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Reliability Allocation and Optimization for a Complex Systems

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Abstract

A system is defined as an assembly of subsystems; each of the subsystems has its own attributes in terms of reliability. The reliability of the system as a total is a function related to the metrics of the reliability of the subsystem. The sum obtained from the costs that have to be paid for all the subsystems form together the costs of the entire system. I examined in this paper the attempts and method of allocating the values of reliability with the purpose of minimizing the costs. The problem applies to mechanical, electrical, computer hardware and software systems, and it includes thus a very wide range of aspects. The reliability of the system must be expressed in order for the problem to be solved, yet it must be expressed in relation to the terms given by the values of the subsystem’s reliability, with the specification of the function of the costs for the subsystems, at the same time allowing the overview of the problem related to optimization. A complex example for this is the examination upon the allocation of the software reliability. In my paper I have also carried out an evaluation of the preliminary distribution methods proposed. One of the possibilities is to use specific and accurate methods for optimization. Generally, if the focus is upon a more developed and complex case, this case will present itself with repeated approaches.

Keywords: Reliability; Allocation; Optimization Reliability.

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1. Introduction

The allocation of the optimal reliability is an problem that was tackled by several authors. Although most It has been paying attention to this issue to the repetition Allocation problem [8,9], a different approach for the problem is taken in this paper. In this research I will not focus on redundancy exclusively and on the minimum requirements, yet I will use a different approach, evaluating and estimating each unit's reliability within the entire system, in order to achieve the minimal costs for ensuring the reliability of the system which will be my paper's main objective. The model that I am using assesses each unit's reliability in relation to the reliability of the cost's increase. Various methods have been used to manage these problems related to customization, yet most of them have been limited as per their applicability to systems composed of units. In this paper the problem of allocation for optimal solutions which are constrained and nonlinear, and the solution that I have found for this problem is the algorithm based on the genetic algorithm method, using also Lagrange multipliers. By this approach which proved to be of high reliability, the components can be allocated in any type of system, either it is complex or simple. Although the non-linear aspects have indicated that the optimal solution is the formulation of the program which presented problems for several authors in the past, we could expect for this approach to be the one that reliability engineers resort to on a large scale. They will give this adequate attention to the implementation of this approach. The elements that contributed to this situation are two elements. On one hand, there is a system reliability equation in light of the analytical model and although this is not a complex problem in the case of simple systems, in the complex ones it can become a wide and complex question. On the other hand, there is a cost function model which also has a reliability aspect involved. Various authors have addressed this field by introducing the sports element in combinations of functions that generally involve costs [1,11]. Engineers must supply mathematical formulas, using specific criteria. Finding the parameters needed for the measurement was a heavy challenge for the engineers, most of the time because no documents were concluded and issued in relation to reliability principles. There were flaws in the attempts to shape and formulate a problem related to distribution in this field and a great achievement was reached when the model was implemented by introducing to the public the Analysis Package Turbo Pascal, a maintenance program operating a system with high reliability. We can now import this equation as a direct contribution to the algorithm in order to make it optimal. The cost function problem has also been approached by means of proposals and by means of the general cost function. This is a function of the parameters and engineers can easily quantify it and use it.

2. System reliability allocation

The allocations of reliability for the hardware or software systems can start immediately after the creation of the models for reliability. The initial values that the system is allocated should be the values specified for the system's reliability metrics or a set of values related to reliability that are more difficultly obtained than the specified ones. The more offensive reliability values than the required ones are assigned sometimes to the system, for the purpose of the functionality aspect of the system to increase and for giving additionally some reliability margin in the further stage of the design process to the parts of the system which could not reach the values that were assigned to them.

Notation

1. C = Total cost of the system.

2. c_i (R_i) = The cost of the component / subsystem i.

3. R_i = The reliability of the component / subsystem i.

4. n = The number of components in the system, that are taken into account for optimization.

5. $R_{i,}$ *min* = The minimum reliability of the component / subsystem i.

6. $R_{i,}$ *max* = The maximum reliability of the component / subsystem i that can be achieved.

7. R_s = The system reliability.

8. R_G = The goal of the system reliability

9. I_R = The importance of the reliability.

2.1. Assumptions of the Model

1. All the components in each stage are assumed to be identical, i.e., all the components have the same reliability.
2. The components are assumed to be statistically independent, i.e., failure of a component does not affect the performance of the other components in any system.
3. A component is either in working condition or non-working condition.

