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Hamiltonian System Mechanics on (2,0)-Jet Bundles

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Abstract

The goal of this paper is to present Hamiltonian system Mechanics on (2,0)-jet bundles . In conclusion, some differential geometrical and physical results on the related mechanic systems have been given.

Keywords: Jet bundle; holomorphic bundle; complex , Hamiltonian Dynamics.

1. Introduction

It is well known that the dynamics of Lagrangian formalisms is characterized by a suitable vector field defined on the tangent and cotangent bundles which are phase-spaces of velocities and momentum of a given configuration manifold. If $\mathcal{M}$ is an m-dimensional configuration manifold [6]. If $\mathbf{H}: \mathbf{T}^*\mathcal{M} \longrightarrow \mathbf{R}$ is a regular Hamiltonian function then there is a unique vector field Z_H on cotangent bundle $\mathbf{T}^*\mathcal{M}$ such that dynamical equations

$$\mathbf{i}_{Z_H}\boldsymbol{\phi} = \mathbf{dH} \qquad (1)$$

where $\boldsymbol{\phi}$ is the symplectic form and H stands for Hamiltonian function. The paths of the Hamiltonian vector field Z_H are the solutions of the Hamiltonian equations shown by

$$\frac{dq^i}{dt} = \frac{\partial H}{\partial p_i} \quad , \quad \frac{dp_i}{dt} = \frac{\partial H}{\partial q^i} \qquad (2)$$

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where q^i and $(q^i, p_i), 1 \leq i \leq m$, are coordinates of $\mathcal{M}$ and $\mathbf{T}^*\mathcal{M}$. The triple $(\mathbf{T}^*\mathcal{M}, \phi, H)$, is called Hamiltonian system on the cotangent bundle $\mathbf{T}^*\mathcal{M}$ with symplectic form ϕ. Let $\mathbf{T}^*\mathcal{M}$ be symplectic manifold with closed symplectic form ϕ. In this paper related to Hamiltonian equations Hamiltonian system Mechanics on (2,0)-jet bundles.

2. The geometry of holomorphic $J^{(2,0)}\mathcal{M}$ bundles

2.1 Definition

Let $\mathcal{M}$ be a complex manifold, $\mathbf{T_c}\mathcal{M} = \acute{\mathbf{T}}\mathcal{M} \oplus \acute{\acute{\mathbf{T}}}\mathcal{M}$, the complexified tangent bundle of $(\mathbf{1}, \mathbf{0})$- and of $(\mathbf{0}, \mathbf{1})$-type vectors, respectively. If $(\mathbf{z^i})_{\mathbf{i}=\overline{\mathbf{1;n}}}$ are complex coordinates, then $\acute{\mathbf{T}}_{\mathbf{z}}\mathcal{M}$ is spanned by $\left\{\frac{\partial}{\partial \mathbf{z^i}}\right\}_{\mathbf{i}=\overline{\mathbf{1;n}}}$ and $\acute{\acute{\mathbf{T}}}_{\mathbf{z}}\mathcal{M}$ is spanned by $\left\{\frac{\partial}{\partial \bar{\mathbf{z}}^{\mathbf{i}}}\right\}_{\mathbf{i}=\overline{\mathbf{1;n}}}$ moreover $\acute{\mathbf{T}}\mathcal{M}$ is a holomorphic vector bundle

let $\mathbf{Z} = (\mathbf{z^i}, \mathbf{X^i} = \boldsymbol{\eta}^{\mathbf{i}(\mathbf{1})} = \frac{\mathbf{dz^i}}{\mathbf{d\theta}}, \mathbf{Y^i} = \boldsymbol{\eta}^{\mathbf{i}(\mathbf{2})} = \frac{\mathbf{d^2z^i}}{\mathbf{d\theta^2}})$ be local complex coordinates in the chart $(\mathbf{U};\, \boldsymbol{\psi})$ from $\mathbf{J}^{(2,0)}\mathcal{M}$;

we shall the following notations [1].

$$Z = \left(z^i, x^i = \eta^{i(1)}, y^i = \eta^{i(2)}\right) = (z^i, X^i, Y^i) \qquad (3)$$

