right) \end{aligned} \quad (7)$$

$$= -\frac{N_0\gamma S_n^\beta}{1+\gamma S_n^\beta} + n\ln\gamma + n\ln\beta + n\ln N_0 + \beta\sum_{i=1}^{n} lns_i -$$

$$\sum_{i=1}^{n} lns_i - 2\sum_{i=1}^{n} \ln\left(1+\gamma S_i^\beta\right) \quad (8)$$

Differentiating the above function with respect to N_0, γ, and β, we have

$$\begin{cases} \dfrac{\partial \ln L(N_0, \boldsymbol{\gamma}, \beta|\underline{S})}{\partial N_0} = -\dfrac{\gamma S_n^\beta}{1+\gamma S_n^\beta} + \dfrac{n}{N_0}. \\ \dfrac{\partial \ln L(N_0, \boldsymbol{\gamma}, \beta|\underline{S})}{\partial \gamma} = \dfrac{n}{\gamma} - \dfrac{N_0 S_n^\beta}{1+\gamma S_n^\beta} + 2\sum_{i=1}^{n} \dfrac{S_i^\beta}{1+\gamma S_i^\beta}. \\ \dfrac{\partial \ln L(N_0, \boldsymbol{\gamma}, \beta|\underline{S})}{\partial \beta} = \dfrac{n}{\beta} + \sum_{i=1}^{n} lns_i - \dfrac{N_0\gamma S_n^\beta ln S_n}{\left(1+\boldsymbol{\gamma} S_n^\beta\right)^2} + 2\sum_{i=1}^{n} \dfrac{\gamma S_i^\beta ln S_i}{1+\gamma S_i^\beta}. \end{cases} \quad (9)$$

The ML estimates can be obtained by setting the three expressions in Eq. (9) to zero as follows:

$$\begin{cases} N_0 = n\left(\frac{1+\gamma S_n^{\beta}}{\gamma S_n^{\beta}}\right). \\ \frac{n}{\gamma} - \frac{n}{\gamma\left(1+\gamma S_n^{\beta}\right)} + 2\sum_{i=1}^{n}\frac{S_i^{\beta}}{1+\gamma S_i^{\beta}} = 0. \\ \frac{n}{\beta} + \sum_{i=1}^{n} lnS_i - \frac{n lnS_n}{1+\gamma S_n^{\beta}} + 2\sum_{i=1}^{n}\frac{\gamma S_i^{\beta} lnS_i}{1+\gamma S_i^{\beta}} = 0. \end{cases} \tag{10}$$

Due to the lack of explicit solutions to the second and third expressions of Eq. (10), we numerically find the estimates the parameters γ and β then by substituting them in the first expression, $\widehat{\mathrm{N}}_0$ is obtained.

To get the variance and covariance matrix for the estimated parameters, we first need to calculate the Fisher information matrix [17], which is:

$$\mathrm{F} = \begin{bmatrix} -\frac{\partial^2 \ln \mathrm{L}}{\partial \mathrm{N}_0^2} & -\frac{\partial^2 \ln \mathrm{L}}{\partial \mathrm{N}_0\,\partial\gamma} & -\frac{\partial^2 \ln \mathrm{L}}{\partial \mathrm{N}_0\,\partial\beta} \\ -\frac{\partial^2 \ln \mathrm{L}}{\partial\gamma\,\partial \mathrm{N}_0} & -\frac{\partial^2 \ln \mathrm{L}}{\partial\gamma^2} & -\frac{\partial^2 \ln \mathrm{L}}{\partial\gamma\,\partial\beta} \\ -\frac{\partial^2 \ln \mathrm{L}}{\partial\beta\,\partial \mathrm{N}_0} & -\frac{\partial^2 \ln \mathrm{L}}{\partial\beta\,\partial\gamma} & -\frac{\partial^2 \ln \mathrm{L}}{\partial\beta^2} \end{bmatrix}, \tag{11}$$

where

$$\frac{\partial^2 \ln\mathrm{L}(\mathrm{N}_0, \boldsymbol{\gamma}, \beta|\underline{\mathrm{S}})}{\partial N_0^2} = \frac{-n}{N_0^2}, \tag{12}$$

