

A New Hybrid Optimization Model under Outsourcing Constraint: A Tunisian Case Study

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Abstract

This paper introduces a new Hybrid Optimization Model under Outsourcing Constraint. It integrates the constraint of outsourcing over the product life cycle of product. It came out of the hybridization of two well-known models. The first is the new product diffusion model which considers the interaction between manufacturing and marketing decisions within a company and ignores other functions like sourcing and distribution. The second model is the configuring a new supply chain (SCC) which selected the option that minimizes the total supply chain cost. The new hybrid model simultaneously determines optimal production and delivery plan since it allows to specify when and how much to subtract. More precisely, our Model under Outsourcing Constraint aim to maximize net profit and facilitate decision-making regarding strategy of Outsourcing. The proposed model is applied in practical case with several scenarios to identify the optimal plan which defines the correct quantities to be produced or distributed by company resources and those to be outsourced to maximize the company's net revenue. A comparison between the results obtained by our model shows that the Hybrid Optimization Model under Outsourcing Constraint are better than the real data obtained by the strategy of the company.

Keyword: Supply chain; delivery; outsourcing.

1. Introduction

Outsourcing is one of the most replicated forms of collaboration today, as it represents a strategy for the organization internally to focus on the mainly process and create a network of external suppliers for the secondary activities. Using subcontracting by a company is justified either by a financial and / or insufficient organizational or by a need for technical failure. The choice of integrating the outsourcing can be benefits for a company from several advantages. It allows the company to focus on more strategic and creative tasks and to have a good control of costs and manufacturing quality.

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It prevents the company from making additional investment in equipment and accessing technologies or processes that are not part of these competencies. Outsourcing can be beneficial, but it is necessary to choose in which process the Supply Chain will be integrated.

Despite the fact that there is a lack of models in the literature that considers the presence of outsourcing, we will be based on two main works to achieve our new model. The model of new product diffusion (NPD) proposed by [1], this is an extension of the classical Bass in which consider the interaction between manufacturing and marketing decisions within a company and ignore other functions like sourcing, procurement and distribution. Graves and Williams [2] proposed a model for configuring a new supply chain (SCC). In their research, supply chain is modeled as a network whose nodes represent functional requirements in the supply chain to satisfy this functional requirement, multiple options can exist. The (SCC) model aim to select the options that minimizes the total supply chain cost is composed by the sum of three relevant costs: goods sold cost, safety stock cost, and pipeline stock cost.

Amini and Li [3] study the impact of the demand trajectory during new product diffusion on the supply chain configuration; this is not treated by the SCC model. These authors consider, also that the problems of the NPD and SCC models are closely related.

Therefore, Amini and his colleagues in [3] developed an integrated optimization model for configuring new product's supply chains. This model consider explicitly the impact of demand dynamics during new product's diffusion and provide both the demand/supply pattern and unit-product cost., some models ignore the demand quantity and the planned period specified when the demand growth .The SCC, NPD and the hybrid optimization model are interested to configure the supply chain for a dynamic quantity of demand which is adapted to the company internal resources. For this reason, it will be a necessity to integrate the outsourcing as a solution to ask for external service delivery to cover the exceeded demand.

The aim of this paper is to integrate the outsourcing constraint in the supply chain configuration for diffusion of new product .The proposed model define when and how much quantity must be outsourced during the production and delivery process. Our model can be considered as a decision support tool that provides a modeling framework, to design a supply chain, adaptive to the changing customer demand during new product's lifetime.

The remainder of this paper is organized as follows. Section 2 presented a related literature. In section 3, we present our mathematical model. Computational results on real testes are interpreted in section 4. Finally, section 5 summarizes the main conclusions.

2. Related Literature

In order to present the specifics of our model that considers the integration of the outsourcing within the supply chain configuration, we will give an overview on the various researches integrating the subcontracting and the configuration of the logistics chain during distribution of new products.

Mohanty and Mishra (2010) [4] searched the method that allows the company making the decision to outsource production in order to satisfy the high demand and maximize the benefit. They used the model TOC (theory of constraint), PL (linear programming) and the standard of accounting. The results obtained by these three methods shown that the analysis using the TOC method is more efficient and easier to implement. But these solutions are not always better than the result obtained by the PL which depends on the margin of the entrepreneur and the calculation time.