2.2 Theorem

A local basis in $\acute{\mathbf{T}}_{\mathbf{z}}\left(\mathbf{J}^{(2,0)}\mathcal{M}\right)$ is $\left\{\frac{\partial}{\partial \mathbf{z^i}}, \frac{\partial}{\partial \mathbf{x^i}}, \frac{\partial}{\partial \mathbf{y^i}}\right\}_{\mathbf{i}=\overline{\mathbf{1;n}}}$ and in $\acute{\acute{\mathbf{T}}}_{\mathbf{z}}\left(\mathbf{J}^{(2,0)}\mathcal{M}\right)$ theirs conjugates $\left\{\frac{\partial}{\partial \bar{\mathbf{z}}^{\mathbf{i}}}, \frac{\partial}{\partial \bar{\mathbf{x}}^{\mathbf{i}}}, \frac{\partial}{\partial \bar{\mathbf{y}}^{\mathbf{i}}}\right\}_{\mathbf{i}=\overline{\mathbf{1;n}}}$: Due to holomorphic changes on $\mathbf{J}^{(2,0)}\mathcal{M}$, that is all of $\frac{\partial \acute{\mathbf{z}}^{\mathbf{i}}}{\partial \bar{\mathbf{z}}^{\mathbf{j}}}, \frac{\partial \acute{\mathbf{x}}^{\mathbf{i}}}{\partial \bar{\mathbf{z}}^{\mathbf{j}}}, \frac{\partial \acute{\mathbf{y}}^{\mathbf{i}}}{\partial \bar{\mathbf{z}}^{\mathbf{j}}}, \frac{\partial \acute{\mathbf{x}}^{\mathbf{i}}}{\partial \bar{\mathbf{x}}^{\mathbf{j}}}, \frac{\partial \acute{\mathbf{y}}^{\mathbf{i}}}{\partial \bar{\mathbf{x}}^{\mathbf{j}}}, \frac{\partial \acute{\mathbf{y}}^{\mathbf{i}}}{\partial \bar{\mathbf{y}}^{\mathbf{j}}}$ j are vanishing, and also theirs conjugates, it follows that local bases from $\acute{\mathbf{T}}_{\mathbf{z}}\left(\mathbf{J}^{(2,0)}\mathcal{M}\right)$ change w.r.t. the transformations by the rules:

$$\frac{\partial}{\partial \mathbf{z^j}} = \frac{\partial \acute{\mathbf{z}}^{\mathbf{i}}}{\partial \mathbf{z^j}}\frac{\partial}{\partial \acute{\mathbf{z}}^{\mathbf{i}}} + \frac{\partial \acute{\mathbf{x}}^{\mathbf{i}}}{\partial \mathbf{z^j}}\frac{\partial}{\partial \acute{\mathbf{x}}^{\mathbf{i}}} + \frac{\partial \acute{\mathbf{y}}^{\mathbf{i}}}{\partial \mathbf{z^j}}\frac{\partial}{\partial \acute{\mathbf{y}}^{\mathbf{i}}} \Rightarrow$$

$$\frac{\partial}{\partial \mathbf{x^j}} = \frac{\partial \acute{\mathbf{x}}^{\mathbf{i}}}{\partial \mathbf{z^j}}\frac{\partial}{\partial \acute{\mathbf{x}}^{\mathbf{i}}} + \frac{\partial \acute{\mathbf{y}}^{\mathbf{i}}}{\partial \mathbf{z^j}}\frac{\partial}{\partial \acute{\mathbf{y}}^{\mathbf{i}}}$$

$$\frac{\partial}{\partial \mathbf{x^j}} = \frac{\partial \acute{\mathbf{y}}^{\mathbf{i}}}{\partial \mathbf{z^j}}\frac{\partial}{\partial \acute{\mathbf{y}}^{\mathbf{i}}} \qquad (4)$$

Infer that $\frac{\partial \acute{\mathbf{z}}^{\mathbf{i}}}{\partial \mathbf{z^j}} = \frac{\partial \acute{\mathbf{x}}^{\mathbf{i}}}{\partial \mathbf{x^j}} = \frac{\partial \acute{\mathbf{y}}^{\mathbf{i}}}{\partial \mathbf{y^j}}$ but in change $\frac{\partial \acute{\mathbf{z}}^{\mathbf{i}}}{\partial \mathbf{z^j}} = \frac{\partial \acute{\mathbf{x}}^{\mathbf{i}}}{\partial \mathbf{x^j}}$ contain the second order derivatives of $\acute{\mathbf{z}}^{\mathbf{i}}$. while $\frac{\partial \acute{\mathbf{x}}^{\mathbf{i}}}{\partial \mathbf{z^j}}$ contains even the 3-th derivatives of $\acute{\mathbf{z}}^{\mathbf{i}}$.