$$\frac{\partial^2 \ln\mathrm{L}(\mathrm{N}_0, \boldsymbol{\gamma}, \beta|\underline{\mathrm{S}})}{\partial \gamma^2} = -\frac{n}{\gamma^2} + \frac{2N_0 S_n^{2\beta}}{\left(1+\gamma S_n^{\beta}\right)^3} - 2\sum_{i=1}^{n}\frac{S_i^{2\beta}}{\left(1+\gamma S_n^{\beta}\right)^2}, \tag{13}$$

$$\frac{\partial^2 \ln\mathrm{L}(\mathrm{N}_0, \boldsymbol{\gamma}, \beta|\underline{\mathrm{S}})}{\partial \beta^2} = -\frac{n}{\beta^2} - \frac{2N_0\gamma^2 S_n^{2\beta}(lnS_n)^2}{\left(1+\gamma S_n^{\beta}\right)^3} + 2\gamma\sum_{i=1}^{n}\frac{S_i^{\beta}(lnS_i)^2}{\left(1+\gamma S_i^{\beta}\right)^2}, \tag{14}$$

$$\frac{\partial^2 \ln\mathrm{L}(\mathrm{N}_0, \boldsymbol{\gamma}, \beta|\underline{\mathrm{S}})}{\partial N_0 \partial \gamma} = \frac{-S_n^{\beta}}{1+\gamma S_n^{\beta}}, \tag{15}$$

$$\frac{\partial^2 \ln\mathrm{L}(\mathrm{N}_0, \boldsymbol{\gamma}, \beta|\underline{\mathrm{S}})}{\partial N_0 \partial \beta} = -\frac{\gamma S_n^{\beta} lnS_n}{\left(1+\gamma S_n^{\beta}\right)^2}, \text{ and} \tag{16}$$

$$\frac{\partial^2 \ln\mathrm{L}(\mathrm{N}_0, \boldsymbol{\gamma}, \beta|\underline{\mathrm{S}})}{\partial \gamma \partial \beta} = \frac{N_0 S_n^{\beta}\left(\alpha S_n^{\beta}-1\right) lnS_n}{1+\alpha S_n^{\beta}} + 2\sum_{i=1}^{n}\frac{S_i^{\beta} lnS_i}{\left(1+\gamma S_i^{\beta}\right)^2}, \tag{17}$$

The asymptotic variance-covariance matrix is obtained by:

$$\Sigma = \mathrm{F}^{-1}$$

$$= \begin{bmatrix} Var(N_0) & \mathrm{Cov}(N_0,\gamma) & \mathrm{Cov}(N_0,\beta) \\ \mathrm{Cov}(N_0,\beta) & Var(\gamma) & \mathrm{Cov}(\gamma,\beta) \\ \mathrm{Cov}(\beta,N_0) & \mathrm{Cov}(\beta,\gamma) & Var(\beta) \end{bmatrix}, \tag{18}$$

So, the $100(1-\alpha)\%$ asymptotic confidence intervals for the parameters N_0, γ, and $\beta$of the NHPP LL model are given, respectively, by:

$$\left(\widehat{N}_0 - Z_{\frac{\alpha}{2}}\sqrt{Var(\widehat{N}_0)}, \widehat{N}_0 + Z_{\frac{\alpha}{2}}\sqrt{Var(\widehat{N}_0)}\right), \tag{19}$$

$$\left(\hat{\gamma} - Z_{\frac{\alpha}{2}}\sqrt{Var(\hat{\gamma})},\ \hat{\gamma} + Z_{\frac{\alpha}{2}}\sqrt{Var(\hat{\gamma})}\right), \text{ and} \tag{20}$$

$$\left(\hat{\beta} - Z_{\frac{\alpha}{2}}\sqrt{Var(\hat{\beta})},\ \hat{\beta} + Z_{\frac{\gamma}{2}}\sqrt{Var(\hat{\beta})}\right), \tag{21}$$

where, $Z_{\alpha/2}$ is the percentile of standard normal distribution with right-tail probability $\alpha/2$, $Var(\widehat{N}_0)$, $Var(\hat{\gamma})$, and $Var(\hat{\beta})$ are, respectively, the diagonal elements of the asymptotic variance and covariance matrix given by Eq. (18).