Lu (2012) [5] studied the problem of companies that manufacture many products in multiple periods of stochastic demands. These companies have, in general, two alternatives for the production: the first is to outsource the production of the components of the product to external suppliers then they make the assembly. The second, the company manufactured the components internally and the assembly is done outside. The authors in [5] seeks way to have the best compromise between these two alternatives by proposing an analytical approach. Numerical experiments show the significant reduction in costs that can be obtained by using the dynamic programming model.

The work of the authors in [3] presents a first attempt to model the interaction between the configuration of the logistics chain and the dissemination of the new product, taking into account the impact of the dynamics of the application during the distribution of the product. They develop a hybrid optimization model while explicitly considering the impact of demand dynamics during new products diffusion. This model simultaneously determines optimal production and sales plan and supply chain configuration. They realized a computational experimental study to analyze the performance of the model under various topologies characterized by problem parameters from both the product diffusion and supply chain configuration perspectives. The comparison between solutions obtained by the integrated model and those found by different heuristic policies show the effectiveness of the model of the authors in [3]. The model does not necessarily lead to maximum amount of sales revenue from the marketing perspective, or minimum amount of costs from the supply chain configuration perspective. It balances the tradeoffs among various cost and revenue components. In the same context, Nagurney and Li [6] develop a supply chain network game theory model with product differentiation, possible outsourcing of production and distribution, and quality and price competition. They developed an algorithm, which provides a discrete-time adjustment process and tracks the evolution of the product flows, quality levels, and prices over time, is proposed and convergence results given. The model can be applied in practice for the case of pharmaceuticals products to fast fashion to high technology products.

Sarker and Giri [7] affirm that the company can be outsourcing part or all of their supply chain operations to reduce the burden of logistics activities and achieve customer satisfaction and overall performance. Their work aims to improve the performance of a supply chain consisting of a monopolistic manufacturer, a third party logistics service provider (TPLSP) and multiple independent retailers.

3. Hybrid Optimization Model under Outsourcing Constraint: “HOMOC”

In this section we formulated our mathematical model that presents an extension of hybrid model of Authors in [3].

3.1 Problem statement

The work of Amini and his colleagues in [3] presents a first attempt to model the interaction between the configuration supply chain management and the diffusion of the new product. This hybrid model has also provided a modeling framework for designing a supply chain that is not only cost-effective but also adaptable to changing market demand.

The objective function is to maximize the total net profit of the life of the new product which is calculated by the difference between the total income of the life cycle product and the supply chain configuration costs. The authors in [3] concluded that equation:

Maximize: Total Net Profit = Total Life - Cycle Revenue – SCC costs.

Where:

$$\text{Total Life-Cycle Revenue} = \sum_{t=0}^T \bar{C}_N \left[\pi y_t - r_t - wI_t - hI_t \right] \quad (1)$$

Supply Chain Configuration Costs = Product stock cost + Safety stock cost

$$\left\{ \begin{array}{l} \text{Product stock cost} = \sum_{i=1}^N h(\bar{C}_i \frac{C_i}{2}) P_i \mu_i \end{array} \right. \quad (2)$$

$$\text{Safety stock cost} = \sum_{i=1}^N h \bar{C}_i k \sigma_i \sqrt{s_i^{in} + p_i - S_i^{out}} \quad (3)$$

This hybrid model doesn't take into account the importance of outsourcing although it represents a potential in the supply chain.

The outsourcing has many advantage, the company can focus on its core business and have a better responsiveness to the increase in market demand. In addition, it allows an increase production volume and flexibility of production processes and reduced risk of technical malfunctions.

Also, the previous model is interested in configuring the supply chain for an unknown quantity of demand without synchronizing with inventory management. For that, it would be important to integrate outsourcing as a solution to cover outdated demands and satisfy the quantities requested.

3.2 Model formulation

Our new model named “Hybrid Optimization Model under Outsourcing Constraint: HOMOC” is based on the model of authors in [3] but by adding two essential constraints in order to integrate outsourcing in the process of

production and delivery (distribution of product).

- Variables: two additional variables are introduced and they are defined as:

$$\alpha \in [0,1] \begin{cases} \text{if } \alpha = 1 ; \text{ Locally produced quantity} \\ \text{if } \alpha = 0 ; \text{ Quantity produced outsourced} \\ \text{if } 0 \leq \alpha \leq 1 ; \text{ Combination between locally and outsourced production} \end{cases}$$

$$\beta \in [0,1] \begin{cases} \text{if } \beta = 1 ; \text{ Locally delivered quantity} \\ \text{if } \beta = 0 ; \text{ Quantity delivered by outsourced} \\ \text{if } 0 \leq \beta \leq 1 ; \text{ Combination between locally and outsourced delivery} \end{cases}$$

The production can be realized locally then we according the variable α or outsourced ($1 - \alpha$). Same for the delivery process, if it's ensured locally we according the variable β if not we have outsourced delivery so ($1 - \beta$).

