Confidence Interval Estimation of the Conditional Reliability Function for Time Domain Data

Lutfiah Ismail Al turk*

Statistics Department, Faculty of Sciences, King Abdulaziz University, Jeddah, Kingdom of Saudi Arabia
Email: lturk@kau.edu.sa

Abstract

The function of conditional reliability gives the probability of successfully implementing another operation following the successful implementation of a previous operation. The prediction of this function can help software developers in determining optimal release times. In this paper, the Maximum Likelihood Estimation (MLE) method is used to estimate the Non-Homogeneous Poisson Process Log-Logistic (NHPP LL) model’s parameters. The upper and the lower bounds of the parameters and conditional reliability function of time domain data are obtained. Real data application is conducted using the coefficient of multiple determination criteria and observed interval length to evaluate the performance of the NHPP LL model and the constructed confidence intervals, respectively. Our results encourage for more assessment of confidence intervals of other measures of reliability of the NHPP models.

Keywords: NHPP log-logistic model; maximum likelihood estimation; confidence interval; conditional reliability function; observed interval length.

1. Introduction

Software reliability is defined as the probability of failure–free operation of a computer program in a specified environment for a specified period of time [1], it received great attention due to its huge impact in our daily life [2,3]. Software reliability models based on Non-Homogeneous Poisson Process (NHPP) of time between failures class have been considered in the literature and validated as an accurate approach for estimating and predicting software reliability [4-7]. Hence, considering the Confidence Intervals (CIs) of software reliability can enhance the precision of the predictions for software testing.

* Corresponding author.
For software reliability data analysis, Yamada and Osaki [8] examined the Maximum Likelihood (ML) estimates using several SRGMs, they founded CI of the mean value function by the conventional NHPP method. Yin and Trivedi [9] obtained the confidence limits for the model parameters using the Bayesian method via implementing the estimation approach of Yamada and Osaki [8]. Huang [10] also followed the approach of Yamada and Osaki [13] to illustrate the CI of the mean value function graphically.

This paper presents CI of the conditional reliability function of a NHPP model that assumes the time between two successive failures follow a Log-Logistic (LL) distribution. The LL distribution was first considered by Fisk [11], it is like the log-normal distribution, but with a little narrower peak and heavier tails. The LL distribution is among the class of survival time parametric models where the hazard function firstly increases and then decreases and at times can be hump-shaped, its mathematical simplicity and practicality has attracted many researchers in the field of survival analysis [12-14].

The paper layout of is as follows: Section 2 describes the conditional reliability function of the NHPP LL model. Section 3 discusses the parameter estimation and reliability prediction with confidence Intervals for the parameters and conditional reliability function of the NHPP LL model based on the times between failures data. Section 4 presents the analysis of three failure data sets, and Section 5 concludes the paper.

2. Conditional Reliability Function of a NHPP Model

A NHPP model aim to estimate the expected number of faults experienced up to a certain point of time. If \( N(t) \) be the cumulative number of faults detected by the time \( t \), \( F(t) \) is the distribution function and denote the expected number of faults that would be detected in a given infinite testing time, then the mean value function of a NHPP model is given by [15]:

\[
\mu(t; N_0, \Theta) = N_0 F(t; \Theta),
\]

where, \( N_0 > 0 \) is the expected number of errors, \( F(t_i; \Theta) \) is the cumulative distribution of \( t_i \), \( i = (1, 2, ..., n) \), \( \Theta \) is its unknown parameters. Accordingly, the mean value function of the NHPP Log Logistic Model (NHPP LL model) is given below:

\[
\mu(t; N_0, \gamma, \beta) = \frac{N_0 \gamma t_i^\beta}{1 + \gamma t_i^\beta},
\]

where \( \beta > 0 \) is the shape parameter, and \( \gamma > 0 \) is positive scale parameter. The corresponding failure intensity function can be found by differentiating Eq. (2) as follows:

\[
\eta(t; N_0, \gamma, \beta) = \frac{N_0 \beta t_i^{\beta-1}}{(1+\gamma t_i)^2},
\]

