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On Pairwise λ-Open Soft Sets and Pairwise Locally Closed Soft Sets

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Abstract

Kandil and his colleagues [10], introduced the notion of $p\lambda$-closed soft set by involving $p\Lambda$-soft set and p-closed soft set. In this paper, we give some additional properties of $p\lambda$-closed soft sets. We also introduce and study a related new class of $PST^*_{1/4}$-spaces which lies between $\boldsymbol{PST^*_0}$ and $PST^*_{1/2}$. Moreover, we show that there exists a very important relation between the notion of $p\lambda$-closed soft sets and the $PST^*_{1/4}$ property, $i = 0, 1/4, 1/2$. In addition, we offer the notion of p-locally closed soft sets and we investigate a related new pairwise soft separation axiom PST^*_L which is independent from $PST^*_{1/4}$. The relationships between the $p\lambda$-closed soft sets and the p-locally closed soft sets are obtained. Furthermore, we introduce the notion of $p\lambda$-open soft sets and we construct supra soft topology associated with the class of $p\lambda$-open soft sets and we present pairwise soft separation axioms related to such soft sets, namely PST^*_λ. We provide some illustrative examples to support the results.

Keywords: Soft set; Soft topology; Soft bitopology; Soft bitopological spaces; Pairwise soft separation axioms; $p\lambda$- closed soft sets; Pairwise $p\lambda$-open soft sets; and p- locally closed soft sets.

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1. Introduction

In 1999, Molodtsov [14] introduced the concept of soft set as a mathematical modeling for dealing with uncertainties inherent in many of real world problems. Shabir and Naz [20] introduced the concept of soft topological spaces and investigate some fundamental properties of such spaces. Many researches (see, for example, [1,3,4,5,6,7,8,15,16,18]) introduced and discuss new notions of soft topological spaces. Ittanagi [2] introduced the notion of soft bitopological space. He also offer some types of soft separation axioms in soft bitopological space. Kandil and his colleagues [10] introduced the notions of $p\Lambda$ -soft sets and $p\lambda$ -closed soft sets in soft bitopological spaces. They the family of all $p\Lambda$ -soft sets defines an Alexandroff soft topology. Kandil and his colleagues [9] introduced the concept of gp -closed soft sets and defined the associated pairwise soft separation axioms, namely, $PST^*_{1/2}$ and PSR^*_0. Recently, Kandil and his colleagues [11] introduced some types of pairwise soft separation axioms, namely, PST^*_0 , PST^*_1 , PST^*_2 and PSR^*_1 . They studied the characterization and implications among these types of separation axioms.

The motivation of the present paper is to introduce new classes of soft sets called p -locally closed soft sets and $p\lambda$ -open soft sets in soft bitopological spaces. It turn out that $p\lambda$ -closed soft sets, $p\lambda$ -open soft sets and p -locally closed soft sets are weaker forms of p -open and p -closed soft sets. We also conclude several important properties of such soft sets. Moreover, we introduce and study a related pairwise soft separation axioms, namely, $PST^*_{1/4}$, PST^*_L and PST^*_λ. We studied the relationships between these types of separation axioms and the other in [9,11].

2. Preliminaries

In this section, we briefly review some concepts and some related results of soft set, soft topological space and soft bitopological space which are needed to used in current paper. For more details about these concepts you can see [2,4,5,6,7,8,9,10,11,14,15,16,19,20,21,22].

Let X be an initial universe, E be a set of parameters and $P(X)$ be the power set of X .

Definition 2.1 *[16] A pair* (F,E) *is called a soft set over* X *, where* F *is a mapping given by* $F : E \to P(X)$ *. A soft set can also be defined by the set of ordered pairs*

$$(F,E) = \{(e, F(e)) : e \in E,\ F : E \to P(X)\}.$$

From now on, $SS(X)_E$ denotes the family of all soft sets over X with a fixed set of parameters E .