2.3 Theorem

On $\boldsymbol{T_c}\left(\mathbf{J}^{(2,0)}\boldsymbol{\mathcal{M}}\right)$ the natural complex structure $\boldsymbol{J^2 = -I}$ acts as follows:

$$J\left(\frac{\partial}{\partial z^j}\right) = \boldsymbol{i}\frac{\partial}{\partial z^j} \quad , \; J\left(\frac{\partial}{\partial x^j}\right) = \boldsymbol{i}\frac{\partial}{\partial x^j} \quad , \; J\left(\frac{\partial}{\partial y^j}\right) = \boldsymbol{i}\frac{\partial}{\partial y^j}$$

$$\boldsymbol{J\left(\frac{\partial}{\partial \bar{z}^j}\right) = -i\frac{\partial}{\partial z^j} \quad , \; J\left(\frac{\partial}{\partial \bar{x}^j}\right) = -i\frac{\partial}{\partial x^j} \quad , \; J\left(\frac{\partial}{\partial \bar{y}^j}\right) = -i\frac{\partial}{\partial y^j}} \quad (5)$$

The dual endomorphism the cotangent space $\mathbf{T_c^*}\left(\mathbf{J}^{(2,0)}\boldsymbol{\mathcal{M}}\right)$at any point p of manifold $\mathbf{J}^{(2,0)}\boldsymbol{\mathcal{M}}$ satifies $\mathbf{J^{2^*} = -I}$ and is defined by

$$\boldsymbol{J^*(dz^j) = idz^j \quad , \quad J^*(dx^j) = idx^j \quad , \quad J^*(dy^j) = idy^j}$$

$$\boldsymbol{J^*(d\bar{z}^j) = -id\bar{z}^j \quad , \quad J^*(d\bar{x}^j) = -id\bar{x}^j \quad , \quad J^*(\bar{y}^j) = -id\bar{y}^j} \quad (6)$$

3. Hamiltonian Dynamical Systems

In this section, we obtain complex Hamiltonian equations for classical mechanics structured on momentum space $\mathbf{T_c^*}\left(\mathbf{J}^{(2,0)}\boldsymbol{\mathcal{M}}\right)$ that is 2m- dimensional cotangent bundle of an m-dimensional configuration manifold $\boldsymbol{\mathcal{M}}$.

Let $\mathbf{T_c^*}\left(\mathbf{J}^{(2,0)}\boldsymbol{\mathcal{M}}\right)$be the momentum space and $\mathbf{Z} = \left(\mathbf{z^i}, \mathbf{x^i} = \boldsymbol{\eta}^{\mathbf{i}^{(1)}}, \mathbf{y^i} = \boldsymbol{\eta}^{\mathbf{i}^{(2)}}\right) = (\mathbf{z^i}, \mathbf{X^i}, \mathbf{Y^i})$, $\mathbf{1 \le i \le m}$ its complex coordinates

Let almost complex structure $\mathbf{J}^*$ and Liouville form $\boldsymbol{\lambda}$ give by

$$\boldsymbol{\omega} = \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{z^i d\bar{z}_i + \bar{z}_i dz^i + x^i d\bar{x}_i + \bar{x}_i dx^i + y^i d\bar{y}_i + \bar{y}_i dy^i}) \quad (7)$$

$$\boldsymbol{\lambda} = (\mathbf{J}^*\boldsymbol{\omega}) = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{J}^*(\mathbf{z^i d\bar{z}_i + \bar{z}_i dz^i + x^i d\bar{x}_i + \bar{x}_i dx^i + y^i d\bar{y}_i + \bar{y}_i dy^i})$$

or

$$\boldsymbol{\lambda} = (\mathbf{J}^*\boldsymbol{\omega}) = \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{z^i J^*(d\bar{z}_i) + \bar{z}_i J^*(dz^i) + x^i J^*(d\bar{x}_i) + \bar{x}_i J^*(dx^i) + y^i J^*(d\bar{y}_i) + \bar{y}_i J^*(dy^i)})$$