- Objective Function: its be formulated as follows:

Total Net Profit = Total Life - Cycle Revenue - Supply Chain Configuration Costs – outsourcing costs (production and distribution).

The objective function is to maximize net income over the life of the new product. The net income is difference between total revenue, product lifecycle, cost of storage and outsourcing costs of production and delivery products.

$$\begin{aligned} \text{Max } Z(\alpha, \beta) = & \left[\sum_{t=0}^T \pi_t y_t - \sum_{t=0}^T \overline{C}_N (\alpha r_{t \text{ loc}} + (1 - \alpha) r_{t \text{ sout}}) \right] - \left[\sum_{i=1}^N h(\overline{C}_i \frac{C_i}{2}) P_i \mu_i + \sum_{i=1}^N h \overline{C}_i k \sigma_i \sqrt{s_i^{\text{in}} + p_i - S_i^{\text{out}}} \right] \\ & - \left[\beta C_{\text{livloc}} + (1 - \beta) C_{\text{livsout}} \right] \end{aligned} \tag{4}$$

As shown this objective function is composed by three formulation.

First Formulation: Total life cycle of the product revenue based on cost of outsourcing production

=

$$\left[\sum_{t=0}^T \pi_t y_t - \sum_{t=0}^T \bar{C}_N (\alpha r_{t \text{ loc}} + (1-\alpha) r_{t \text{ sout}}) \right] \tag{5}$$

Where:

$$\left\{ \begin{array}{l} \bar{C}_N : \text{Cumulative cost means of sums } N \\ \pi_t y_t : \text{Selling price ratio Sales of product at time } t \\ \alpha r_{t \text{ loc}} : \text{Production cost at time } t \end{array} \right.$$

$$(1-\alpha) r_{t \text{ sout}} : \text{Outsourced production at time } t$$

$$WL_t : \text{Waiting cost rate per unit backlogged per unit time}$$

$$hI_t : \text{Inventory holding cost rate per time period}$$

$$\text{Second Formulation: Storage cost} = \sum_{i=1}^N h \left(\bar{C}_i - \frac{C_i}{2} \right) P_i \mu_i + \sum_{i=1}^N h k \bar{C}_i \sigma_i \sqrt{s_i^{\text{in}} + p_i - S_i^{\text{out}}} \tag{6}$$

We note:

$$\bar{C}_i - \frac{C_i}{2} : \text{Average value of goods over its lead time } p_i$$

$$s_i^{\text{in}} + p_i - S_i^{\text{out}} \geq 0 : \text{Net replenishment time of a function}$$

With:

$$\left\{ \begin{array}{l} p_i : \text{Lead time of function } i \\ \mu : \text{Demand } \mu_i \text{ from its immediate successors} \\ K : \text{Incidents or } z\text{-value} \end{array} \right.$$

σ_i : Standard deviation demand

$$\text{Third Formulation: Diffusion cost outsourced} = \beta C_{livloc} + (1 - \beta) C_{livsout} \quad (7)$$

We note:

$$\left\{ \begin{array}{l} \beta C_{livloc} : \text{Local Delivery Cost at time } t \end{array} \right.$$

$(1 - \beta) C_{livsout}$: Outsourced delivery cost at time t

Constraints

A set of Constraints is added to our formulation. These Constraints are presented and described in this section.

$$\bar{c}_N = \sum_{i=0}^N \frac{c_i}{N} ; \quad \forall i \in \{0, 1, \dots, N\} \quad (8)$$

$$R = \sum_{t=0}^T r_t ; \quad \forall t \in \{0, 1, \dots, T\} \quad (9)$$

$$Y = \sum_{t=0}^T y_t ; \quad \forall t \in \{0, 1, \dots, T\} \quad (10)$$

$$I_t = R - Y ; \quad \forall t \in \{0, 1, \dots, T\} \quad (11)$$

$$p_{i+1} = p_{i-1} + p_i ; \quad \forall i \in \{0, 1, \dots, N-1\} \quad (12)$$