For two soft sets $(F,E),(G,E) \in SS(X)_E$, (F,E) is called a soft subset of (G,E) , denoted by $(F,E) \tilde{\subseteq} (G,E)$, if $F(e) \subseteq G(e)$, $\forall e \in E$. In this case, (G,E) is called a soft superset of (F,E). In addition, the union of soft sets (F,E) and (G,E), denoted by $(F,E)\tilde{\cup}(G,E)$, is the soft set (H,E) which defined as $H(e)=F(e)\cup G(e)$, $\forall e\in E$. Moreover, the intersection of soft sets (F,E) and (G,E), denoted by $(F,E)\tilde{\cap}(G,E)$, is the soft set (M,E) which defined as, $M(e)=F(e)\cap G(e)$, $\forall e\in E$. The complement of a soft set (F,E), denoted by $(F,E)^c$, is defined as, $(F,E)^c=(F^c,E)$, where $F^c:E\to P(X)$ is a mapping given by $F^c(e)=X\setminus F(e)$, $\forall e\in E$. The difference of soft sets (F,E) and (G,E), denoted by $(F,E)\setminus(G,E)$, is the soft set (H,E), which defined as, $H(e)=F(e)\setminus G(e)$, $\forall e\in E$. Clearly, $(F,E)\setminus(G,E)=(F,E)\tilde{\cap}(G,E)^c$. A soft set (F,E) is called a null soft set, denoted by $(\tilde{\phi},E)$, if $F(e)=\phi$, $\forall e\in E$. Moreover, a soft set (F,E) is called an absolute soft set, denoted by $(\tilde{X},E)$, if $F(e)=X$, $\forall e\in E$. Clearly, we have $(\tilde{\phi},E)^c=(\tilde{X},E)$ and $(\tilde{X},E)^c=(\tilde{\phi},E)$. Moreover, a soft set (G,E) is said to be a finite soft set if $G(e)$ is a finite set for all $e\in E$. Otherwise, it is called an infinite soft set.

Definition 2.2 *([15,17,21]) A soft set (F,E) over X is said to be a soft point in $(\tilde{X},E)$ if there exist $x\in X$ and $e\in E$ such that $F(e)=\{x\}$ and $F(e')=\phi$ for each $e'\in E\setminus\{e\}$. This soft point is denoted by (x_e,E) or x_e, i.e., $x_e:E\to P(X)$* is a mapping defined as

$$x_e(a)=\begin{cases}\{x\} & if \quad e=a,\\ \phi & if \quad e\neq a\end{cases} \qquad \text{for all } a\in E.$$

A soft point (x_e,E) is said to be belonging to the soft set (G,E), denoted by $x_e\tilde{\in}(G,E)$, if $x_e(e)\subseteq G(e)$, i.e., $\{x\}\subseteq G(e)$. Clearly, $x_e\tilde{\in}(G,E)$ if and only if $(x_e,E)\tilde{\subseteq}(G,E)$. In addition, two soft points x_{e_1}, y_{e_2} over X are said to be equal if $x=y$ and $e_1=e_2$. Thus, $x_{e_1}\neq y_{e_2}$ iff $x\neq y$ or $e_1\neq e_2$. The family of all soft points in $(\tilde{X},E)$ is denoted by $\xi(X)_E$.

Proposition 2.1 *[21] The union of any collection of soft points can be considered as a soft set and every soft set can be expressed as a union of all soft points belonging to it, i.e., $(G,E)=\bigcup\{(x_e,E):x_e\tilde{\in}(G,E)\}$.*

Proposition 2.2 *[21] Let (G,E), (H,E) be two soft sets over X. Then,*

1) $x_e\tilde{\in}(G,E)\Leftrightarrow x_e\tilde{\notin}(G,E)^c$.

2) $x_e\tilde{\in}(G,E)\tilde{\cup}(H,E)\Leftrightarrow x_e\tilde{\in}(G,E)$ or $x_e\tilde{\in}(H,E)$.