3.2. Confidence interval estimation of the conditional reliability function

According to the invariance property of the ML estimators, the estimate of the conditional reliability of the NHPP LL model is obtained by:

$$\widehat{\mathrm{R}}\left(\mathrm{t_i}; \widehat{\mathrm{N}}_0, \hat{\boldsymbol{\gamma}}, \hat{\beta} \middle| \mathrm{x_n}\right) = \exp\left\{-\widehat{\mathrm{N}}_0\hat{\gamma}\left(\frac{(\mathrm{t_i}+\mathrm{x_n})^{\hat{\beta}} - \mathrm{x_n^{\hat{\beta}}}}{\left(1+\hat{\gamma}\mathrm{x_n^{\hat{\beta}}}\right)\left(1+\hat{\gamma}(\mathrm{t}+\mathrm{x_n})^{\hat{\beta}}\right)}\right)\right\}, \tag{22}$$

and its variance is defined as:

$$V(\hat{R}) = \left(\frac{\partial R}{\partial N_0}\right)^2\Big|_{N_0=\widehat{N}_0} V(\widehat{N}_0) + \left(\frac{\partial R}{\partial \gamma}\right)^2\Big|_{\gamma=\hat{\gamma}} V(\hat{\gamma}) + \left(\frac{\partial R}{\partial \beta}\right)^2\Big|_{\beta=\hat{\beta}} V(\hat{\beta}) +$$

$$2\left(\frac{\partial R}{\partial \gamma}\right)\left(\frac{\partial R}{\partial N_0}\right)\Big|_{\gamma=\hat{\gamma},\, N_0=\widehat{N}_0} \mathrm{Cov}\,(\hat{\gamma},\widehat{N}_0) + 2\left(\frac{\partial R}{\partial \gamma}\right)\left(\frac{\partial R}{\partial N_0}\right)\Big|_{\gamma=\hat{\gamma},\, N_0=\widehat{N}_0} \mathrm{Cov}\,(\hat{\beta},\widehat{N}_0) +$$

$$2\left(\frac{\partial R}{\partial \gamma}\right)\left(\frac{\partial R}{\partial \beta}\right)\Big|_{\gamma=\hat{\gamma},\ \beta=\hat{\beta}} \mathrm{Cov}\,(\hat{\gamma},\hat{\beta}), \tag{23}$$

where

$$\frac{\partial R}{\partial N_0} = -\gamma\left(\frac{(t_i+x_n)^{\beta} - x_n^{\beta}}{\left(1+\gamma x_n^{\beta}\right)\left(1+\gamma(t+x_n)^{\beta}\right)}\right) exp\left\{-\mathrm{N}_0\gamma\left(\frac{(t+x_n)^{\beta} - x_n^{\beta}}{\left(1+\gamma x_n^{\beta}\right)\left(1+\gamma(t+x_n)^{\beta}\right)}\right)\right\}, \tag{24}$$

$$\frac{\partial R}{\partial \gamma} = \mathrm{N}_0 \left(\frac{(t_i+x_n)^\beta - x_n^\beta}{\left(1+\gamma x_n^\beta\right)\left(1+\gamma(t_i+x_n)^\beta\right)} \right) \left(\frac{\left((1+x_n)^\beta\left(1+2\gamma x_n^\beta\right)+x_n^\beta\right)}{\left(1+\gamma x_n^\beta\right)\left(1+\gamma(t_i+x_n)^\beta\right)} - 1 \right) \times$$

$$exp\left\{ -\mathrm{N}_0 \gamma \left(\frac{(t_i+x_n)^\beta - x_n^\beta}{\left(1+\gamma x_n^\beta\right)\left(1+\gamma(t_i+x_n)^\beta\right)} \right) \right\}, \text{ and} \tag{25}$$

$$\frac{\partial R}{\partial \beta} = \frac{\alpha \mathrm{N}_0}{\left(1+\gamma x_n^\beta\right)\left(1+\gamma(t_i+x_n)^\beta\right)} \times$$

$$\left(\frac{\gamma\left((t_i+x_n)^\beta - x_n^\beta\right)\left(x_n^\beta\left(1+\gamma(1+x_n)^\beta\right) ln x_n + (t_i+x_n)^\beta\left(1+\gamma x_n^\beta\right) ln(t_i+x_n)\right)}{\left(1+\gamma x_n^\beta\right)\left(1+\gamma(t_i+x_n)^\beta\right)} - (t_i+x_n)^\beta ln(t_i+x_n) - x_n^\beta ln x_n^\beta\right) \times$$

$$exp\left\{ -\mathrm{N}_0 \gamma \left(\frac{(t_i+x_n)^\beta - x_n^\beta}{\left(1+\gamma x_n^\beta\right)\left(1+\gamma(t_i+x_n)^\beta\right)} \right) \right\}. \tag{26}$$