$$s_i^{in} + p_i - s_i^{out} \geq 0 ; \quad \forall i \in \{0, 1, \dots, N\} \quad (13)$$

$$S_i^{in} \geq S_i^{out} ; \quad \forall i \in \{0, 1, \dots, N\} \quad (14)$$

$$S_i^{in}, S_i^{out} \geq 0 ; \quad \forall i \in \{0, 1, \dots, N\} \quad (15)$$

$$0 \leq \alpha \leq 1 ; \quad 0 \leq \beta \leq 1 \quad (16)$$

Constraints (8) represent the cumulative cost of the finished product (cumulative direct cost). Constraints (9) and (10) calculate the cumulative production, demand and sales of the product in each time period, respectively. Constraints (11) calculate the inventory of finished product in each time period as the difference between cumulative production and cumulative sales.

Constraints (12) calculate the cumulative periodicity of execution of a function i . Constraint (13) states that the inbound service time of a function must be no less than the outbound service time of its immediate predecessors.

Constraint (14) ensures that the time of execution of function i depend on delivery time. Constraints (15) insure that the inbound and the outbound service times must be positive.

Finally, constraints (16) are respectively related to the quantity produced and delivered by the company or by outsourced company.

4. Experimental results and discussion

Our model is applied to the case of a Tunisian company " Printing Reliure d'Art (IRA) ", it's an industrial company specialized in printing books, revues and magazine and other printing works.

The objective is to validate our contribution, check the improvement of the results and help to make the best decision in the company.

4.1 The context of the study

The company "IRA" is specialized in printing works, it has a wide range of articles that can be classified into three categories: the first contains all types of books presenting the most important category for the company.

The second is called City works contains business cards, headers and flayers.

The last category includes all types of leaflets.

We collected from the company the measurements by function for the three types of categories, which are:

- The book category;
- The leaflets category;
- The city works category.

For the item book, we have identified the values of this product during cycle of production at each level function (N) from $N=0$ until $N=10$. So we have the following parameters:

- Fixed parameters: they are constants whatever the case and the category they do not change and presented in the table (1).

Table 1: Fixed parameters

Les paramètres	Les valeurs
Profit for sale price	0.35
h : Cost of stock / unit	0.004
K : incidents	0.1
μ_i : interne demande	0.1
σ_i : Average demand	0.1

- Variable parameters: they vary according to the case. For example for pound 1, they are grouped in the following table (2).

Table 2: Variable parameters according to category

Parameters	Values according to category		
	Book	Leaflets	City works
N (fonctions)	10	5	4
T (time)	10	5	4
Labor cost	0.03 ; 0.03 ; 0.1 ; 0.035 ; 0.035 ; 0.08 ; 0.14 ; 0.21 ; 0.25 ; 0.06	0.01 ; 0.02 ; 0.01 ; 0.02 ; 0.01	0.01 ; 0.02 ; 0.01
Completion rate	0 ; 0.05 ; 0.1 ; 0.1 ; 0.05 ; 0.05 ; 0.2 ; 0.25 ; 0.15 ; 0.05	0.05 ; 0.75 ; 0.10 ; 0.05 ; 0.05	0.01 ; 0.02 ; 0.01
Waste rate	0 ; 0 ; 0 ; 0 ; 0 ; 0.01 ; 0.02 ; 0.03 ; 0.02 ; 0	0.01 ; 0.02 ; 0.01 ; 0.01 ; 0.02 ; 0.01 ; 0.01	0.01 ; 0.02 ; 0.01
Periodicity	0 ; 0 ; 10 ; 0 ; 10 ; 5 ; 10 ; 4 ; 1	1 ; 1 ; 0 ; 1 ; 0	0 ; 1 ; 0
S_i^{in}	0 ; 10 ; 10 ; 10 ; 11 ; 11 ; 16 ; 26 ; 30 ; 31	0 ; 0 ; 0 ; 0 ; 0	0 ; 0 ; 0
S_i^{out}	0 ; 0 ; 0 ; 1 ; 0 ; 5 ; 5 ; 10 ; 5 ; 1	0 ; 0 ; 0 ; 0 ; 0	0 ; 0 ; 0
Selling price	1.7	0.250	0.060
Quantity produced	20000	5000	2500
Rate of local production cost	0.65	0.20	0.20
Outsourced cost of production	0.55	0.25	0.25
Unit cost of local delivery	0.168	0.005	0
Unit cost of outsourced delivery	0.136	0.01	0
Machine cost	0.41	0.13	0.13
Number of days	30	2	2

Table (2) defines the parameters according to the main production process (supplying - processing and delivery).