3) $x_e\tilde{\in}(G,E)\tilde{\cap}(H,E)\Leftrightarrow x_e\tilde{\in}(G,E)$ and

$x_e \tilde{\in} (H,E)$.

4) $(G,E) \tilde{\subseteq} (H,E) \Leftrightarrow [x_e \tilde{\in} (G,E) \Rightarrow x_e \tilde{\in} (H,E)]$

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For more details for soft point you can see in [15,21,17].

Definition 2.3 *[20] Let η be a collection of soft sets over a universe X with a fixed set of parameters E, i.e., $\eta \subseteq SS(X)_E$. The collection η is called a soft topology on X if it satisfies the following axioms:*

1) $(\tilde{X}, E), (\tilde{\phi}, E) \in \eta$,
2) The union of any number of soft sets in η belongs to η,
3) The intersection of any two soft sets in η belongs to η.

The triple (X, η, E) is called a soft topological space. Any member of η is said to be an open soft set in (X, η, E). A soft set (F, E) over X is said to be a closed soft set in (X, η, E), if its complement $(F, E)^c$ is an open soft set in (X, η, E). We denote the family of all closed soft sets by η^c.

Definition 2.4 *[20] Let (X, η, E) be a soft topological space and $(F, E) \in SS(X)_E$. The soft closure of (F, E), denoted by $scl_\eta(F, E)$, is the intersection of all closed soft super sets of (F, E), i.e,*

$$scl_\eta(F,E) = \bigcap \{(H,E) \in SC_\eta(X) : (F,E) \tilde{\subseteq} (H,E)\}).$$

Clearly $scl_\eta(F, E)$ is the smallest closed soft set over X which contains (F, E).

Definition 2.5 *[13] A soft set (G, E) in a soft topological space (X, η, E) is called a generalized closed soft set [briefly, g-closed soft set] if $scl_\eta(G,E) \tilde{\subseteq} (H,E)$ whenever $(G,E) \tilde{\subseteq} (H,E)$ and $(H,E) \in \eta$.*

Definition 2.6 *[16] A soft topological space (X, η, E) is said to be a soft T_0 [briefly, ST_0] if for each $x_\alpha, y_\beta \in \xi(X)_E$ with $x_\alpha \neq y_\beta$, there exists $(G,E) \in \eta$ such that $x_\alpha \tilde{\in} (G,E)$, $y_\beta \tilde{\notin} (G,E)$ or $y_\beta \tilde{\in} (G,E)$, $x_\alpha \tilde{\notin} (G,E)$.*

Definition 2.7 *[13] A soft topological space (X, η, E) is called a soft $T_{1/2}$ [briefly,S $T_{1/2}$] if every g-closed soft set is a closed soft set.*

Theorem 2.1 *[9] A soft topological space (X, η, E) is a soft $T_{1/2}$ if and only if every soft point either open* *soft set or closed soft set.*

Definition 2.8 *[4] Let* μ *be a collection of soft sets over* X *[i.e.,* $\mu \subseteq SS(X)_E$*]. The collection* μ *is called a supra soft topology on* X *if it satisfies the following axioms:*

1) $(\tilde{X}, E)$, $(\tilde{\phi}, E) \in \mu$,
2) The union of any number of soft sets in μ belongs to μ.

Definition 2.9 *[2] A quadrable system* (X, η_1, η_2, E) *is called a soft bitopological space [briefly, sbts], where* η_1, η_2 *are arbitrary soft topologies on* X *with a fixed set of parameters* E.