3. Model Formulation

Consider a system consisting of n element. The goal is to the allocate the reliability of each units so that Match the reliability of the system reliability goal system (R_s) the cost function of obtaining the minimum value. This non-linear and the formulation of the problem as follows:

$$p: min \quad C = \sum_{i=1}^{n} ci(Ri) \tag{1}$$

s.t

$R_s \geq R_G$,

$$0 \leq R_i \leq 1,\ i = 1, 2, ..., n.$$

Where R_s is the overall reliability of the system which can be calculated by different methods, a review of these methods can be found in [5]. R_G is the goal reliability of the system. C is the cost function which is determined with an empirical relation as follows:

$$c_i = K_i \ln\left(\frac{1}{1 - R_i}\right), \quad K_i > 0. \tag{2}$$

This expression aims to obtain the least cost for the total system, which is subjected to R_G a limit of the system reliability which is lower. The first step is to achieve the analytical reliability function for the system (with respect to the reliability of the components in the system. There are various approaches for the achievement of the reliability equations for the system. A review for these approaches can be found in the reference. The Turbo Pascal software will be used in this paper, whereas it is conceived in order to solve problems for the analytical reliability function of the system. The following step is to achieve the relationship for each component's cost, as a performance of the system's reliability.

The starting reliability for that component in particular is 70 as the achievable reliability and 99. The expression of eq. (1) in relation to each parameter and the explanation for these parameters with full details is given. The cost function proposed, which is given by the eq. (2) meets the requirements below (1):

1. The cost is a function of the reliability of one component that increases steadily.
2. The cost of the component with a high level of reliability is extremely high.
3. The cost of the component with a high level of reliability is extremely low.
4. The derivation of the cost (related to reliability) is a function of the reliability that is increasing steadily.

One can easy observe that the cost function of the eq. (2) can be easily implemented, having only two elements that are required to be inserted (additionally to the component's failure distribution). That is the feasibility and the highest reliability that can be achieved. One should also observe that this function with aspects of penalty, within eq. (2) doesn't have any dimensions, yet it functions as a factor for weight, presenting the difficulty encountered while increasing the reliability of the component from its previous values. I hereby presented and illustrated the way in which the model can be applied for a series system and for a complex one, in Section.

After analyzing the cost function of the eq. (2), the notes below are derived and concluded:

1) There is a relation between the increase of the cost and the difference between the assigned reliability and the lowest one (the current one), addressing the maximum reliability that can be achieved.
2) The range of improvement has as one of its functions the cost. The cost represents the difference between the initial reliability of the components and the highest reliability that can be achieved.
3) The eq. (2) shows an exponent approaching infinity while the reliability of the components comes closer to the highest value that it can achieve. This involves an easier way to reach a higher reliability

of a component as from 70 to 75, than to reach a higher reliability for a component from 95 to 96.

3.1. Reliability systems (network model)

We introduce a graphical network model in which it is possible to determine whether a system is working correctly by determining whether a successful path exists in the system. The system fails when no such path exists. The considered network models which have been mentioned in the previous sections provide a picture of the reliability of complex systems. The system in Fig. 1 cannot be split into a group of series and parallel systems. This is primarily due to the fact that the components *A* and *D* each allow two paths emerging from them, whereas *B* has only one; S_1, S_2, S_3, S_4 and S_5 are called subsystems or arcs. There exist several methods for obtaining the reliability of a complex system, as, for example, decomposition method [12].

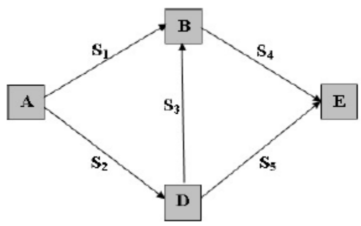

Figure 1: A complex system (network model).