$$\boldsymbol{\lambda} = \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{-iz^i d\bar{z}_i + i\bar{z}_i dz^i - ix^i d\bar{x}_i + i\bar{x}_i dx^i - iy^i d\bar{y}_i + i\bar{y}_i dy^i})$$

or

$$\boldsymbol{\lambda} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{i}(\mathbf{-z^i d\bar{z}_i + \bar{z}_i dz^i - x^i d\bar{x}_i + \bar{x}_i dx^i - y^i d\bar{y}_i + \bar{y}_i dy^i})) \quad (8)$$

such that $\boldsymbol{\omega}$ complex 1-form on $\mathbf{T_c^*}\left(\mathbf{J}^{(2,0)}\boldsymbol{\mathcal{M}}\right)$.

If $\mathbf{\Phi} = -\mathbf{d\lambda}$ is closed Kahlerian form, then $\mathbf{\Phi}$ is also a symplectic structure on $\mathbf{T_c^*}\left(\mathbf{J^{(2,0)}}\boldsymbol{\mathcal{M}}\right)$.

$$\mathbf{\Phi} = -\mathbf{d\lambda} = -\mathbf{d}\left(\frac{\mathbf{1}}{\mathbf{2}}\mathbf{i}\left(-\mathbf{z^i d\bar{z}_i} + \mathbf{\bar{z}_i dz^i} - \mathbf{x^i d\bar{x}_i} + \mathbf{\bar{x}_i dx^i} - \mathbf{y^i d\bar{y}_i} + \mathbf{\bar{y}_i dy^i}\right)\right)$$

$$\mathbf{\Phi} = -\mathbf{d\lambda} = -\mathbf{id}\left(-\mathbf{z^i d\bar{z}_i} + \mathbf{\bar{z}_i dz^i}\right) - \mathbf{id}\left(-\mathbf{x^i d\bar{x}_i} + \mathbf{\bar{x}_i dx^i}\right) - \mathbf{id}\left(-\mathbf{y^i d\bar{y}_i} + \mathbf{\bar{y}_i dy^i}\right)$$

$$\mathbf{\Phi} = -\mathbf{d\lambda} = -\mathbf{i}\left(\mathbf{d\bar{z}_i} \wedge \mathbf{dz^i}\right) - \mathbf{i}\left(\mathbf{d\bar{x}_i} \wedge \mathbf{dx^i}\right) - \mathbf{i}\left(\mathbf{d\bar{y}_i} \wedge \mathbf{dy^i}\right) \qquad (\mathbf{9})$$

Let $\mathbf{T_c^*}\left(\mathbf{J^{(2,0)}}\boldsymbol{\mathcal{M}}\right)$ be momentum space with closed Kaehlerian form $\mathbf{\Phi}$ .Consider that Hamiltonian vector field $\mathbf{Z_H}$ associated Hamiltonian energy $\mathbf{H}$ is given by

$$\mathbf{Z} = \mathbf{Z_H} = \mathbf{Z^i}\frac{\partial}{\partial \mathbf{z^i}} + \mathbf{\bar{Z}_i}\frac{\partial}{\partial \mathbf{\bar{z}_i}} + \mathbf{X^i}\frac{\partial}{\partial \mathbf{x^i}} + \mathbf{\bar{X}_i}\frac{\partial}{\partial \mathbf{\bar{x}_i}} + \mathbf{Y^i}\frac{\partial}{\partial \mathbf{y^i}} + \mathbf{\bar{Y}_i}\frac{\partial}{\partial \mathbf{\bar{y}_i}} \qquad \mathbf{1} \le \mathbf{i} \le \mathbf{m}$$