Definition 2.10 *[10] Let* (X, η_1, η_2, E) *be a sbts. A soft set* (G, E) *over* X *is said to be a pairwise open soft set in* (X, η_1, η_2, E) *[briefly,* p*-open soft set] if there exist an open soft set* (G_1, E) *in* η_1 *and an open soft set* (G_2, E) *in* η_2 *such that* $(G, E) = (G_1, E) \tilde{\cup} (G_2, E)$. *A soft set* (G, E) *over* X *is said to be a pairwise closed soft set in* (X, η_1, η_2, E) *[briefly,* p*-closed soft set] if its complement is a* p*-open soft set in* (X, η_1, η_2, E). *Clearly, a soft set* (F, E) *over* X *is a* p*-closed soft set in* (X, η_1, η_2, E) *if there exist an* η_1*-closed soft set* (F_1, E) *and an* η_2*-closed soft set* (F_2, E) *such that* $(F, E) = (F_1, E) \tilde{\cap} (F_2, E)$.

The family of all p-open (p-closed) soft sets in a sbts (X, η_1, η_2, E) is denoted by η_{12} (η_{12}^c), respectively.

Theorem 2.2 *[10] Let* (X, η_1, η_2, E) *be a sbts. The family of all* p*-open soft sets* η_{12} *is a supra soft topology on* X, *where*

$$\eta_{12} = \{(G, E) = (G_1, E) \tilde{\cup} (G_2, E) : (G_i, E) \in \eta_i, i = 1,2\}.$$

The triple (X, η_{12}, E) is the supra soft topological space associated to the soft bitopological space (X, η_1, η_2, E).

Definition 2.11 *[10] Let* (X, η_1, η_2, E) *be a sbts and let* $(G, E) \in SS(X)_E$. *The pairwise soft closure of* (G, E), *denoted by* $scl_{12}(G, E)$, *is defined by*

$$scl_{12}(G, E) = \bigcap \{(F, E) \in \eta_{12}^c : (G, E) \tilde{\subseteq} (F, E)\}.$$

Clearly, $scl_{12}(G, E)$ is the smallest p-closed soft set contains (G, E).For more details about the properties of pairwise soft closure operator see in [10].

Definition 2.12 *[10] Let* (X,η_1,η_2,E) *be a sbts and let* $(G,E)\in SS(X)_E$ *. The pairwise soft interior of* (G,E) *, denoted by* $sint_{12}(G,E)$ *, is defined by*

$$sint_{12}(G,E)=\bigcup\{(H,E)\in\eta_{12}:(H,E)\tilde{\subseteq}(G,E)\}.$$

Clearly, $sint_{12}(G,E)$ is the largest p -open soft set contained in (G,E) . For more details about the properties of pairwise soft interior operator you can see see [10].

Definition 2.13 *[10] Let* (X,η_1,η_2,E) *be a sbts and let* $(G,E)\in SS(X)_E$ *. The pairwise soft kernel of* (G,E) *[briefly,* $sker_{12}(G,E)$ *], is the intersection of all* p *-open soft supersets of* (G,E) *, i.e.,*

$$sker_{12}(G,E)=\bigcap\{(H,E)\in\eta_{12}:(G,E)\tilde{\subseteq}(H,E)\}.$$

Definition 2.14 *[10] A soft set* (G,E) *is said to be a pairwise* Λ *- soft set in a soft bitopological space* (X,η_1,η_2,E) *[briefly,* $p\Lambda$ *-soft set] if* $sker_{12}(G,E)=(G,E)$ *.*

Theorem 2.3 *[10] Every* p *-open soft set is a* $p\Lambda$ *-soft set.*

Theorem 2.4 *[10] Let* (X,η_1,η_2,E) *be a sbts. Then, the class of all* $p\Lambda$ *-soft sets is an Alexandroff soft topology on* X *. This soft topology we denoted by* $\eta_{p\Lambda}$ *. The triple* $(X,\eta_{p\Lambda},E)$ *is the soft topological space associated to the soft bitopological space* (X,η_1,η_2,E) *, induced by the family of all* $p\Lambda$ *-soft sets.*