The conditional reliability function at time \( t \) of a NHPP model is exponential, given by:

\[
R(t; N_0, \Theta|x_n) = \exp\left\{-\left(\mu(t + x_n; N_0, \Theta) - \mu(t; N_0, \Theta)\right)\right\},
\]

where
R(t_i; N_0, \Theta|\mathbf{x}_n) is a monotone non-increasing function of t_i; R(0; N_0, \Theta|\mathbf{x}_n) = 0 and R(\infty; N_0, \Theta|\mathbf{x}_n) = 1.

Consequently, the conditional reliability function of the NHPP LL model is given by:

\[
R(t_i; N_0, \gamma, \beta|\mathbf{x}_n) = \exp \left\{ -N_0 \gamma \left( \frac{(t_i + x_n)^\beta - x_n^\beta}{(1 + y_s)^\beta} \right) \right\}.
\]  

(5)

More details about the NHPP LL model can be found in Al turk [16].

3. Confidence Interval Estimation of the NHPP LL Model

In this paper the MLE method will be applied to the time-interval between failures class of non-homogeneous Poisson process (NHPP) models.

3.1. Confidence interval estimation of the parameters

Suppose that we have n observations represents the cumulative time to failures denoted by s_1, s_2, ..., s_n, then by considering Eqs. (2) and (3) the mean value and intensity functions of the NHPP LL model the log-likelihood function of N_0, \gamma, and \beta can be written as:

\[
L(N_0, \gamma, \beta|\mathbf{s}) = e^{-\mu(t_1; N_0, \gamma, \beta)} \prod_{i=1}^{n} \eta(t_i; N_0, \gamma, \beta).
\]  

(6)

Taking the natural logarithm of Eq. (6) we obtain:

\[
\ln L(N_0, \gamma, \beta|\mathbf{s}) = -\mu(t_1; N_0, \gamma, \beta) + \sum_{i=1}^{n} \ln \eta(t_i; N_0, \gamma, \beta).
\]  

(7)

\[
= -\frac{N_0 \gamma s_i^\beta}{1 + y_s} + \sum_{i=1}^{n} \ln \left( \frac{N_0 \gamma s_i^\beta - 1}{1 + y_s} \right)
\]  

(8)

\[
= -\frac{N_0 \gamma s_i^\beta}{1 + y_s} + n \ln \gamma + n \ln \beta + n \ln N_0 + \beta \sum_{i=1}^{n} \ln s_i - 
\]  

\[
\sum_{i=1}^{n} \ln s_i - 2 \sum_{i=1}^{n} \ln (1 + y_s s_i^\beta).
\]  

(9)

Differentiating the above function with respect to N_0, \gamma, and \beta, we have

\[
\left\{ \begin{array}{c}
\frac{\partial \ln L(N_0, \gamma, \beta|\mathbf{s})}{\partial N_0} = -\frac{\gamma s_i^\beta}{1 + y_s} + \frac{n}{N_0}, \\
\frac{\partial \ln L(N_0, \gamma, \beta|\mathbf{s})}{\partial \gamma} = \frac{n}{\gamma} - \frac{N_0 s_i^\beta}{1 + y_s} + 2 \sum_{i=1}^{n} \frac{s_i^\beta}{1 + y_s}, \\
\frac{\partial \ln L(N_0, \gamma, \beta|\mathbf{s})}{\partial \beta} = \frac{n}{\beta} + \sum_{i=1}^{n} \ln s_i - \frac{N_0 \gamma s_i^\beta \ln s_i}{(1 + y_s s_i^\beta)} + 2 \sum_{i=1}^{n} \frac{s_i^\beta \ln s_i}{(1 + y_s s_i^\beta)}.
\end{array} \right.
\]  