4.2 Experimental result

The objective of our work is to construct a guide for the decision-maker to help him tacking the best decision in the supply chain configuration projects and whether to include or not the outsourcing constraint in the

production process and in the delivery.

The compilation takes values between 0 and 1 for α and β with a range of 0.01 hence for α value it affects 100 β values and contrary. In fact, we will have several results (net profit) for each article. After compilation and comparison of the results, it's the Maximum value of the net profit with the optimal values of α and β corresponding to it.

- The book category

After illustrating the used parameters in our practical case, we present a summary table of the results with the various scenarios studied.

Table 3: The obtained results for the various scenarios

Case1 : $\alpha = 1 ; \beta = 1$	Solution1 : Max Z (1, 1)= 6423,97
Case2 : $\alpha = 1 ; \beta = 0$	Solution 2 : Max Z (1, 0)= 6875,877
Case3 : $\alpha = 0 ; \beta = 0$	Solution 3 : Max Z (0, 0)=11823,07
Case4 : $\alpha = 0 ; \beta = 1$	Solution 4 : Max Z (0,1)=8879,320
Case 5 : $0 < (\alpha, \beta) < 1$	Solution 5 : Max Z (0,0.78)= 9630,06

We will interpret the different cases presented in the table:

Case 1 : Local Production and Local delivery

This table shows the case where the production and delivery are locally carried out by the company in this case we have $\alpha=1$ and $\beta=1$. This configuration allows to achieve a net profit equal to $Z (1, 1) = 6423.97$ TND.

Table 12

	Life cycle cost	Storage cost	Delivery cost	Net Profit
$\alpha = 1; \beta = 1$	10573.441	309.471	3840	6423,97

This profit despite it being positive and beneficial but it does not generate the best solution for the company.

Case 2: Local Production and outsourced Delivery

The main role of company is the process of production therefore $\alpha=1$, while the process of delivery and distribution is delegated to a to a service provider which $\beta=0$. This combination gives $\text{Max } Z (1, 0) = 6875.877$

TND.

Table 13

	Life cycle cost	Storage cost	Delivery cost	Net Profit
$\alpha = 1; \beta = 0$	9704.201	208.324	2620	6875.877

This result presented a benefit of 12% of net result (local production and delivery). The rate of net profit is calculated as following: $(6875.877 - 6423.97) / 6423.97 = 7.03\%$. This result is higher than that found in case 1 thus outsourced delivery generated additional profit to the company.

Case 3: Outsourced Production and outsourced delivery

This case presents a complete outsourcing for the process of production and delivery ($\alpha=0, \beta=0$). The company's role is limited to the life cycle management of the product.

Table 14

	Life cycle cost	Storage cost	Delivery cost	Net Profit
$\alpha = 0; \beta = 0$	14599.27	105.380	2630	11823.89

This combination presents to society the most profitable case since the generated profit which equal to: $\text{Max } Z(0, 0) = 11823.89$ TND.

Case 4 : Outsourced Production and local delivery

We will reverse the roles presented in the case 2, where the company will focus on distribution of product $\beta=1$ while the production will be entrusted to an external company under contract therefore $\alpha=0$. So, we obtained $\text{Max } Z(0, 1) = 8879,320$ TND.

Table 15

	Life cycle cost	Storage cost	Delivery cost	Net Profit
$\alpha = 0; \beta = 1$	12327,82	128.5	3320	8879,320

The result obtained in this case is better than that in 1 and 2 but case 3 represents until the best combination.

We assume that we can find a better option to maximize net profit by identifying a percentage of the production that will be processed by the company and outsourcing the rest, product. Since, we will vary the variables α and β randomly in the case 5.

Case 5: Variation of $(0 \leq (\alpha, \beta) \leq 1)$

The case 5 presents a random variation of the two variables α and β between 0 and 1 in order to identify the best combination between local or outsourced production and the same for delivery. This case is executed in two steps, during the first step we set the value of α between 0 and 1 and vary β between 0 and 1 until identified the best combination, such as $Z(0, 0.78) = 9630.06$ TND. Then, we reversed the roles by varying α between 0 and 1 and fixing the value of β between 0 and 1. This gave a variety of results represented in table (4).