From the isomorphism given in, we calculate by

$$\begin{aligned}\mathbf{i_{Z_H}}\,\mathbf{\Phi} &= \mathbf{i_{Z_H}}(-\mathbf{d\lambda}) \\ &= \left(\mathbf{Z^i}\frac{\partial}{\partial \mathbf{z^i}} + \mathbf{\bar{Z}_i}\frac{\partial}{\partial \mathbf{\bar{z}_i}} + \mathbf{X^i}\frac{\partial}{\partial \mathbf{x^i}} + \mathbf{\bar{X}_i}\frac{\partial}{\partial \mathbf{\bar{x}_i}} + \mathbf{Y^i}\frac{\partial}{\partial \mathbf{y^i}} + \mathbf{\bar{Y}_i}\frac{\partial}{\partial \mathbf{\bar{y}_i}}\right)\left(-\mathbf{i}\left(\mathbf{d\bar{z}_i} \wedge \mathbf{dz^i}\right) - \mathbf{i}\left(\mathbf{d\bar{x}_i} \wedge \mathbf{dx^i}\right)\right. \\ &\quad \left. - \mathbf{i}\left(\mathbf{d\bar{y}_i} \wedge \mathbf{dy^i}\right)\right)\end{aligned}$$

$$\mathbf{i_{Z_H}}\,\mathbf{\Phi} = \mathbf{i\bar{Z}_i dz^i} + \mathbf{iZ^i d\bar{z}_i} + \mathbf{i\bar{X}_i dx^i} + \mathbf{iX^i d\bar{x}_i} + \mathbf{i\bar{Y}_i dy^i} + \mathbf{iY^i d\bar{y}_i} \qquad (\mathbf{10})$$

On the other hand, we obtain as

$$\mathbf{dH} = \frac{\partial \mathbf{H}}{\partial \mathbf{z^i}}\mathbf{dz^i} + \frac{\partial \mathbf{H}}{\partial \mathbf{\bar{z}_i}}\mathbf{d\bar{z}_i} + \frac{\partial \mathbf{H}}{\partial \mathbf{x^i}}\mathbf{dx^i} + \frac{\partial \mathbf{H}}{\partial \mathbf{\bar{x}_i}}\mathbf{d\bar{x}_i} + \frac{\partial \mathbf{H}}{\partial \mathbf{y^i}}\mathbf{dy^i} + \frac{\partial \mathbf{H}}{\partial \mathbf{\bar{y}_i}}\mathbf{d\bar{y}_i} \qquad (\mathbf{11})$$

the differential of Hamiltonian energy. From $\mathbf{i_{Z_H}}\mathbf{\Phi} = \mathbf{dH}$, we find as

$$\begin{aligned}\mathbf{i_{Z_H}}\mathbf{\Phi} = \mathbf{dH} &= \mathbf{\bar{Z}_i dz^i} + \mathbf{iZ^i d\bar{z}_i} + \mathbf{i\bar{X}_i dx^i} + \mathbf{iX^i d\bar{x}_i} + \mathbf{i\bar{Y}_i dy^i} + \mathbf{iY^i d\bar{y}_i} \\ &= \frac{\partial \mathbf{H}}{\partial \mathbf{z^i}}\mathbf{dz^i} + \frac{\partial \mathbf{H}}{\partial \mathbf{\bar{z}_i}}\mathbf{d\bar{z}_i} + \frac{\partial \mathbf{H}}{\partial \mathbf{x^i}}\mathbf{dx^i} + \frac{\partial \mathbf{H}}{\partial \mathbf{\bar{x}_i}}\mathbf{d\bar{x}_i} + \frac{\partial \mathbf{H}}{\partial \mathbf{y^i}}\mathbf{dy^i} + \frac{\partial \mathbf{H}}{\partial \mathbf{\bar{y}_i}}\mathbf{d\bar{y}_i} \qquad (\mathbf{12})\end{aligned}$$

Or

$$\mathbf{Z_H} = \frac{\mathbf{1}}{\mathbf{i}}\frac{\partial \mathbf{H}}{\partial \mathbf{\bar{z}_i}}\frac{\partial}{\partial \mathbf{z^i}} - \frac{\mathbf{1}}{\mathbf{i}}\frac{\partial \mathbf{H}}{\partial \mathbf{z^i}}\frac{\partial}{\partial \mathbf{\bar{z}_i}} + \frac{\mathbf{1}}{\mathbf{i}}\frac{\partial \mathbf{H}}{\partial \mathbf{\bar{x}_i}}\frac{\partial}{\partial \mathbf{x^i}} - \frac{\mathbf{1}}{\mathbf{i}}\frac{\partial \mathbf{H}}{\partial \mathbf{x^i}}\frac{\partial}{\partial \mathbf{\bar{x}_i}} + \frac{\mathbf{1}}{\mathbf{i}}\frac{\partial \mathbf{H}}{\partial \mathbf{\bar{y}_i}}\frac{\partial}{\partial \mathbf{y^i}} - \frac{\mathbf{1}}{\mathbf{i}}\frac{\partial \mathbf{H}}{\partial \mathbf{y^i}}\frac{\partial}{\partial \mathbf{\bar{y}_i}} \qquad (\mathbf{13})$$