The ML estimates can be obtained by setting the three expressions in Eq. (9) to zero as follows:
\[
\begin{aligned}
N_0 &= n \left( \frac{1 + \gamma s_n^\beta}{y s_n^\beta} \right), \\
\frac{n}{\gamma} - \frac{n}{y (1 + y s_n^\beta)} + 2 \sum_{i=1}^{n} \frac{s_i^\beta}{1 + y s_i^\beta} &= 0, \\
\frac{n}{\beta} + \sum_{i=1}^{n} \ln s_i - \frac{n \ln s_n}{1 + y s_n^\beta} + 2 \sum_{i=1}^{n} \frac{y s_i^\beta \ln s_i}{1 + y s_i^\beta} &= 0.
\end{aligned}
\]

(10)

Due to the lack of explicit solutions to the second and third expressions of Eq. (10), we numerically find the estimates the parameters \( \gamma \) and \( \beta \) then by substituting them in the first expression, \( \hat{N}_0 \) is obtained.

To get the variance and covariance matrix for the estimated parameters, we first need to calculate the Fisher information matrix [17], which is:

\[
F = \begin{bmatrix}
-\frac{\partial^2 \ln L}{\partial \delta^2} & \frac{\partial^2 \ln L}{\partial \delta \partial \gamma} & \frac{\partial^2 \ln L}{\partial \delta \partial \beta} \\
\frac{\partial^2 \ln L}{\partial \gamma \partial \delta} & -\frac{\partial^2 \ln L}{\partial \gamma^2} & \frac{\partial^2 \ln L}{\partial \gamma \partial \beta} \\
\frac{\partial^2 \ln L}{\partial \beta \partial \delta} & \frac{\partial^2 \ln L}{\partial \beta \partial \gamma} & -\frac{\partial^2 \ln L}{\partial \beta^2}
\end{bmatrix}
\]

(11)

where

\[
\frac{\partial^2 \ln L(N_0, \gamma, \beta | S)}{\partial \delta^2} = -\frac{n}{\delta^2},
\]

(12)

\[
\frac{\partial^2 \ln L(N_0, \gamma, \beta | S)}{\partial \gamma^2} = -\frac{n}{\gamma^2} + \frac{2 N_0 s_n^{2 \beta}}{(1 + y s_n^\beta)^2} - 2 \sum_{i=1}^{n} \frac{s_i^\beta}{(1 + y s_i^\beta)^2},
\]

(13)

\[
\frac{\partial^2 \ln L(N_0, \gamma, \beta | S)}{\partial \beta^2} = -\frac{n}{\beta^2} - \frac{2 N_0 s_n^{2 \beta} (\ln s_n)^2}{(1 + y s_n^\beta)^2} + 2 \gamma \sum_{i=1}^{n} \frac{s_i^\beta (\ln s_i)^2}{(1 + y s_i^\beta)^2},
\]

(14)

\[
\frac{\partial^2 \ln L(N_0, \gamma, \beta | S)}{\partial N_0 \partial \gamma} = -\frac{s_n^\beta}{1 + y s_n^\beta},
\]

(15)

\[
\frac{\partial^2 \ln L(N_0, \gamma, \beta | S)}{\partial N_0 \partial \beta} = \frac{y s_n^\beta \ln s_n}{1 + y s_n^\beta},
\]

(16)

\[
\frac{\partial^2 \ln L(N_0, \gamma, \beta | S)}{\partial \gamma \partial \beta} = \frac{N_0 s_n^\beta (\ln s_n - 1) \ln s_n}{1 + y s_n^\beta} + 2 \sum_{i=1}^{n} \frac{s_i^\beta \ln s_i}{(1 + y s_i^\beta)},
\]

(17)

The asymptotic variance-covariance matrix is obtained by:

\[
\Sigma = F^{-1}
\]
\[
\begin{bmatrix}
\text{Var}(N_0) & \text{Cov}(N_0, \gamma) & \text{Cov}(N_0, \beta) \\
\text{Cov}(N_0, \gamma) & \text{Var}(\gamma) & \text{Cov}(\gamma, \beta) \\
\text{Cov}(\beta, N_0) & \text{Cov}(\beta, \gamma) & \text{Var}(\beta)
\end{bmatrix}
\]
\text{(18)}

