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Reliability Prediction Updating Through Computational Bayesian for Mixed and Non-mixed Lifetime Data Using More Flexible New Extra Modified Weibull Model

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Abstract

A new lifetime reliability model with four parameters is proposed. We call it the extra modified Weibull model (EMWM), which is an extension of the modified Weibull model (MWM), capable of modeling a different shapes of hazard function. The new model is developed by introducing fourth parameter in MWM called indicator parameter. The main advantage of an indicator (fourth) parameter is that it gives the new model mixture and non-mixture options, besides different shapes of hazard function including bathtub. The model parameters can be estimated based on a Bayesian generalized posterior method that serves as a tool for model identification, and it gives an efficient computational updating approach with new ways of predicting and measuring behavior. To have insight of the new indicator parameter and to see its importance, we have considered three data sets [Murthy and his colleagues [1], Badar and Priest [2], and Aarset [3]) which have been studied in the past. A prediction updating of the earlier studies of the data sets through the generalized posterior summaries using Markov Chain Monte Carlo (MCMC) Gibbs sampling approach are presented for the proposed model for the different parameters. The behavior of the parameters would help the users to have more clarity about the role of the indicator parameter, and hence may be useful for certain sets of data. The proposed model is fully adaptive to the available failure data and gives reliability engineers and scientists another option for modeling the life time data. We provide description of the mathematical properties of the new model along with failure rate function.

Keywords: bayesian analysis; extra modified weibull model; gibbs sampling; indicator parameter; markov chain monte carlo; mixture model; modified weibull model.

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1. Introduction

In reliability analysis, hazard rate plays a very important role to characterize life phenomena. In literature, many modification and generalizations of Weibull model are suggested in recent years. [see Al-Saleh and Agarwal [4], Nasiru [5], Almalki, and Nadarajah [6], and Benaicha [7] which can accommodate increasing, decreasing, unimodal and bathtub shaped hazard functions. These models are also tested to see whether they fit real life data. From the practical point of view, it is always important to consider bi-model shaped models with as few parameters as possible. So far none of the modified or generalized Weibull model is available in the literature which could identify and detect the problems involving uncertain events in mixed and non-mixed data. In this paper, a new fourth parameter (called indicator parameter) is introduced to the three parameters modified Weibull model [see Lai and his colleagues [8], and Ng [9]]. This parameter not only gives hazard function bathtub as well as bi-model but also helps to identify and detect the problems involving uncertain events in mixed and non-mixed data. The extra modified Weibull model may be useful in the reliability of a product of more generalize setting. For example, a product may be quite reliable and possibly work for some period of time, and then all of a sudden a fail appears to occur quickly, and then it begins to improve for sometimes, before a complete wear-out. This phenomenon causes a drastic shift in the behavior of the product under study. The new model includes many lifetime models such as, the Weibull model (WM), the modified Weibull model (MWM) as special cases. Further, the proposed extra modified Weibull model (EMWM) leads to an efficient computational Bayesian generalized posterior updating prediction. The Markov chain Monte Carlo (MCMC) Gibbs sampling methods are used to demonstrate a good fit for the real data in comparison to earlier well-known models used for given data sets. In section 2, we have proposed extra modified Weibull model, and studied some of its statistical properties. In section 3, the procedure to estimate the parameters of extra modified Weibull model using Bayesian generalized posterior methodology is developed. We have also estimated the hazard and reliability functions for EMWM and examined the issue of model compatibility using predictive results. Three data sets, available in literature, are used to show the importance and usefulness of EMWM by updating their prediction within the Bayesian framework.