Theorem 2.5 *[10] Let* (X,η_1,η_2,E) *be a sbts. Then,*

$$\eta_1\cup\eta_2\subseteq\eta_{12}\subseteq\eta_{p\Lambda}\subseteq SS(X)_E.$$

Definition 2.15 *[10] A soft set* (G,E) *is said to be a pairwise* λ *-closed soft set in a sbts* (X,η_1,η_2,E) *[briefly,* $p\lambda$ *-closed soft set] if* $(G,E)=(F,E)\tilde{\cap}(H,E)$ *, where* (F,E) *is a* p *-closed soft set and* (H,E) *is a* $p\Lambda$ *-soft set. The family of all* $p\lambda$ *-closed soft sets we denoted by* $P\lambda CS(X,\eta_1,\eta_2)_E$ *.*

Theorem 2.6 *[10] Let* (X,η_1,η_2,E) *be a sbts. Then,*

1) Every p -closed soft set is a $p\lambda$ -closed soft set.
2) Every $p\Lambda$ -soft set is a $p\lambda$ -closed soft set.

Definition 2.16 *[12] Let* (X,η_1,η_2,E) *be a sbts and let* $(G,E)\in SS(X)_E$ *. The pairwise soft sub Kernel of*

(G,E) *[briefly,* $sker_{12}^{*}(G,E)$*] is defined by*

$$sker_{12}^{*}(G,E)=\bigcup\{(F,E)\in\eta_{12}^{c}:(F,E)\tilde{\subseteq}(G,E)\}.$$

For more details you can see in [12].

Definition 2.17 *[12] A soft set* (G,E) *is said to be a pairwise* $\vee$*-soft set [briefly,* $p\vee$*-soft set] in a sbts* (X,η_1,η_2,E) *if* $sker_{12}^{*}(G,E)=(G,E)$*. We denote the family of all* $p\vee$*-soft sets by* $P\vee S(X,\eta_1,\eta_2)_E$ .Clearly, (G,E) is a $p\Lambda$-soft set if and only if $(G,E)^c$ is a $p\vee$-soft set.

Corollary 2.1 *[12] Let* (X,η_1,η_2,E) *be a sbts. The family of all* $p\vee$*-soft sets is an Alexandroff soft topology on* X*. This soft topology we denoted by* $\eta_{p\vee}$.

Theorem 2.7 *[12] For any sbts* (X,η_1,η_2,E)*, we have* $\eta_{p\Lambda}=\eta_{p\vee}^{c}$.

Definition 2.18 *[11] A soft bitopological space* (X,η_1,η_2,E) *is said to be a pairwise soft* $\boldsymbol{T_0^*}$ *[briefly,* $\boldsymbol{PST_0^*}$*] if for each* $x_\alpha, y_\beta\in\xi(X)_E$ *with* $x_\alpha\neq y_\beta$*, there exists* $(G,E)\in\eta_{12}$ *such that* $x_\alpha\tilde{\in}(G,E)$*,* $y_\beta\tilde{\notin}(G,E)$ *or* $y_\beta\tilde{\in}(G,E)$*,* $x_\alpha\tilde{\notin}(G,E)$.

Theorem 2.8 *[11] Let* (X,η_1,η_2,E) *be a sbts. Then*

(X,η_1,η_2,E) is a $\boldsymbol{PST_0^*}$ if and only if every soft point $x_e\in\xi(X)_E$ is a $p\lambda$-closed soft set.

Lemma 2.1 *[11] Let* (X,η_1,η_2,E) *be a sbts. Then*

(X,η_1,η_2,E) is a $\boldsymbol{PST_0^*}$ if and only if for all $x_\alpha, y_\beta\in\xi(X)_E$, $x_\alpha\neq y_\beta$ there exists $(G,E)\in\eta_{12}\cup\eta_{12}^{c}$ such that $x_\alpha\tilde{\in}(G,E)$ and $y_\beta\tilde{\notin}(G,E)$.