So, the \(100(1-\alpha)\%\) asymptotic confidence intervals for the parameters \(N_0, \gamma,\) and \(\beta\) of the NHPP LL model are given, respectively, by:

\[
\begin{align*}
\hat{N}_0 &- Z_{\alpha} \sqrt{\text{Var}(\hat{N}_0)}, \hat{N}_0 + Z_{\alpha} \sqrt{\text{Var}(\hat{N}_0)}, \\
\hat{\gamma} &- Z_{\alpha} \sqrt{\text{Var}(\hat{\gamma})}, \hat{\gamma} + Z_{\alpha} \sqrt{\text{Var}(\hat{\gamma})}, \text{and} \\
\hat{\beta} &- Z_{\alpha} \sqrt{\text{Var}(\hat{\beta})}, \hat{\beta} + Z_{\alpha} \sqrt{\text{Var}(\hat{\beta})}.
\end{align*}
\text{(19)-(21)}
\]

where, \(Z_{\alpha/2}\) is the percentile of standard normal distribution with right-tail probability \(\alpha/2, \text{Var}(\hat{N}_0), \text{Var}(\hat{\gamma}), \text{and Var}(\hat{\beta})\) are, respectively, the diagonal elements of the asymptotic variance and covariance matrix given by Eq. (18).

### 3.2. Confidence interval estimation of the conditional reliability function

According to the invariance property of the ML estimators, the estimate of the conditional reliability of the NHPP LL model is obtained by:

\[
\hat{R}(t_i, \hat{N}_0, \hat{\gamma}, \hat{\beta} | x_n) = \exp \left\{ -\hat{N}_0 \hat{\gamma} \left( \frac{(t_i + x_n)^{\hat{\beta}} - x_n^{\hat{\beta}}}{(1 + \gamma x_n^{\hat{\beta}})(1 + \gamma(t_i + x_n)^{\hat{\beta}})} \right) \right\},
\text{(22)}
\]

and its variance is defined as:

\[
V(\hat{R}) = \left( \frac{\partial R}{\partial N_0} \right)^2 \bigg|_{N_0 = \hat{N}_0} \text{Var}(\hat{N}_0) + \left( \frac{\partial R}{\partial \gamma} \right)^2 \bigg|_{\gamma = \hat{\gamma}} \text{Var}(\hat{\gamma}) + \left( \frac{\partial R}{\partial \beta} \right)^2 \bigg|_{\beta = \hat{\beta}} \text{Var}(\hat{\beta}) + \\
2 \left( \frac{\partial R}{\partial \gamma} \right) \left( \frac{\partial R}{\partial N_0} \right) \bigg|_{\gamma = \hat{\gamma}, N_0 = \hat{N}_0} \text{Cov}(\hat{\gamma}, \hat{N}_0) + \\
2 \left( \frac{\partial R}{\partial \beta} \right) \left( \frac{\partial R}{\partial \beta} \right) \bigg|_{\gamma = \hat{\gamma}, \beta = \hat{\beta}} \text{Cov}(\hat{\gamma}, \hat{\beta}),
\text{(23)}
\]

where

\[
\frac{\partial R}{\partial N_0} = -\gamma \left( \frac{(t_i + x_n)^{\hat{\beta}} - x_n^{\hat{\beta}}}{(1 + \gamma x_n^{\hat{\beta}})(1 + \gamma(t_i + x_n)^{\hat{\beta}})} \right) \exp \left\{ -N_0 \gamma \left( \frac{(t_i + x_n)^{\hat{\beta}} - x_n^{\hat{\beta}}}{(1 + \gamma x_n^{\hat{\beta}})(1 + \gamma(t_i + x_n)^{\hat{\beta}})} \right) \right\},
\text{(24)}
\]
The coefficient of multiple determination $R^2$ is used in our application to evaluate the model performance. Its formula is as follows [18]:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \bar{y}(u_i))^2}{\sum_{i=1}^{n}(y_i - \bar{y}(x_i))^2}$$

(28)
It takes the values from 0 to 1. The larger value of $R^2$ indicates better model performance. Though, the observed interval length is used to compare the CIs. The smaller the length, the better the confidence interval.