2. The new extra modified Weibull model (EMWM)

The distribution function of proposed extra modified Weibull model with parameters α, β, γ >0, and κ ≥ 0, is defined as:

$$F(x) = 1 - \frac{\exp\left(-\alpha x^{\beta} e^{\gamma x}\right)}{(1+x)^{\kappa}}, \quad x > 0. \tag{2.1}$$

The probability density function (pdf) of (2.1) is:

$$f(x) = \frac{\exp\left(-\alpha x^{\beta} e^{\gamma x}\right)\cdot\left(\alpha x^{\beta}(1+x)(\beta \frac{1}{x} + \gamma) e^{\gamma x} + \kappa\right)}{(1+x)^{k}}, \quad \alpha, \beta, \gamma > 0, \kappa \geq 0, x > 0. \tag{2.2}$$

with failure rate (hazard function) λ(x)

$$(2.3) \qquad \lambda(x) = \frac{\left(\alpha x^{\beta}(1+x)(\beta\frac{1}{x}+\gamma)e^{\gamma x}+\kappa\right)}{(1+x)}, \ \alpha, \beta, \gamma > 0, \kappa \geq 0, x > 0.$$

In some particular cases the parameter k of equation (2.1) can be seen as providing not only an extra flexibility to the model, but can also be considered as an indicator parameter that helps to express proposed probability model [equation (2.1)] as an exact form of mixture of models under certain conditions as shown in the following figures.

Figure 1(a,e) represent the salient characteristics of the distribution (2.2) and the failure rate λ(x) for different values of α, β, γ, and κ.

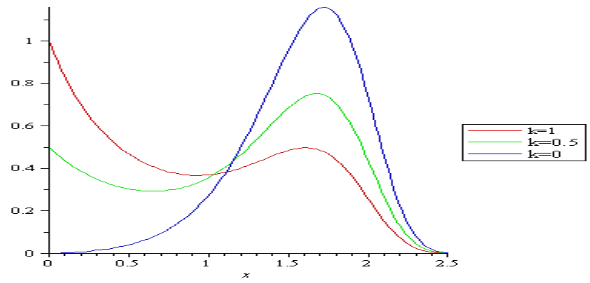

Figure 1(a): The pdf (2.2) for α=1, β=2, γ=13, and different values of κ

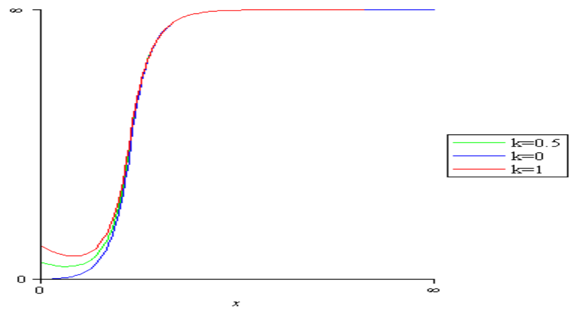

Figure 1(b): The failure rate (2.3) for α=1, β=2, γ=13, and different values of κ

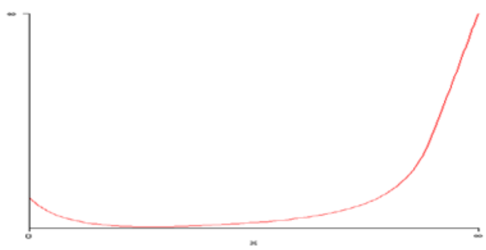

Figure 1(c): The failure rate (2.3) for α=0.1, β=2, γ=0.001, and κ=2

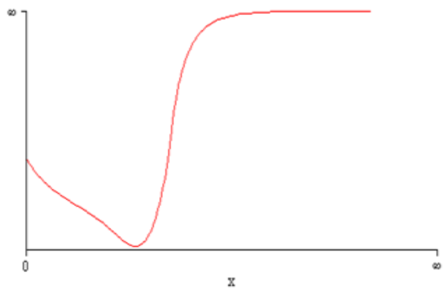

Figure 1(d): The failure rate (2.3) for α=0.00095, β=2, γ=2, and κ=2

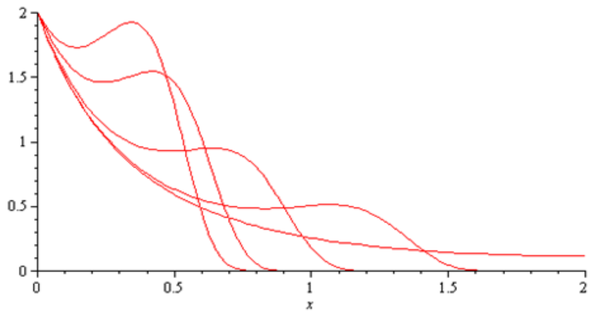

Figure 1(e): The pdf (2.2) for β=2, γ=2, κ =2, and different values of α= 0.00095, 0.01, 0.1, 0.5, 1.0