We employ the central limit theorem of the conditional reliability function and gets the asymptotic $100(1-\alpha)\%$ confidence bounds for the actual values as follows:

$$\left(\hat{R} - Z_{\frac{\alpha}{2}}\sqrt{Var(\hat{R})}, \hat{R} + Z_{\frac{\gamma}{2}}\sqrt{Var(\hat{R})}\right). \tag{27}$$

where, $Z_{\alpha/2}$ is the percentile of standard normal distribution with right-tail probability $\alpha/2$, $\hat{\lambda}$ is obtained from Eq. (22), and $Var(\hat{\lambda})$ is defined by Eq.(23).

4. Numerical Application

A numerical application is illustrated in this section. The confidence interval estimation of the parameters and conditional reliability function of the NHPP LL model is investigated based on three real data sets. The NTDS data is obtained from Goel and Okumoto [17], it consists of 34 failures. The CSR2 and SYS2 data sets are from Lyu [3], the number of failures in these data sets are 129, and 86 failures, respectively. The three data sets are shown in Tables [1-3].

Table 1: NTDS data, 34 failures.

9	12	11	4	7	2	5	8	5	7	1	6	1	9	4	1	3	3	6	1
11	33	7	91	2	1	87	47	12	9	135	258	16	35						

The coefficient of multiple determination R^2 is used in our application to evaluate the model performance. Its formula is as follows [18]:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}\left(y_i - \hat{m}(t_i)\right)^2}{\sum_{i=1}^{n}\left(y_i - \sum_{k=1}^{n} y_k/n\right)^2}. \tag{28}$$

It takes the values from 0 to1. The larger value of R^2 indicates better model performance. Though, the observed interval length is used to compare the CIs. The smaller the length, the better the confidence interval.

Table 2: CSR2 data, 129 failures

760	758	303	6	22	14	42	4	84	15	221	14	15
41	1	153	409	54	24	44	180	397	19	145	36	54
1337	163	8	1	17	16	87	19	29	0	5	360	10
11	100	252	460	179	3	24	253	163	54	137	328	3
9	12	18	9	75	15	366	428	212	115	264	269	276
1	999	30	495	472	344	550	131	47	92	863	991	35
9549	249	607	83	614	352	673	4179	111	75	407	288	894
1314	845	55	409	36	15	1960	60	19	20	79	24	1737
7984	10	20	338	250	1682	212	287	56	4973	3500	59	98
2439	1812	6203	385	3500	4892	687	62	2796	3268	3845	76	

Table 3: SYS2 data, 86 failures.

479	266	277	554	1034	249	693	597	117	170	117	1274	469
1174	693	1908	135	277	596	757	437	2230	437	340	405	535
277	363	522	613	277	1300	821	213	1620	1601	298	874	618
2640	5	149	1034	2441	460	565	1119	437	927	4462	714	181
1485	757	3154	2115	884	2037	1481	559	490	593	1769	85	2836
213	1866	490	1487	4322	1418	1023	5490	1520	3281	2716	2175	3505
725	1963	3979	1090	245	1194	994	3902					

4.1. Numerical results

The ML estimates and CIs at 95% significance level of the parameters N_0, γ, and β are assessed using Eqs. (19), (20), and (21) corresponding to the last failure number of each data sets. For the comparison purpose the observed interval lengths of the CIs are computed as follows:

$$2Z_{\alpha/2}[Var(\widehat{N_0})]^{1/2},\ 2Z_{\alpha/2}[Var(\hat{\gamma}))]^{1/2}\ ,2Z_{\alpha/2}[Var(\hat{\beta})]^{1/2}.$$

Also, to assess the model performance the coefficient of multiple determination criteria is computed for each of the selected data set, the results are summarized in Table 4.

Table 4: Estimated Parameter Values of the NHPP LL Model and 95% Confidence Intervals.