Table 4: The obtained results when α and $\beta \in [0,1]$

A	B	Max Z (α, β)
1	0.65	4325.197
0	0.78	9630.06
0.35	1	7820.743
0.77	0	6630.277

The result can be presented also in the figure (1). The variation of the net profit $\text{Max Z}(\alpha, \beta)$ for item book, we conclude that the Shape of the curve reaches its maximum when $\alpha=0$ and $\beta=0.78$.

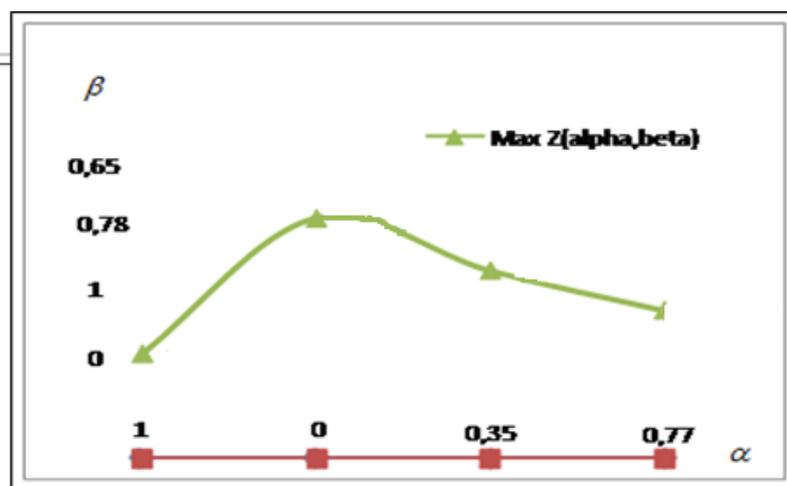


Figure 1: Variation of α and β

- The leaflets category

In this section, the product category were changed in order to assess the effect and effectiveness of outsourcing on differ items in the company. For the leaflet, we tested the following 3 scenarios:

Scenario 1: α and β are fixed either at 0 or 1

The solutions showed that when the company produces this article locally the profitability is higher compared to the results obtained by outsourcing. Net profit is maximized when production and delivery will do locally this the case 1 as show in table (5) providing the various cases for the leaflets category.

Table 5: The obtained results by Scenario 1

	α	B	Max Z (α, β)
Cas1	1	1	175,73
Cas2	1	0	150,73
Cas3	0	1	162,7
Cas4	0	0	137,7

Scenario 2: we set α (either $\alpha = 0$, or $\alpha = 1$) and we vary β and vice versa.

Our model is used to evaluate whether the outsourcing of the production or delivery is necessary. Its role is to determine whether these activities are to be carried out internally or externally. It is thus perfectly applicable for the cases presented in table (6).

Table 6: The best solution for the leaflets category

A	B	Max Z (α, β)
1	0.40	188.68
0	0,7	155,2
0,38	1	167,65
0,15	0	139,66

Firstly, the company decides to make its production locally and outsources a part of its delivery ($\beta= 0.4$) so this represent the best solution.

Secondly, IRA has changed its strategy and has resorted to the subcontracting of all the production and part of the delivery.

Scenario 3: Variation of α and β between 0 and 1

The company will carry out a part of its activities (production / delivery) while remaining responsible for the design.

Regarding the table (7), the case giving the optimal solution is that which has the index relative to

$\alpha_{sout} = 0.8$ and $\beta_{sout} = 0.7$. So, concerning the locally production and delivery we have $\alpha_{loc} = 0.2 < \beta_{loc} = 0.3$.

In this case, the company uses outsourced production or it uses inventory management to satisfy the demand of its customers.

Table 7: Variation of α and β for the scenario 3

A	B	Max Z (α, β)
0.18	0.33	148.3
0.53	0.23	150.36
0.23	0.54	154.2
0.5	0.5	156.72
0.8	0.7	165.63

The city works category

The city works category consists of three basic elements: header, visiting card and flayer.

Based on the strategy of the IRA Company where each customer takes care of its delivery, we will eliminate the cost of delivery in our mathematical modeling.

For all the articles manufactured and presented below we will take the case where α varied and β not applied (NA) in delivery.

$$\begin{cases} C_{livloc} = 0 \\ C_{livosout} = 0 \end{cases} \implies [\beta C_{livloc} + (1 - \beta) C_{livosout}] = 0$$

Header category: for

this item we canceled the delivery variable in our basic model.