**Table 2: CSR2 data, 129 failures**

<table>
<thead>
<tr>
<th>760</th>
<th>758</th>
<th>303</th>
<th>6</th>
<th>22</th>
<th>14</th>
<th>42</th>
<th>4</th>
<th>84</th>
<th>15</th>
<th>221</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>41</td>
<td>1</td>
<td>153</td>
<td>409</td>
<td>54</td>
<td>24</td>
<td>44</td>
<td>180</td>
<td>397</td>
<td>19</td>
<td>145</td>
<td>36</td>
<td>54</td>
</tr>
<tr>
<td>1337</td>
<td>163</td>
<td>8</td>
<td>1</td>
<td>17</td>
<td>16</td>
<td>87</td>
<td>19</td>
<td>29</td>
<td>0</td>
<td>5</td>
<td>360</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>100</td>
<td>252</td>
<td>460</td>
<td>179</td>
<td>3</td>
<td>24</td>
<td>253</td>
<td>163</td>
<td>54</td>
<td>137</td>
<td>328</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
<td>18</td>
<td>9</td>
<td>75</td>
<td>15</td>
<td>366</td>
<td>428</td>
<td>212</td>
<td>115</td>
<td>264</td>
<td>269</td>
<td>276</td>
</tr>
<tr>
<td>1</td>
<td>999</td>
<td>30</td>
<td>495</td>
<td>472</td>
<td>344</td>
<td>550</td>
<td>131</td>
<td>47</td>
<td>92</td>
<td>863</td>
<td>991</td>
<td>35</td>
</tr>
<tr>
<td>9549</td>
<td>249</td>
<td>607</td>
<td>83</td>
<td>614</td>
<td>352</td>
<td>673</td>
<td>4179</td>
<td>111</td>
<td>75</td>
<td>407</td>
<td>288</td>
<td>894</td>
</tr>
<tr>
<td>1314</td>
<td>845</td>
<td>55</td>
<td>409</td>
<td>36</td>
<td>15</td>
<td>1960</td>
<td>60</td>
<td>19</td>
<td>20</td>
<td>79</td>
<td>24</td>
<td>1737</td>
</tr>
<tr>
<td>7984</td>
<td>10</td>
<td>20</td>
<td>338</td>
<td>250</td>
<td>1682</td>
<td>212</td>
<td>287</td>
<td>56</td>
<td>4973</td>
<td>3500</td>
<td>59</td>
<td>98</td>
</tr>
<tr>
<td>2439</td>
<td>1812</td>
<td>6203</td>
<td>385</td>
<td>3500</td>
<td>4892</td>
<td>687</td>
<td>62</td>
<td>2796</td>
<td>3268</td>
<td>3845</td>
<td>76</td>
<td></td>
</tr>
</tbody>
</table>