Data set	Failure Number	C.I. Lower	C.I. Lower	C.I. lower	R^2 Criteria
		$\hat{N}_0$	$\hat{\gamma}$	$\hat{\beta}$	
		C.I. Upper	C.I. Upper	C.I. upper	
		Observed Interval Length	Observed Interval Length	Observed Interval Length	
NTDS data	34	53.3639	0.0387	0.4484	0.7724
		53.614	0.0387	0.4947	
		53.8641	0.0387	0.5411	
		0.5002	1e-04	0.0927	
CSR2 data	129	204.4341	0.024600	0.2936	0.6393
		205.0794	0.024601	0.303	
		205.7247	0.024603	0.3123	
		1.2907	3.32e-06	0.0188	
SYS2 data	86	199.9347	0.0199058	0.2704	0.6052
		201.0775	0.0199064	0.2745	
		202.2204	0.0199069	0.2786	
		2.2857	1.18e-06	0.0082	

For the last three failure numbers, Table 5 illustrates the estimated conditional reliability of the NHPP LL model which is calculated using Eq. (22) and the corresponding 95% CIs with their observed interval lengths which are found using Eq. (27) and $2Z_{\alpha/2}[Var(\hat{R})]^{1/2}$, respectively.

Table 5: 95% Confidence Intervals of the conditional reliability function of the NHPP LL model.

Data set	Time to failure	Estimated reliability at Time t	C.I. lower	C.I. upper	Observed Interval Length
NTDS data	258	0.117	0.091	0.143	0.0536
	16	0.113	0.087	0.139	0.0528
	35	0.1047	0.0787	0.1307	0.052
CSR2 Data	3268	0.1592	0.1797	0.2002	0.0413
	3845	0.1671	0.1466	0.1876	0.0412
	76	0.1669	0.1464	0.1874	0.041
SYS2 data	1194	0.168	0.1345	0.2016	0.0679
	994	0.1652	0.1316	0.1987	0.0675
	3902	0.1545	0.121	0.1881	0.6052

Figures [1-3] demonstrate the estimated conditional reliability function and the corresponding 95% CIs for each selected data sets.

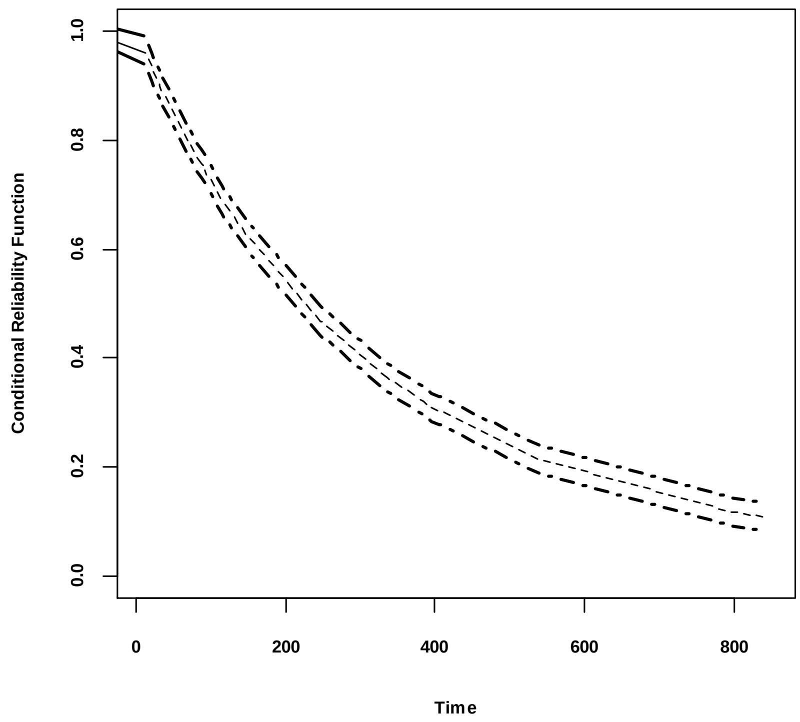

Figure 1: Estimated reliability with 95% interval based on the NHPP LL model, NTDS data