Definition 2.19 *[9] Let* (X,η_1,η_2,E) *be a sbts and let* $(G,E)\in SS(X)_E$*. A soft set* (G,E) *is said to be a generalized pairwise closed soft set [briefly,* gp*-closed soft set] if* $scl_{12}(G,E)\tilde{\subseteq}(H,E)$ *whenever* $(G,E)\tilde{\subseteq}(H,E)$ *and* (H,E) *is a* p*-open soft set.*

Definition 2.20 *[9] A soft bitopological space* (X,η_1,η_2,E) *is called a pairwise soft* $T_{1/2}^{*}$ *[briefly,* $PST_{1/2}^{*}$*] if every* gp*-closed soft set is a* p*-closed soft set.*

Theorem 2.9 *[9] Let* (X,η_1,η_2,E) *be a soft bitopological space. Then,* (X,η_1,η_2,E) *is a* $PST^*_{1/2}$ *if and only if every soft point either* p *-open soft set or* p *-closed soft set.*

Theorem 2.10 *[9] In any sbts* (X,η_1,η_2,E) *, every soft point either* p *-closed soft set or its complement is a* gp *-closed soft set.*

3. More on $p\lambda$ −closed soft sets

In this section, we give some additional properties of $p\lambda$ -closed soft sets and introduce and study a related new pairwise soft separation axiom $PST^*_{1/4}$.

Theorem 3.1 *Let* (X,η_1,η_2,E) *be a sbts and* $(G,E)\in SS(X)_E$ *. The arbitrary intersection of* $p\lambda$ *-closed soft sets is a* $p\lambda$ *-closed soft set.*

Proof. Let $\{(G_i,E): i\in\Delta\}\subseteq P\lambda CS(X,\eta_1,\eta_2)_E$. Then, $(G_i,E)\in P\lambda CS(X,\eta_1,\eta_2)_E$ $\forall\, i\in\Delta$. Thus, for all $i\in\Delta$ there exist $(F_i,E)\in\eta_{12}^c$ and $(H_i,E)\in\eta_{p\Lambda}$ such that

$$(G_i,E)=(F_i,E)\tilde{\cap}(H_i,E)\;\Rightarrow\;\bigcap_{i\in\Delta}(G_i,E)=\bigcap_{i\in\Delta}[(F_i,E)\tilde{\cap}(H_i,E)]$$

$$=[\bigcap_{i\in\Delta}(F_i,E)]\bigcap[\bigcap_{i\in\Delta}(H_i,E)].$$

Since $(F_i,E)\in\eta_{12}^c$ $\forall\, i\in\Delta$, then $\bigcap_{i\in\Delta}(F_i,E)\in\eta_{12}^c$. Also, since $(H_i,E)\in\eta_{p\Lambda}$ $\forall\, i\in\Delta$, then $\bigcap_{i\in\Delta}(H_i,E)\in\eta_{p\Lambda}$ [for $\eta_{p\Lambda}$ is an Alexandroff soft topology]. Therefore, $\bigcap_{i\in\Delta}(G_i,E)$ is a $p\lambda$ -closed soft set.

Remark 3.1 *The union of any two* $p\lambda$ *-closed soft sets may not be a* $p\lambda$ *-closed soft set as shown in the following example.*

Example 3.1 *Let* $X=\{x,y\}$, $E=\{e_1,e_2\}$ *and let*

$$\eta_1=\{(\tilde{\phi},E),(\tilde{X},E),(G_1,E),(G_2,E)\},\ \eta_2=\{(\tilde{\phi},E),(\tilde{X},E),(H,E)\},$$

where

$$(G_1,E)=\{(e_1,\{x\}),(e_2,\{y\})\},$$

$(G_2, E) = \{(e_1, \{y\}), (e_2, \{x\})\}$,

$(H, E) = \{(e_1, \{x\}), (e_2, \{x\})\}$.