Hence, we will calculate the costs of the entire product lifecycle without distribution.

Then our model will be limited to:

$$MaxZ(\alpha, \beta) = \left[\sum_{t=0}^T \pi_t y_t - \sum_{t=0}^T \overline{C}_N (\alpha r_{t \text{ loc}} + (1-\alpha) r_{t \text{ sout}}) \right] - \left[\sum_{i=1}^N h(\overline{C}_i - \frac{C_i}{2}) P_i \mu_i + \sum_{i=1}^N hk\overline{C}_i \sigma_i \sqrt{s_i^{in} + p_i - S_i^{out}} \right]$$

The result presented in table (8) show that the maximum is reached when the production process is carried out locally.

In this case the net profit has the value $Max Z = (1, NA) = 44.86$ when $(\alpha = 1)$.

Table 8: Best result for the Header category

A	B	Max Z (α, β)
1	NA	44.86
0	NA	41.99
0.35	NA	43
0.18	NA	43.51
0.42	NA	43.20
0.10	NA	42.28
0.63	NA	43.80
0.82	NA	44.29
0.50	NA	43.43

Business Card Category : For this case we also note that the highest gain is realized when the company uses its internal capacities for the production process.

Table 9: Best result for the Business Card category

A	B	Max Z (α, β)
1	NA	21.16
0	NA	24.67
0.15	NA	20.08
0.25	NA	20.14
0.63	NA	20.66
0.10	NA	19.93
0.83	NA	20.93
0.76	NA	20.84

As show in the table (9), the company changed its strategy regarding this article. We noticed that when it is in charge of the production it loses a lot of time and money since this article is specific for each customer. Each one has its own design and its specificities.

Customer satisfaction for this item is average because the company is not a specialist for creating cards so preferences it gives this task to a subtracted. Making it possible to realize more profit by comparing it with its old strategy

Category flayer: We can conclude from table (10) the net profit is maximized when the company takes over the production process.

Table 10: Best result for the Category flayer

	A	B	Max Z (α, β)
1		NA	86.08
0		NA	80.61
0.15		NA	81.43
0.25		NA	81.98
0.63		NA	84.85
0.35		NA	82.92

4.3 Comparison between the results of our model and the real data

We present a summary table (11) in which we present the net benefits for each item.

Table 11: Comparison between our model and the real data

Category of Product	Strategy IRA	Our Model		
	Net profit	α	B	Net profit
Book	5344.948	1	0	6875,877
Leaflets	175.70	1	0.40	188.68
Header	44.86	1	NA	44.86
Business Card	21.16	0	NA	24.67
Flayer	86.08	1	NA	86.08

The table summarizes the results in terms of net profit for the different categories. After the integration of outsourcing in the configuration of the logistics chain precisely in the production and delivery process, there is an increase in the profit values of the company. Therefore shows the importance of the strtegy of outsourcing, this amounts to the following reasons: firstly, it ensures the satisfaction of orders while giving short-term

contracts and including certain fixed guarantees. Secondly, concerning strategic optimization, it reflects the fundamental mission of the company and the relevance of pursuing other activities. Thirdly, at the production level, using outsourcing makes it possible to increase the volume of production and to reduce the risks of non-performance and technical failures. Outsourcing ensures distribution for manufacturing companies that cannot do it by themselves. Service providers have the resources to manage distribution networks. Finally, the quality of service supplied by the service provider is higher when the distribution service is not the core business of the company and when it is difficult to have the required skills internally.

5. Conclusions

In this paper, we develop a new Hybrid Optimization Model under Outsourcing Constraint. It integrates the constraint of outsourcing over the product life cycle of new product. Our model considers the impact of outsourcing in the process of production and delivery of product, it aims to maximize net profit and facilitate decision-making regarding strategy of outsourcing. It simultaneously determines optimal production and delivery plan with well-determined quantities in supply chain configuration.

The application of our model in the case of Tunisian company aim to search for several scenarios the optimal plan, which defines the correct quantities, produced or distributed by the company resources and those to be outsourced to maximize the company's net revenue. A comparison between the results obtained by our model shows that the Hybrid Optimization Model under Outsourcing Constraint are better than the real data obtained by the strategy of the company.

We can conclude that outsourcing is an effective solution for the company but the following factors should be considered : the influence of delays, the influence of prices, the influence of variability in demand and the management of inventories.

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