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Divide Column and Subtract One Assignment Method for Solving Assignment Problem

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Abstract

Assignment problem is an important problem in mathematics and is also discuss in real physical world. In this paper we attempt to introduce a new proposed approach for solving assignment problem with algorithm and solution steps. We examine a numerical example by using new method and compute by existing two methods. Also we compare the optimal solutions among this new method and two existing methods. The proposed method is a systematic procedure, easy to apply for solving assignment problem.

Keywords: Assignment problem; Hungarian assignment method (HA-method); Matrix one's assignment method (MOA-method); Divide Column and Subtract One Assignment method; Optimization.

1. Introduction

The assignment problem is one of the main problems while assigning task to the worker. It is one of the fundamental combinatorial optimization problems in the branch of optimization or operation research in Mathematics. In practical field we are sometime faced with type of problem which consists of jobs to machines, drivers to trucks, men to offices etc. in which the assignees possess varying degree of efficiency, called as cost or effectiveness. The basic assumption of this type of problem is that one person can perform one job at a time. An assignment plan is optimal if it minimizes the total or maximizes the profit. This type of linear assignment problems can be solved by the very well-known Hungarian method which was derived by the two mathematicians D. Konig and E. Egervary.

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Although the name "Assignment Problem" seems to have first appeared in 1952 paper by Votaw and Orden [1], what is generally recognized to be the beginning of the development of practical solution methods for and variations on the classic assignment problem was the publication in 1955 of Kuhn's article on the Hungarian method for its solution [2]. Different methods have been presented for assignment problem and various articles have been published on the see [3,4,5] for the history of this method.

In this paper we developed a solution method for assignment problem. The corresponding method has been formulated and numerical example has been considered to illustrate the method. Finally we compare the optimal solutions among new method and two existing methods.

2. Mathematical Formulation of Assignment Problem

Each assignment problem has a matrix associated with it. Generally the row contains the objects or people we wish to assign, and the column comprise the jobs or tasks we want them assigned to.

Consider a problem of assignment of n resources to m activities so as to minimize the overall cost or time in such a way that each resource can associate with one and only one job. The cost matrix (C_{ij}) is given as under:

Table 1: Approach of Assignment Problem

		Activity				
		A_1	A_2	......	A_n	Available
Resource	R_1	C_{11}	C_{12}	......	C_{1n}	1
	R_2	C_{21}	C_{22}	......	C_{2n}	1
	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
	R_n	C_{n1}	C_{n2}	......	C_{nn}	1
Required		1	1	......	1	

The cost matrix is same as that of a transportation problem except that availability at each of the resource and the requirement at each of the destinations is unity.

Let x_{ij} denote the assignment of i^{th} resource to j^{th} activity, such that

$$x_{ij} = \begin{cases} 1 \; ; & \text{if resource i is assigned to activity j} \\ 0 \; ; & \text{otherwise} \end{cases}$$

Then the mathematical formulation of the assignment problem is

Minimize $z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}\, x_{ij}$

Subject to the constraints

$\sum_{i=1}^{n} x_{ij} = 1$ and $\sum_{j=1}^{n} x_{ij} = 1$: $x_{ij} = 0$ or 1

For all $i = 1,2,\ldots\ldots n$ and $j = 1,2,\ldots\ldots n$.

3. New Approach for Solving Assignment Problem

In this section we introduce a new approach for solving Assignment problem with the help of HA-method and MOA-method but different from them. This new method is easy procedure to solve Assignment problem. Also an example is solved by this method and the result is compared to HA-method and MOA-method.

Now we consider the assignment matrix where c_{ij} is the cost of assigning ith job to jth machine.

$$
\begin{array}{c|ccccc|}
 & 1 & 2 & 3 & \ldots & n \\
\hline
1 & C_{11} & C_{12} & C_{13} & \ldots\ldots & C_{1n} \\
2 & C_{21} & C_{22} & C_{23} & \ldots\ldots & C_{2n} \\
. & . & . & . & \ldots\ldots & . \\
. & . & . & . & \ldots\ldots & . \\
. & . & . & . & \ldots\ldots & . \\
. & & & & & \\
n & C_{n1} & C_{n2} & C_{n3} & \ldots\ldots & C_{nn} \\
\hline
\end{array}
$$

Figure 1

3.1 Proposed Method: Divide Column and Subtract One Assignment Method

The proposed algorithm of proposed method is as follows:

Step 1: Find the smallest number (cost) of each column. Divide each column by its smallest number.

Step 2: Then subtract 1 from every number (except zero) and we get zero in every row.

Now make assignment in terms of zeros. If there are some rows and columns without assignment, then we cannot get the optimum solution. Then we go to the next step.

Step 3: Draw the minimum number of lines passing through all the zeros by using the following procedure:

1. Mark (√) rows that do not have assignments.
2. Mark (√) columns that have crossed zeros in that marked rows.
3. Mark (√) rows that have assignments in marked columns.
4. Repeat (b) and (c) till no more rows or columns can be marked.
5. Draw straight lines through all unmarked rows and marked columns.

If the number of lines drawn is equal to the number of rows or columns, then the current solution is optimal solution. Otherwise go to next step.

Step 4: Select the smallest number of the reduced matrix not covered by the lines. Subtract this smallest number from all numbers not covered by a straight line and this same smallest number is added to every number including zeros lying at the intersection of any two lines. Other numbers covered by lines remain unchanged. Again make assignment in terms of zeros.

Step 5: If optimal solution is not found, then repeat steps (4) and (5) successively till an optimum solution is obtained.