Then, (X, η_1, η_2, E) is a sbts. Consequently,

$\eta_{12} = \{(\tilde{\phi}, E), (\tilde{X}, E), (G_1, E), (G_2, E), (H, E), (P_1, E), (P_2, E)\}$,

where

$(P_1, E) = \{(e_1, \{x\}), (e_2, X)\}$,

$(P_2, E) = \{(e_1, X), (e_2, \{x\})\}$.

It is easy to prove that

$$\eta_{p\wedge} = \eta_{12} \bigcup \{(M_1, E), (M_2, E)\}$$

where

$(M_1, E) = \{(e_1, \{x\}), (e_2, \phi)\}$, $(M_2, E) = \{(e_1, \phi), (e_2, \{x\})\}$.

Let $(K, E) = \{(e_1, \phi), (e_2, \{y\})\}$, then (K, E) is a p-closed soft set. It is clear that (K, E) and (G_2, E) are $p\lambda$-closed soft sets but $(K, E) \tilde{\cup} (G_2, E) = \{(e_1, \{y\}), (e_2, X)\}$ is not $p\lambda$-closed soft set.

Moreover, $P\lambda CS(X, \eta_1, \eta_2)_E = \eta_{12} \bigcup \eta_{12}^c \bigcup \{(M_1, E), (M_2, E)\}$.

Lemma 3.1 *Let* (X, η_1, η_2, E) *be a sbts and let* (x_e, E) *be a soft point in* $(\tilde{X}, E)$.

If $sker_{12}(x_e, E)^c = (x_e, E)^c$, then $(x_e, E)^c$ is a p-open soft set.

Proof. Assume that $(x_e, E)^c$ is not a p-open soft set, then the only p-open soft superset of $(x_e, E)^c$ is $(\tilde{X}, E)$. Therefore, $sker_{12}(x_e, E)^c = (\tilde{X}, E)$, a contradiction.

Theorem 3.2 *Let* (X, η_1, η_2, E) *be a sbts. Then*

(X, η_1, η_2, E) is a $PST^*_{1/2}$ iff every soft set on X is a $p\lambda$-closed soft set.

Proof. For any soft set (G,E) over X, let $(M,E)=\bigcup\{(x_e,E)\in\eta_{12}: x_e\,\tilde{\in}\,(G,E)^c\}$. Then, (M,E) is a p-open soft set. Moreover, every soft point in (M,E) is a p-open soft set. We set $(F,E)=\bigcap\{(y_\alpha,E)^c: y_\alpha\,\tilde{\in}\,(M,E)\}$, then (F,E) is a p-closed soft set. Now, we take $(N,E)=(G,E)^c\,\tilde{\cap}\,(M,E)^c$. Let $w_\gamma\,\tilde{\in}\,(N,E)$. Then $w_\gamma\,\tilde{\in}\,(G,E)^c$ and $w_\gamma\,\tilde{\in}\,(M,E)^c$. Therefore, (w_γ,E) is a p-closed soft set because if (w_γ,E) is not p-closed soft set, then by $PST^*_{1/2}$ property and Theorem 2.9 we conclude that (w_γ,E) is p-open soft set. It follows that $(w_\gamma,E)\,\tilde{\subseteq}\,(M,E)$ which contradicts with $w_\gamma\,\tilde{\in}\,(M,E)^c$. Consequently, every soft point in (N,E) is a p-closed soft set. Put $(H,E)=\bigcap\{(z_\beta,E)^c: z_\beta\,\tilde{\in}\,(N,E)\}$ implies

$$sker_{12}(H,E)=sker_{12}[\bigcap\{(z_\beta,E)^c: z_\beta\,\tilde{\in}\,(N,E)\}]$$

$$=\bigcap\{sker_{12}(z_\beta,E)^c: z_\beta\,\tilde{\in}\,(N,E)\}$$ [by Theorem 3.32 in [10]]

$$=\bigcap\{(z_\beta,E)^c: z_\beta\,\tilde{\in}\,(N,E)\}$$ [for $(z_\beta,E)^c$ is p-open soft set]