**Table 3: SYS2 data, 86 failures.**

<table>
<thead>
<tr>
<th>479</th>
<th>266</th>
<th>277</th>
<th>554</th>
<th>1034</th>
<th>249</th>
<th>693</th>
<th>597</th>
<th>117</th>
<th>170</th>
<th>117</th>
<th>1274</th>
<th>469</th>
</tr>
</thead>
<tbody>
<tr>
<td>1174</td>
<td>693</td>
<td>1908</td>
<td>135</td>
<td>277</td>
<td>596</td>
<td>757</td>
<td>437</td>
<td>2230</td>
<td>437</td>
<td>340</td>
<td>405</td>
<td>535</td>
</tr>
<tr>
<td>277</td>
<td>363</td>
<td>522</td>
<td>613</td>
<td>277</td>
<td>1300</td>
<td>821</td>
<td>213</td>
<td>1620</td>
<td>1601</td>
<td>298</td>
<td>874</td>
<td>618</td>
</tr>
<tr>
<td>2640</td>
<td>5</td>
<td>149</td>
<td>1034</td>
<td>2441</td>
<td>460</td>
<td>565</td>
<td>1119</td>
<td>437</td>
<td>927</td>
<td>4462</td>
<td>714</td>
<td>181</td>
</tr>
<tr>
<td>1485</td>
<td>757</td>
<td>3154</td>
<td>2115</td>
<td>884</td>
<td>2037</td>
<td>1481</td>
<td>559</td>
<td>490</td>
<td>593</td>
<td>1769</td>
<td>85</td>
<td>2836</td>
</tr>
<tr>
<td>213</td>
<td>1866</td>
<td>490</td>
<td>1487</td>
<td>4322</td>
<td>1418</td>
<td>1023</td>
<td>5490</td>
<td>1520</td>
<td>3281</td>
<td>2716</td>
<td>2175</td>
<td>3505</td>
</tr>
<tr>
<td>725</td>
<td>1963</td>
<td>3979</td>
<td>1090</td>
<td>245</td>
<td>1194</td>
<td>994</td>
<td>3902</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 4.1. Numerical results

The ML estimates and CIs at 95% significance level of the parameters $N_0$, $\gamma$, and $\beta$ are assessed using Eqs. (19), (20), and (21) corresponding to the last failure number of each data sets. For the comparison purpose the observed interval lengths of the CIs are computed as follows:

$$2Z_{\alpha/2}[\text{Var}(\tilde{N}_0)]^{1/2}, 2Z_{\alpha/2}[\text{Var}(\tilde{\gamma})]^{1/2}, 2Z_{\alpha/2}[\text{Var}(\tilde{\beta})]^{1/2}.$$  

Also, to assess the model performance the coefficient of multiple determination criteria is computed for each of the selected data set, the results are summarized in Table 4.
Table 4: Estimated Parameter Values of the NHPP LL Model and 95% Confidence Intervals.

<table>
<thead>
<tr>
<th>Data set</th>
<th>Failure Number</th>
<th>( N_0 )</th>
<th>( \hat{\gamma} )</th>
<th>( \hat{\beta} )</th>
<th>( \text{C.I. Lower} )</th>
<th>( \text{C.I. Upper} )</th>
<th>( \text{Observed Interval Length} )</th>
<th>( \text{Observed Interval Length} )</th>
<th>( \text{R}^2 )</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>NTDS data</td>
<td>34</td>
<td>53.3639</td>
<td>0.0387</td>
<td>0.4484</td>
<td>0.53614</td>
<td>0.0387</td>
<td>0.4947</td>
<td>0.5411</td>
<td>0.7724</td>
<td>0.6393</td>
</tr>
<tr>
<td>CSR2 data</td>
<td>129</td>
<td>204.4341</td>
<td>0.024600</td>
<td>0.2936</td>
<td>205.0794</td>
<td>0.024601</td>
<td>0.303</td>
<td>0.3123</td>
<td>0.6052</td>
<td>0.6052</td>
</tr>
<tr>
<td>SYS2 data</td>
<td>86</td>
<td>199.9347</td>
<td>0.0199058</td>
<td>0.2704</td>
<td>201.0775</td>
<td>0.0199064</td>
<td>0.2745</td>
<td>0.2786</td>
<td>0.0927</td>
<td>0.0927</td>
</tr>
</tbody>
</table>

For the last three failure numbers, Table 5 illustrates the estimated conditional reliability of the NHPP LL model which is calculated using Eq. (22) and the corresponding 95% CIs with their observed interval lengths which are found using Eq. (27) and \( 2Z_{\alpha/2}[Var(\hat{R})]^{1/2} \), respectively.

Table 5: 95% Confidence Intervals of the conditional reliability function of the NHPP LL model.