The figure 1(a) shows the different shapes for differing values of indicator parameter (k =0, 0.5 and 1.0) when α=1, β=2, γ=13. It can be seen the role of k which provides not only an extra flexibility to the model, but its usefulness as mixture of models. For k=0 it is MWM.

The figure I (e) gives the different shapes for varying values of α for β=2, γ=2, κ =2. It can be seen that pdf of proposed model not only shows an exact form of mixture of models under certain conditions, but also provides the density function more flexibility over the positive range.

The figure 1(b) shows the shapes of hazard function for differing values of indicator parameter κ when α=1, β=2, γ=13, while figures 1(c and d) give different shapes for differing values of α and γ , when β=2, and κ=2. These shapes may give an idea to the users about the behavior of hazard function for varying values of parameters.

3. Bayesian updating predictions data analysis

In this section we discuss the problem of determining whether a given data set can be adequately modeled by EMWM. We will demonstrate it within the framework of Bayesian approach by assuming that the true failure data follow the EMWM.

Further we assume the following prior specification for the parameters α~uniform, β~uniform, γ~uniform, and κ~gamma, independent.

A Markov Chain Monte Carlo (MCMC) Gibbs sampling approach implemented in using Openbugs@ computer software can give an analysis of estimates of each parameter. A burn in of 1000 updates followed by a further 20000 updates is implemented.

The table 3.1, represents the estimates of α, β, γ, for MWM (k = 0) for the three data sets (see Appendix), where as the table 3.2, represents the estimates of α, β, γ, and the fourth indicator parameter κ for EMWM for all three data sets, along with standard deviation, mean and MC error.

Table 3.1: A Bayesian summary **for (modified Wiebull model) for data sets 1-3**

	Data set 1			**Data set 2**			**Data set 3**		
	Mean	**SD**	**MC err**	**Mean**	**SD**	**MCerr**	**Mean**	**SD**	**MC err**
α	**0.4952**	**0.2916**	**0.00318**	**0.02931**	**0.00932**	**2.02E-4**	**0.07145**	**0.02971**	**9.09E-4**
β	**0.4978**	**0.2879**	**0.00289**	**0.946**	**0.05436**	**0.00107**	**0.355**	**0.1066**	**0.00368**
γ	**0.5001**	**0.2887**	**0.00292**	**0.9778**	**0.02638**	**6.11E-4**	**0.02271**	**0.00463 3**	**1.2E-4**

Table 3.2: A Bayesian summary **(extra-modified Wiebull model) for data sets 1-3**

	Data set 1			**Data set 2**			**Data set 3**		
	Mean	**SD**	**MC err**	**Mean**	**SD**	**MC err**	**Mean**	**SD**	**MC err**
α	**0.05701**	**0.05063**	**0.0031**	**0.01574**	**0.01515**	**8.27E-4**	**0.01132**	**0.00908**	**8.01E-4**
β	**0.5361**	**0.2413**	**0.01587**	**0.531**	**0.2923**	**0.01863**	**0.1972**	**0.1727**	**0.01605**
γ	**0.1581**	**0.07925**	**0.00516**	**0.3724**	**0.1534**	**0.00940**	**0.05366**	**0.01317**	**0.00120**
k	**0.7197**	**0.1496**	**0.00564**	**1.481**	**0.1364**	**0.00488**	**0.07692**	**0.1496**	**0.00457**

In order to have more clarity, we have also given figures 2, 3 and 4 which show the estimates α, β, γ, for $\kappa = 0$ (MWM), and for $\kappa \geq 0$ (EMWM) for data sets 1, 2 and 3 respectively. On examination of the above tables 3.1 and 3.2, the following observations can be noted.