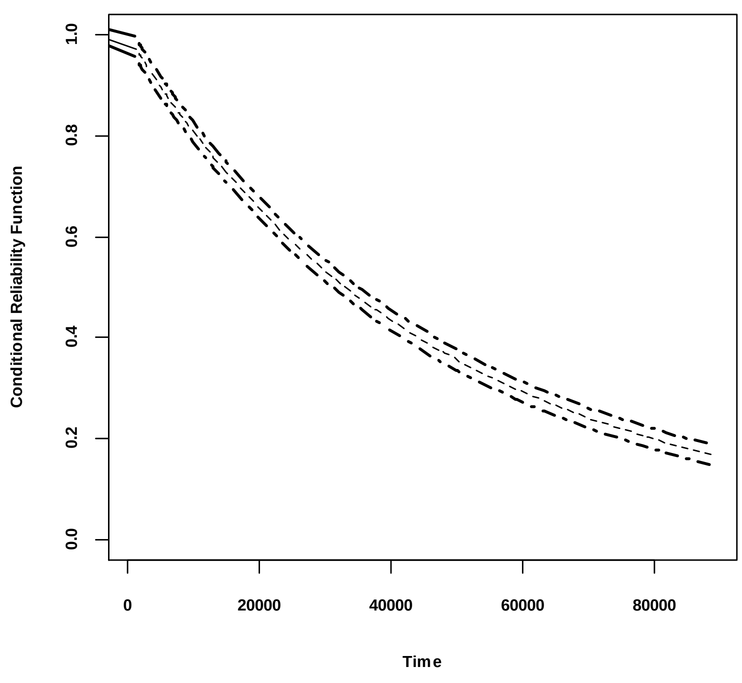

Figure 2: Estimated reliability with 95% interval based on the NHPP LL model, CSR2 Data

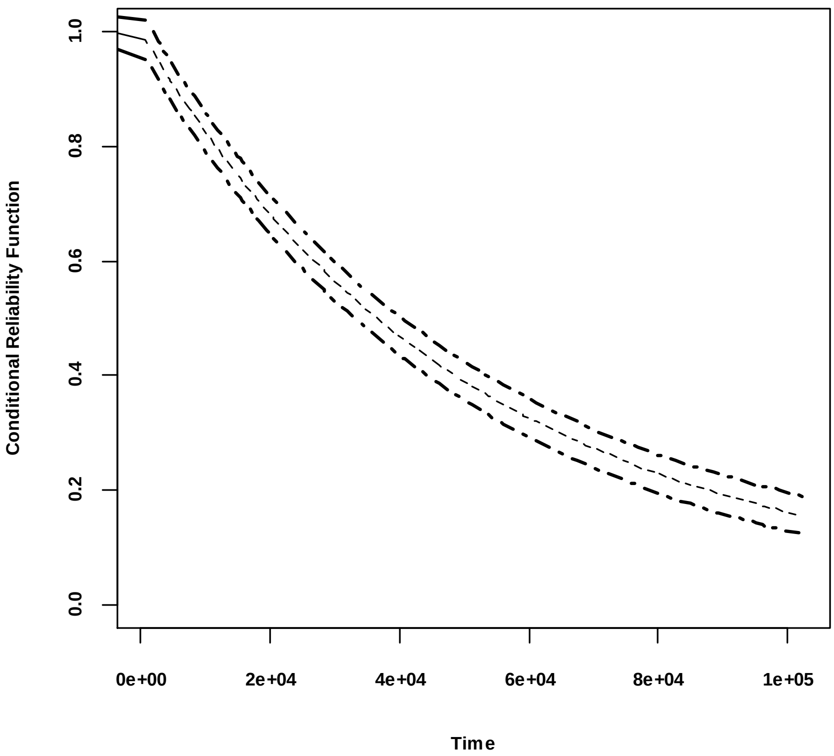

Figure 3: Estimated reliability with 95% interval based on the NHPP LL model, SYS2 data

From Tables 4 and 5, it can be noticed that the CIs estimated by the MLE method have small observed interval length which indicates the accuracy of these CIs. The assessment results, in Table 4, show that the estimator $\hat{\gamma}$ has the shortest observed interval length of the three selected data sets. Also, according to the model accuracy using the R^2criteria the results in Table 4 show that the NHPP model fits best the NTDS data, then CSR2 Data and SYS2 data take the second and third rank, respectively. Regarding the lengths of the CIs presented in Table 5 it can be seen that as the number of detected failures increases narrower intervals of the conditional reliability function are obtained. The estimator $\widehat{R}$ has the shortest observed interval length for the CSR2 Data.

5. Conclusion

It is essential to the software reliability measurement to obtain the confidence bounds for the reliability metrics at any future time t. The reliability function of a software system is an important metric for describing the system's reliability. Our main contribution in this paper is to construct CI for the conditional reliability function of a NHPP model based on the LL distribution.

CIs of the parameters and conditional reliability function of the NHPP LL model have been constructed based on the MLE method and evaluated via the observed interval length. The model performance has been checked using the R^2 criteria.

The application results demonstrate reasonable results for the CIs of the conditional reliability, which can help in improving the decision-making quality of software testing and debugging. Future research may find CIs for other reliability metrics of the NHPP models.

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