3.2. The Inclusion-Exclusion Method

Let Ai denote the event that all elements in the i^th minimal path are functional, so we can say that A_i represents the event that minimal path i works. $\bar{A}_i$ denote the complement of this event. The probability that the minimal path i works can be expressed as

$$P(A_i) = \prod_{k \in \text{element of k}} p_i. \tag{3}$$

Let *np* be the number of minimal path sets. A system with np minimal paths works if and only if at least one of the minimal paths works. System success corresponds to the event $\cup_{i=1}^{np} A_i.$. The reliability of the system is equal to the probability of the union of the n minpaths, namely

$$R_s(G) = P(\cup_{i=1}^{np} A_i).$$

$$S_k = \sum P(A_{i1} \cap A_{i2} \cap \ldots \cap A_{ik}). \quad (4)$$

Then, S_k represents the sum of the probabilities' that any k minimal paths are simultaneously working. By the Inclusion-Exclusion principle (see [9]), the reliability of the system, which is equal to the probability of the union of the np minpaths, can be expressed as:

$$R_s = \sum_{k=1}^{np} (-1)^{(k-1)} S_k. \quad (5)$$

If there are np path sets, then this calculation involves 2^{np}-1 terms. In some cases, two different intersections of Ai's will correspond to the same event so that these different intersections of Ai's will have the same probability. If one intersection consists of an odd number of Ai's and another intersection consists of an even number of Ai's, they will cancel. Satyanarayana [10] and Satyanarayana and Prabhaka in [4] give algorithms that generate only the non cancelling terms.

The path sets are:

$P_1=\{S_1,S_4\}$; $P_2=\{S_2,S_5\}$; $P_3=\{S_2,S_3,S_4\}$;

Then the reliability of complex system (network model) as shown in fig. 1 is:

$$R_s = P_r(P_1,P_2,P_3) = P_r(P_1 \cup P_2 \cup P_3) = P_r(P_1) + P_r(P_2) + P_r(P_3) - P_r(P_1 P_2) - P_r(P_1 P_3) - P_r(P_2 P_3) + P_r(P_1 P_2 P_3)$$

$$= R_1R_4 + R_2R_5 + R_2R_3R_4 - R_1R_2R_4R_5 - R_1R_2R_3R_4 - R_2R_3R_4R_5 + R_1R_2R_3R_4R_5. \quad (6)$$

3.3. Reliability Importance

Importance measures provide a method of identifying the relative importance of each component in a system with respect to the overall reliability of the system. Component importance analysis discussed in [13] and [14] can analyze the system structure and help to diagnose the weaknesses of the system.

$$I_{R,i}(t) = \frac{\partial R_S(t)}{\partial R_i(t)}. \quad (7)$$

The value of the reliability importance given by eq. (7) depends both on the reliability of a component as well as on its corresponding position in the system.

4. Applications

The examples below show the steps used and the procedures addressed for the solving of the allocation problem. The model presented an algorithm of nonlinear programming is used for reaching the solution. The algorithm represents the part of the Turbo Pascal software implemented. (Hooke-Jeeves) is resorted to for the purpose of solving the examples below.

4.1. Application to a Complex System

Consider the system shown in Fig. 1. All components have the same initial reliability of 90% at a given time. A system reliability goal of 90% (at the same given time for the components) is sought. The equation for the system reliability obtained from the Inclusion-Exclusion Method is given by,

$$Rs = R_1 R_4 + R_2 R_5 + R_2 R_3 R_4 - R_1 R_2 R_3 R_4 - R_1 R_2 R_4 R_5 - R_2 R_3 R_4 R_5 + R_1 R_2 R_3 R_4 R_5$$

Before solving the problem of allocating the reliability, a preliminary analysis can be done for assessing the result of the solution. The result can be obtained by the calculation of the importance of reliability for each of the system's components. By means of equation 3, the importance of the reliability was calculated and the potential results were shown in fig 2.