<table>
<thead>
<tr>
<th>Data set</th>
<th>Time to failure</th>
<th>Estimated reliability at Time t</th>
<th>( \text{C.I. Lower} )</th>
<th>( \text{C.I. Upper} )</th>
<th>Observed Interval Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>NTDS data</td>
<td>258</td>
<td>0.117</td>
<td>0.091</td>
<td>0.143</td>
<td>0.0536</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>0.113</td>
<td>0.087</td>
<td>0.139</td>
<td>0.0528</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>0.1047</td>
<td>0.0787</td>
<td>0.1307</td>
<td>0.052</td>
</tr>
<tr>
<td>CSR2 Data</td>
<td>3268</td>
<td>0.1592</td>
<td>0.1797</td>
<td>0.2002</td>
<td>0.0413</td>
</tr>
<tr>
<td></td>
<td>3845</td>
<td>0.1671</td>
<td>0.1466</td>
<td>0.1876</td>
<td>0.0412</td>
</tr>
<tr>
<td></td>
<td>76</td>
<td>0.1669</td>
<td>0.1464</td>
<td>0.1874</td>
<td>0.041</td>
</tr>
<tr>
<td>SYS2 data</td>
<td>1194</td>
<td>0.168</td>
<td>0.1345</td>
<td>0.2016</td>
<td>0.0679</td>
</tr>
<tr>
<td></td>
<td>994</td>
<td>0.1652</td>
<td>0.1316</td>
<td>0.1987</td>
<td>0.0675</td>
</tr>
<tr>
<td></td>
<td>3902</td>
<td>0.1545</td>
<td>0.121</td>
<td>0.1881</td>
<td>0.6052</td>
</tr>
</tbody>
</table>
Figures [1-3] demonstrate the estimated conditional reliability function and the corresponding 95% CIs for each selected data set.

**Figure 1:** Estimated reliability with 95% interval based on the NHPP LL model, NTDS data

**Figure 2:** Estimated reliability with 95% interval based on the NHPP LL model, CSR2 Data
From Tables 4 and 5, it can be noticed that the CIs estimated by the MLE method have small observed interval length which indicates the accuracy of these CIs. The assessment results, in Table 4, show that the estimator $\hat{\gamma}$ has the shortest observed interval length of the three selected data sets. Also, according to the model accuracy using the $R^2$ criteria the results in Table 4 show that the NHPP model fits best the NTDS data, then CSR2 Data and SYS2 data take the second and third rank, respectively. Regarding the lengths of the CIs presented in Table 5 it can be seen that as the number of detected failures increases narrower intervals of the conditional reliability function are obtained. The estimator $\hat{R}$ has the shortest observed interval length for the CSR2 Data.

5. Conclusion

It is essential to the software reliability measurement to obtain the confidence bounds for the reliability metrics at any future time $t$. The reliability function of a software system is an important metric for describing the system’s reliability. Our main contribution in this paper is to construct CI for the conditional reliability function of a NHPP model based on the LL distribution.

CIs of the parameters and conditional reliability function of the NHPP LL model have been constructed based on the MLE method and evaluated via the observed interval length. The model performance has been checked using the $R^2$ criteria.

The application results demonstrate reasonable results for the CIs of the conditional reliability, which can help in improving the decision-making quality of software testing and debugging. Future research may find CIs for other reliability metrics of the NHPP models.
References

Author Information

Lutfiah Ismail Al turk is currently working as associate professor of mathematical statistics in Statistics Department at Faculty of Sciences, King AbdulAziz University, Jeddah, Kingdom of Saudi Arabia. Lutfiah Ismail Al turk obtained her B.Sc degree in statistics and computer science from Faculty of Sciences, King AbdulAziz University in 1993 and M.Sc (mathematical statistics) degree from Statistics Department, Faculty of Sciences, King AbdulAziz University in 1999. She received her Ph.D in mathematical statistics from university of Surrey, UK in 2007. Her current research interests include software reliability modeling and statistical machine learning.

Email: lturk@kau.edu.sa


Address: P.O. Box 42713 Jeddah 21551. Kingdom of Saudi Arabia.