4. Numerical Comparison of Existing Methods with Proposed Method

4.1. Solve the following assignment problem using Proposed Method

Consider the following assignment problem. Assign the five jobs to the five machines as to minimize the total cost.

	1	2	3	4	5
A	12	8	7	15	4
B	7	9	1	14	10
C	9	6	12	6	7
D	7	6	14	6	10
E	9	6	12	10	6

Figure 2

Solution:

Step 1: Find the smallest number (cost) of each column. Divide each column by its smallest number.

	1	2	3	4	5
A	1.71	1.33	7	2.5	1
B	1	1.5	1	2.33	2.5
C	1.29	1	12	1	1.75
D	1	1	14	1	2.5
E	1.29	1	12	1.67	1.5

Figure 3

Step 2: Then subtract 1 from every number (except zero) and we get zero in every row.

	1	2	3	4	5
A	0.71	0.33	6	1.5	0
B	0	0.5	0	1.33	1.5
C	0.29	0	11	0	0.75
D	0	0	13	0	1.5
E	0.29	0	11	0.67	0.5

Figure 4

Step 3: Make initial assignment.

	1	2	3	4	5
A	0.71	0.33	6	1.5	$\boxed{0}$
B	$\cancel{0}$	0.5	$\boxed{0}$	1.33	1.5
C	0.29	$\cancel{0}$	11	$\boxed{0}$	0.75
D	$\boxed{0}$	$\cancel{0}$	13	$\cancel{0}$	1.5
E	0.29	$\boxed{0}$	11	0.67	0.5

Figure 5

Here we see that all zeros are either assigned or crossed out. That is, the total assigned zero's is 5 which is equal to the number of rows or columns. And the solution is (1,5), (2,3), (3,4), (4,1), (5,2).

So the total cost = 4 + 1 + 6 + 7 + 6 = 24.

4.2. Solve the following assignment problem using Hungarian Assignment Method

Consider the following assignment problem. Assign the five jobs to the five machines as to minimize the total cost.

	1	2	3	4	5
A	12	8	7	15	4
B	7	9	1	14	10
C	9	6	12	6	7
D	7	6	14	6	10
E	9	6	12	10	6

Figure 6

Solution:

Step 1: Select the minimum element in each row and subtract this element from every element in that row.

	1	2	3	4	5
A	8	4	3	11	0
B	6	8	0	13	9
C	3	0	6	0	1
D	1	0	8	0	4
E	3	0	6	4	0

Figure 7

Step 2: Select the minimum element in each column and subtract this element from every element in that column.

	1	2	3	4	5
A	7	4	3	11	0
B	5	8	0	13	9
C	2	0	6	0	1
D	0	0	8	0	4
E	2	0	6	4	0

Figure 8

Step 3: Make initial assignment.

	1	2	3	4	5
A	7	4	3	11	$\boxed{0}$
B	5	8	$\boxed{0}$	13	9
C	2	$\cancel{0}$	6	$\boxed{0}$	1
D	$\boxed{0}$	$\cancel{0}$	8	$\cancel{0}$	4
E	2	$\boxed{0}$	6	4	$\cancel{0}$

Figure 9

Here we see that all zeros are either assigned or crossed out. That is, the total assigned zero's is 5 which is equal to the number of rows or columns. And the solution is (1,5), (2,3), (3,4), (4,1), (5,2).

And the total cost = 4 + 1 + 6 + 7 + 6 = 24.

4.3. Solve the following assignment problem using Matrix One's Assignment Method

Consider the following assignment problem. Assign the five jobs to the five machines as to minimize the total cost.

	1	2	3	4	5
A	12	8	7	15	4
B	7	9	1	14	10
C	9	6	12	6	7
D	7	6	14	6	10
E	9	6	12	10	6

Figure 10

Solution:

Step 1: Find the minimum cost of each row and then divide each element of each row of the matrix by its minimum cost.

	1	2	3	4	5
A	3	2	1.75	3.75	1
B	7	9	1	14	10
C	1.5	1	2	1	1.17
D	1.17	1	2.33	1	1.67
E	1.5	1	2	1.67	1

Figure 11

Step 2: Find the minimum cost of each column and then divide each element of the column by its minimum cost.

	1	2	3	4	5
A	2.57	2	1.75	3.75	1
B	6	9	1	14	10
C	1.29	1	2	1	1.17
D	1	1	2.33	1	1.67
E	1.29	1	2	1.67	1

Figure 12

Step 3: Now make initial assignment.

	1	2	3	4	5
A	2.57	2	1.75	3.75	$\boxed{1}$
B	6	9	$\boxed{1}$	14	10
C	1.29	$\cancel{1}$	2	$\boxed{1}$	1.17
D	$\boxed{1}$	$\cancel{1}$	2.33	$\cancel{1}$	1.67
E	1.29	$\boxed{1}$	2	1.67	$\cancel{1}$

Figure 13

Hence we can assign the ones and the solution is (1,5), (2,3), (3,4), (4,1), (5,2).

Hence the minimum cost= 4 + 1 + 6 + 7 + 6 =24.

Table 2: Comparison of Optimal Values of three Methods

Example	**HA-Method**	**MOA-Method**	**Proposed Method**	**Optimum**
01	**24**	**24**	**24**	**24**

Therefore we conclude that this new Proposed Method is effective for solving assignment problem.

5. Conclusion

In this paper, we presented a new method for solving Assignment problem. Initially, we explained the proposed algorithm and showed the efficiency of it by numerical example. And we get the optimal solution which is same as the optimal solutions of HA-method and MOA-method. Therefore this paper introduces a different approach which is easy to solve Assignment problem.

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