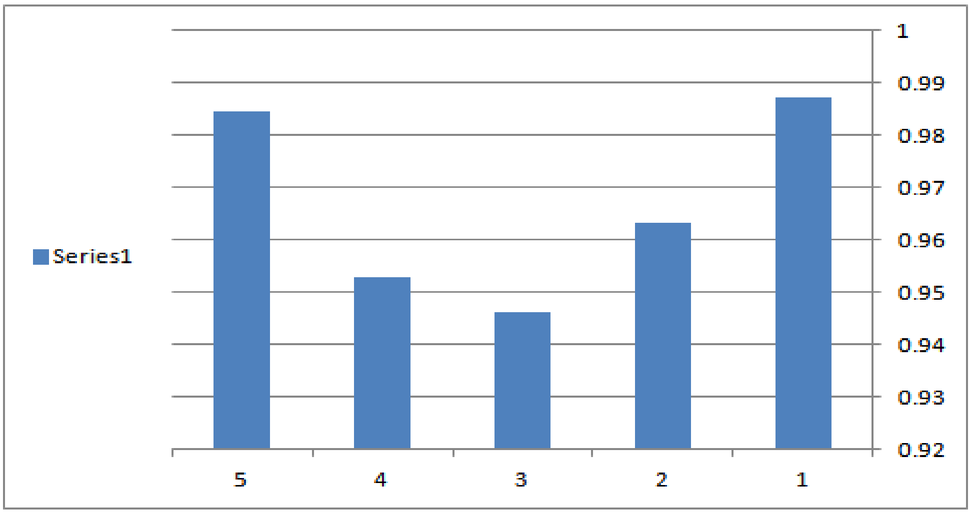

Figure 2: Reliability Importance for the system of Fig. 1.

Due to the fact that the same improvement range and the same level of feasibility is used for all the components of the system, the reliability is assigned in relation to the position of each of the system's components. The way in which the reliability of the system affects each component due to its position can be calculated with the importance of the reliability. Thus, one will expect that the components which have a high level of reliability will have allocated the same value of reliability. It is obvious that the first and the seventh component are crucial and have the same value of the importance. Subsequently, it is easy to see that the reliability of the system will be first allocated to the first and to the seventh component, having the same value of reliability. In a similar way, for all the other components, the results for the values of reliability which are the highest that can be achieved are shown by Table 1. This table shows that the optimization results are consistent with the preliminary assessment, based on the importance of the reliability for every component. Additionally, the impact of the highest reliability that can be achieved on the optimization solution can also be seen. The reliability is firstly assigned to the first and to the fifth component. Components second, third and forth show a slight improvement. Thus, the process of assignment is focused on the components left in the system. This example illustrates that the complexity of the system's reliability equation can increase, but at the same time the allocation problem can still be solved successfully. In the same manner, systems with redundancy and standby components can also be optimized as long as the system's reliability equation can be obtained analytically in the form of as equation (3).

Table 1: An example of a table

No.	Case 1	Case 2
Component 1	0.994	0.9874
Component 2	0.923	0.9633
Component 3	0.932	0.9463
Component 4	0.941	0.9029
Component 5	0.992	0.9846

5. Conclusions

I concluded this paper aiming to discuss the problem of allocation reliability for each unit of the units forming the system in general. A non-linear programming problem was proposed related to the cost function. I have chosen to use this function because its configuration is more simple, having parameters that can be more easily quantified. The concentration should focus more on the research aiming to obtain these jobs, based on the current and factual data related to costs. Using the method could be applied to any system, showing an increased number of complications. The model is used with high efficiency results for larger projects and systems. As long as we can derive the reliability function of the system by using the analytical method, we can use this model in order to find the solution for the reliability allocation problem. Phrasing it differently, this can be personalized for the reliability of some subsystems or of all subsystems of the system. The methodology that I have shown in this paper represents a useful instrument for aiding the researchers in making a decision. The cost function parameters proposed could be changed, in order to allow the researchers to inquire upon the distribution of various scenarios. Consequently, we could design together with the reliability engineers a plan on deciding the way in which we could achieve the lowest level of allocated credibility needed for each component of the system. For more description, A complex system allocation problem was solved It was shown that different components have different importance and effect on overall cost of the system. For more description, A complex system allocation problem was solved It was shown that different components have different importance and effect on overall cost of the system. This paper, it is estimated in the minimum model Reliability requirements for multiple components within the a system that will achieve the objective value of the reliability of the system. General behavior of it is supposed to cost as a function of the component's reliability this matter. And then reduce the system cost by solving for optimum reliability element, which satisfies requirement target system reliability. Once reliability the requirement for each component is estimated, one can then decide whether to achieve this reliability through fault tolerance or Error terminate the contract. Made a very encouraging model the results can be applied to any type of system, simple or complex, and a variety of distributions. Feature this model is that it is very flexible, and requires very little processing time. These advantages make the proposed reliability allocation great solution system design tool.

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