When we see the posterior means of the estimate α for k=0 [Table 3.1] and for k≥0 [Table 3.2], we notice that there is a drastic shift of the posterior mean to the left as k moves away from zero, while there is sharp declined of the posterior standard deviation (s.d.), except for data 2 (s.d. is not far away in two cases), for k≥0. On comparison of the MC error for k=0 and for k≥0 shows there is no difference in both cases. More or less similar observations can be noted for the posterior mean of the estimate of β (except for data 1) as well as for the posterior mean of the estimate of γ. However, the s.d. increases for the case k≥0, except for data 1. On comparing the posterior mean of the estimate k [table 3.2], we notice a drastic shift to the right from k=0, while for data 3 the shift is not much (may be non-mixing data).

In brief, the values of the posterior means vary to some extent across the results for known k=0 (MWM) and unknown k≥0. However, the estimated prediction is better in the case when k is unknown compared to k=0. It shows that the parameter k behaving as an indicator parameter to indicate the presence of mixed data. Therefore, in the above example, when k is unknown, the updating Bayesian analysis using EMWM for the failure mixed data seems more successful than the MWM (k=0). The proposed new class of models EMWM offers more flexibility for Bayesian methods to choose among the existing classes of MWM models. Hence, the hazard and reliability function are estimated using EMWM for the failure mixed data instead of MWM (see Figures 5,6, and 7).

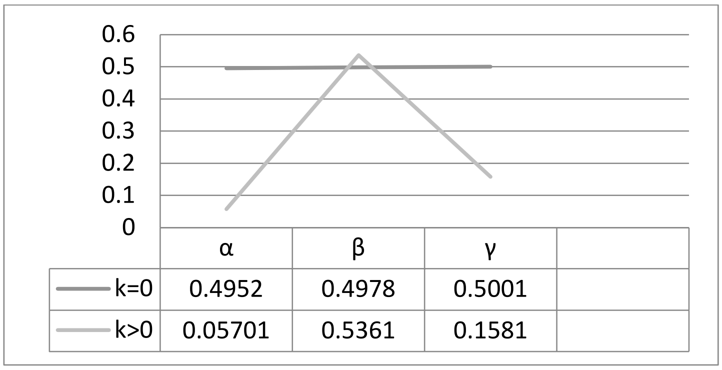

Figure 2: The behavior of estimates α, β, γ, for known κ=0 (MWM) and for unknown κ ≥ 0 (EMWM) **for data set 1**

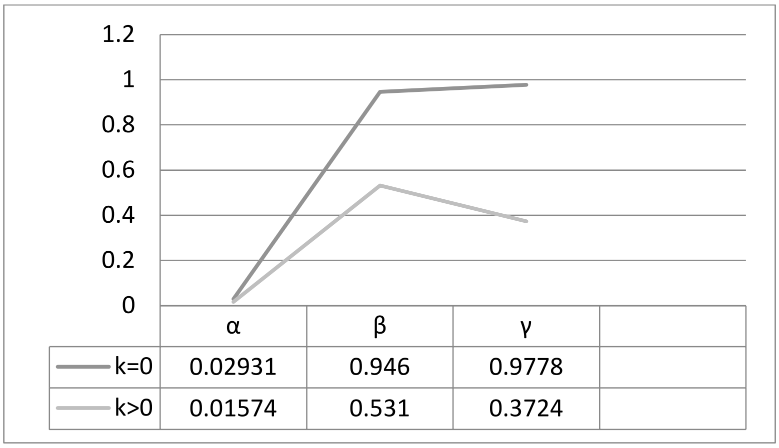

Figure 3: The behavior of estimates α, β, γ, for known κ=0 (MWM) and for unknown κ ≥ 0 (EMWM) **for data set 2**