$$\mathbf{1} \le \mathbf{i} \le \mathbf{m}$$

Let $\{\mathbf{Z} = (\mathbf{z^i}, \mathbf{\bar{z}_i}, \mathbf{x^i}, \mathbf{\bar{x}_i}, \mathbf{y^i}, \mathbf{\bar{y}_i}) : \mathbf{1} \le \mathbf{i} \le \mathbf{m}\}$ be the complex coordinates in the momentum space. Suppose that the curve

$$\boldsymbol{\alpha}: \mathbf{I} \subset \mathbf{C} \rightarrow \mathbf{T}\boldsymbol{\mathcal{M}}$$

be an integral curve of Hamiltonian vector field $\mathbf{Z_H}$, i.e.,

$$\mathbf{Z_H}\big(\boldsymbol{\alpha}(\mathbf{t})\big) = \dot{\boldsymbol{\alpha}} \quad , \mathbf{t} \in \mathbf{I}.$$

In the local coordinates we have

$$\boldsymbol{\alpha}(\mathbf{t}) = \Big(\mathbf{z^i}(\mathbf{t}), \bar{\mathbf{z}}_\mathbf{i}(\mathbf{t}), \mathbf{x^i}(\mathbf{t}), \bar{\mathbf{x}}_\mathbf{i}(\mathbf{t}), \mathbf{y^i}(\mathbf{t}), \bar{\mathbf{y}}_\mathbf{i}(\mathbf{t})\Big),$$

And

$$\dot{\boldsymbol{\alpha}}(\mathbf{t}) = \frac{\mathbf{dz^i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\mathbf{z^i}} + \frac{\mathbf{d\bar{z}_i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\mathbf{\bar{z}_i}} + \frac{\mathbf{dx^i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\mathbf{x^i}} + \frac{\mathbf{d\bar{x}_i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\mathbf{\bar{x}_i}} + \frac{\mathbf{dy^i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\mathbf{y^i}} + \frac{\mathbf{d\bar{y}_i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\mathbf{\bar{y}_i}} \qquad (\mathbf{14})$$

the Hamiltonian vector field on momentum space $\mathbf{T_c^*}\big(\mathbf{J}^{(2,0)}\boldsymbol{\mathcal{M}}\big)$ with closed Kaehlerian form $\boldsymbol{\phi}$.Now, from $\mathbf{Z_H}\big(\boldsymbol{\alpha}(\mathbf{t})\big) = \dot{\boldsymbol{\alpha}}$,

$$\frac{1}{i}\frac{\partial H}{\partial \bar{z}_i}\frac{\partial}{\partial z^i} - \frac{1}{i}\frac{\partial H}{\partial z^i}\frac{\partial}{\partial \bar{z}_i} + \frac{1}{i}\frac{\partial H}{\partial \bar{x}_i}\frac{\partial}{\partial x^i} - \frac{1}{i}\frac{\partial H}{\partial x^i}\frac{\partial}{\partial \bar{x}_i} + \frac{1}{i}\frac{\partial H}{\partial \bar{y}_i}\frac{\partial}{\partial y^i} - \frac{1}{i}\frac{\partial H}{\partial y^i}\frac{\partial}{\partial \bar{y}_i}$$
$$= \frac{dz^i}{dt}\frac{\partial}{\partial z^i} + \frac{d\bar{z}_i}{dt}\frac{\partial}{\partial \bar{z}_i} + \frac{dx^i}{dt}\frac{\partial}{\partial x^i} + \frac{d\bar{x}_i}{dt}\frac{\partial}{\partial \bar{x}_i} + \frac{dy^i}{dt}\frac{\partial}{\partial y^i} + \frac{d\bar{y}_i}{dt}\frac{\partial}{\partial \bar{y}_i}$$

then we infer the following equations

$$\frac{1}{i}\frac{\partial H}{\partial \bar{z}_i}\frac{\partial}{\partial z^i} = \frac{dz^i}{dt}\frac{\partial}{\partial z^i} \quad \rightarrow \frac{dz^i}{dt} = \frac{1}{i}\frac{\partial H}{\partial \bar{z}_i}$$

$$-\frac{1}{i}\frac{\partial H}{\partial z^i}\frac{\partial}{\partial \bar{z}_i} = \frac{d\bar{z}_i}{dt}\frac{\partial}{\partial \bar{z}_i} \rightarrow \frac{d\bar{z}_i}{dt} = -\frac{1}{i}\frac{\partial H}{\partial z^i}$$

$$\frac{1}{i}\frac{\partial H}{\partial \bar{x}_i}\frac{\partial}{\partial x^i} = \frac{dx^i}{dt}\frac{\partial}{\partial x^i} \rightarrow \frac{dx^i}{dt} = \frac{1}{i}\frac{\partial H}{\partial \bar{x}_i}$$

$$-\frac{1}{i}\frac{\partial H}{\partial x^i}\frac{\partial}{\partial \bar{x}_i} = \frac{d\bar{x}_i}{dt}\frac{\partial}{\partial \bar{x}_i} \rightarrow \frac{d\bar{x}_i}{dt} = -\frac{1}{i}\frac{\partial H}{\partial x^i}$$

$$\frac{1}{i}\frac{\partial H}{\partial \bar{y}_i}\frac{\partial}{\partial y^i} = \frac{dy^i}{dt}\frac{\partial}{\partial y^i} \rightarrow \frac{dy^i}{dt} = \frac{1}{i}\frac{\partial H}{\partial \bar{y}_i}$$

$$-\frac{1}{i}\frac{\partial H}{\partial y^i}\frac{\partial}{\partial \bar{y}_i} = \frac{d\bar{y}_i}{dt}\frac{\partial}{\partial \bar{y}_i} \rightarrow \frac{d\bar{y}_i}{dt} = -\frac{1}{i}\frac{\partial H}{\partial y^i}$$

which are called complex Hamiltonian equations on momentum space $\mathbf{T_c^*}\big(\mathbf{J}^{(2,0)}\boldsymbol{\mathcal{M}}\big)$. we have the complex

Hamiltonian equations given by

$$\frac{d\bar{z}^i}{dt} = \frac{1}{i}\frac{\partial H}{\partial z_i} \quad , \quad \frac{dz_i}{dt} = -\frac{1}{i}\frac{\partial H}{\partial \bar{z}^i}$$

$$\frac{dx^i}{dt} = \frac{1}{i}\frac{\partial H}{\partial \bar{x}_i} \quad , \quad \frac{d\bar{x}_i}{dt} = -\frac{1}{i}\frac{\partial H}{\partial x^i}$$

$$\frac{dy^i}{dt} = \frac{1}{i}\frac{\partial H}{\partial \bar{y}_i} \quad , \quad \frac{d\bar{y}_i}{dt} = -\frac{1}{i}\frac{\partial H}{\partial y^i} \tag{15}$$

Thus, by complex Hamiltonian equations ,we may call the equations obtained in (15) on $\mathbf{T}_{\mathbf{c}}^{*}\left(\mathbf{J}^{(2,0)}\boldsymbol{\mathcal{M}}\right)$. Then the quartet $(\mathbf{T}_{\mathbf{c}}^{*}\left(\mathbf{J}^{(2,0)}\boldsymbol{\mathcal{M}}\right), \boldsymbol{\phi}_{\mathbf{H}}, \mathbf{Z}_{\mathbf{H}})$ is named mechanical system with

4. Conclusions

The solutions of the Hamiltonian equations determined by (15) on the mechanical system $(\mathbf{T}_{\mathbf{c}}^{*}\left(\mathbf{J}^{(2,0)}\boldsymbol{\mathcal{M}}\right), \boldsymbol{\phi}_{\mathbf{L}}, \mathbf{Z}_{\mathbf{H}})$are the paths of vector field $\mathbf{Z}_{\mathbf{H}}$ on $\mathbf{T}_{\mathbf{c}}^{*}\left(\mathbf{J}^{(2,0)}\boldsymbol{\mathcal{M}}\right)$.

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