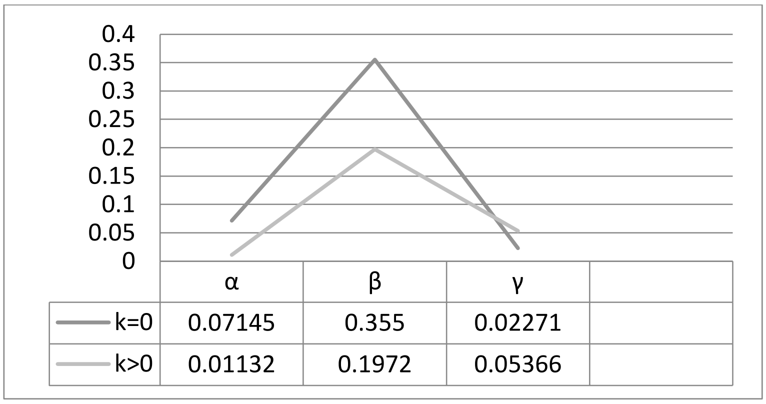

Figure 4: The behavior of estimates α, β, γ, for known κ=0 (MWM) and for κ ≥ 0 (EMWM) **for data set 3**

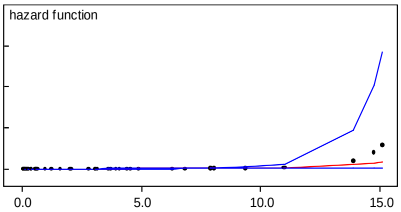

Figure 5(a): The estimated hazard function for EMWM (κ≥ 0) for failure mixed data set 1

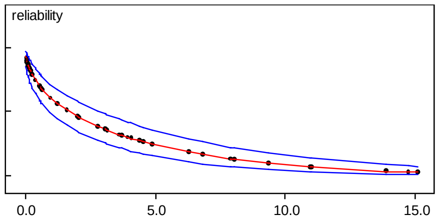

Figure 5(b): The estimated reliability function for EMWM (κ≥ 0) for failure mixed data set 1

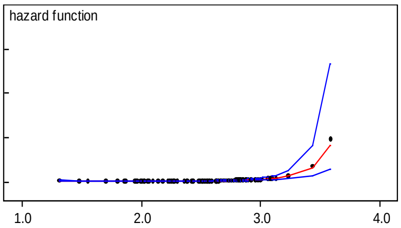

Figure 6(a): The estimated hazard function for EMWM (κ≥ 0) for failure mixed data set 2

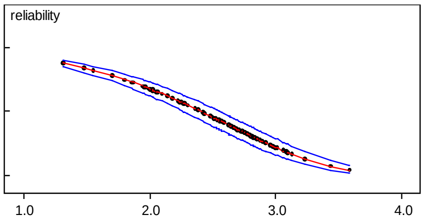

Figure 6(b): The estimated reliability function for EMWM (κ≥ 0) for failure mixed data set 2

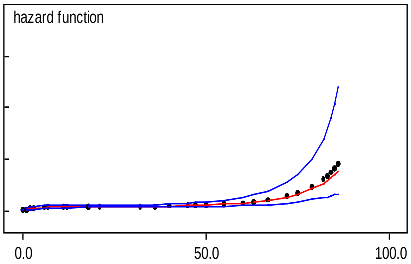

Figure 7(a): The estimated hazard function for EMWM (k≥0) for failure non-mixed data set 3

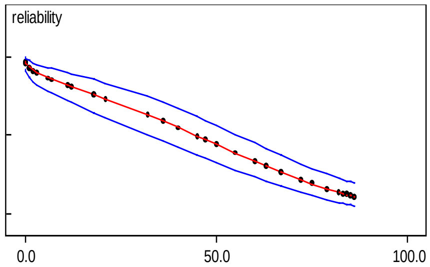

Figure 7(b): The estimated reliability function for EMWM (κ≥ 0) for failure non-mixed data set 3

Figures 5(a) to 7(a) give the shape of the estimated hazard function for EMWM (κ≥ 0) for data sets 1, 2 and 3 while figures 5(b) to 7(b) give the shape of the estimated reliability function. For data set 1 the mean of estimated κ is 0.7197 (< 1); while for data sets 2 and 3 it is 1.481, (> 1) and 0.07692 (close to 0) respectively. **T**he estimated hazard function for data set 1 shows sharp piece wise upward trend, and for data set 2 this upward trend is more sharper, while for data set 3 (non-mixed and κ close to zero), and it is concave. The estimated reliability function for EMWM for data set 3 (non-mixed and κ close to zero) gives a downward more or less a straight line, while for data sets 1 and 2 (mixed) it is slightly concave or convex. These shapes give the importance of role of indicator parameter κ.

4. Conclusion

In this paper we have shown the importance and usefulness of indicator parameter κ of EMWM through three data sets, which are available and used by authors in the past. By giving the shapes of pdf of proposed model [fig 1(a) and 1(e)], it is shown that under certain conditions, the pdf acts as an exact form of mixture of models and has more flexibility over the positive range. The salient characteristics of the failure rate for different values of α, β, γ, and κ are also shown which includes bathtub. The estimated reliability function for EMWM for three data sets are computed, which indicates the role of indicator parameter. The shape, for k close to zero in EMWM, is more or less same as that of the shape when in the past the author used MWM. The figures 2, 3 and 4 give the behavior of estimates α, β, γ, for (MWM) and for κ (EMWM). It can be seen that for data set 3 this relationship is very close between two model (k=0 for MWM and k =0.07 for EMWM) . The present study not only helps to identify and detect for problems involving uncertain events in mixed and non-mixed data, but also gives an efficient computational updating approach with new ways of predicting and measuring behavior. However, EMWM should be verified for further research.

Acknowledgments

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5. Appendix

A.1 Data set 1

The data set given in Table A.1 is taken from [1] Murthy and his colleagues (2004), p180. It represents the failure times of 50 components(per 1000h).

Table A.1: data set

0.036	0.058	0.061	0.074	0.078	0.086	0.102	0.103	0.114	0.116
0.148	0.183	0.192	0.254	0.262	0.379	0.381	0.538	0.570	0.574
0.590	0.618	0.645	0.961	1.228	1.600	2.006	2.054	2.804	3.058
3.076	3.147	3.625	3.704	3.931	4.073	4.393	4.534	4.893	6.274
6.816	7.896	7.904	8.022	9.337	10.940	11.020	13.880	14.730	15.080

A.2 Data set 2

The following strength of carbon bers data (Table A.2) is reported by [2] Badar, and Priest (1982). This data represent the strength measured in GPa, for single carbon bers and impregnated 1000-carbon-ber tows, which were tested under tension.

Table A.2: Strength data of carbon bers at gauge lengths of 20 mm

1.312	1.314	1.479	1.552	1.700	1.803	1.861	1.865	1.944	1.958
1.966	1.997	2.006	2.021	2.027	2.055	2.063	2.098	2.140	2.179
2.224	2.240	2.253	2.270	2.272	2.274	2.301	2.301	2.359	2.382
2.382	2.426	2.434	2.435	2.478	2.490	2.511	2.514	2.535	2.554
2.566	2.570	2.586	2.629	2.633	2.642	2.648	2.684	2.697	2.726
2.770	2.773	2.800	2.809	2.818	2.821	2.848	2.880	2.954	3.012
3.067	3.084	3.090	3.096	3.128	3.233	3.433	3.585	3.585	2.526
2.546	2.628	2.628	2.669	2.669	2.710	2.731	2.731	2.731	2.752
2.752	2.793	2.834	2.834	2.854	2.875	2.875	2.895	2.916	2.916
2.957	2.977	2.998	3.060	3.060	3.060	3.080			

A.3 Data set 3

The non-mixed data set representing failure times and shown in (Table A.3 see [3] Aarset (1987))

Table A.3: Data set

0.1	0.2	1.0	1.0	1.0	1.0	1.0	2.0	3.0	6.0
7.0	11.0	12.0	18.0	18.0	18.0	18.0	18.0	21.0	32.0
36.0	40.0	45.0	45.0	47.0	50.0	55.0	60.0	63.0	63.0
67.0	67.0	67.0	67.0	72.0	75.0	79.0	82.0	82.0	83.0
84.0	84.0	84.0	85.0	85.0	85.0	85.0	85.0	86.0	86.0

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