$$=(H,E).$$

Hence, (H,E) is a $p\Lambda$-soft set. Now, since $(M,E)\,\tilde{\subseteq}\,(G,E)^c$, then $y_\alpha\,\tilde{\in}\,(G,E)^c\ \ \forall\, y_\alpha\,\tilde{\in}\,(M,E)$ implies $(G,E)\,\tilde{\subseteq}\,(y_\alpha,E)^c\ \ \forall\, y_\alpha\,\tilde{\in}\,(M,E)$. It follows that $(G,E)\,\tilde{\subseteq}\,(F,E)$. Similarly, since $(N,E)\,\tilde{\subseteq}\,(G,E)^c$, then $(G,E)\,\tilde{\subseteq}\,(H,E)$. Consequently, $(G,E)\,\tilde{\subseteq}\,(H,E)\,\tilde{\cap}\,(F,E)$. On the other hand, let $z_e\,\tilde{\in}\,(H,E)\,\tilde{\cap}\,(F,E)$. Assume that $z_e\,\tilde{\notin}\,(G,E)$, then $z_e\,\tilde{\in}\,(G,E)^c$. We have two cases:

Case(1): If (z_e,E) is p-open soft point, then $z_e\,\tilde{\in}\,(M,E)$ it follows that $(F,E)\,\tilde{\subseteq}\,(z_e,E)^c$ which contradicts with $z_e\,\tilde{\in}\,(F,E)$.

Case(2): If (z_e,E) is not p-open soft point, then $z_e\,\tilde{\notin}\,(M,E)$ it follows that $z_e\,\tilde{\in}\,(M,E)^c$. Therefore, $z_e\,\tilde{\in}\,(N,E)$ which implies that $(H,E)\,\tilde{\subseteq}\,(z_e,E)^c$ which contradicts with $z_e\,\tilde{\in}\,(H,E)$. In both cases we have a contradiction. Hence, $z_e\,\tilde{\in}\,(G,E)$. Consequently, $(G,E)=(H,E)\,\tilde{\cap}\,(F,E)$. From the above we conclude that, (G,E) is a $p\lambda$-closed soft set.

Conversely, let (x_e,E) be a soft point in X. Then, $(x_e,E)^c$ is a soft set over X. Therefore, by hypothesis, $(x_e,E)^c$ is a $p\lambda$-closed soft set. Now, if (x_e,E) is a p-open soft set, then we are done. If (x_e,E) is not

p-open soft set, then $(x_e, E)^c$ is not p-closed soft set. But $(x_e, E)^c$ is a $p\lambda$-closed soft set, then there exist a $p\Lambda$-soft set (H, E) and a p-closed soft set (F, E) such that $(x_e, E)^c = (H, E) \tilde{\cap} (F, E)$. But the only p-closed soft superset of $(x_e, E)^c$ is $(\tilde{X}, E)$. Therefore, $(x_e, E)^c = (H, E)$. Consequently, $sker_{12}(x_e, E)^c = (x_e, E)^c$. Hence, by Lemma 3.1, $(x_e, E)^c$ is a p-open soft set. Therefore, (x_e, E) is a p-closed soft set.

Definition 3.1 *A soft topological space (X, η, E) is said to be a soft $T_{1/4}$ [briefly, $ST_{1/4}$] if for every finite soft set (G, E) over X and every $x_e \tilde{\notin} (G, E)$, there exists a soft set $(M^{x_e}, E) \in \eta \bigcup \eta^c$ such that $x_e \tilde{\notin} (M^{x_e}, E)$ and $(G, E) \tilde{\subseteq} (M^{x_e}, E)$.*

Example 3.2 Let $X = Z$, where Z denote the set of all integer numbers, E be a non-empty set of parameters such that $E = E_1 \cup \{d\}$ and $d \notin E_1$ and let

$\eta_d = \{(\tilde{\phi}, E)\} \bigcup \{(G, E) \in SS(Z)_E : 0_d \tilde{\in} (G, E) \text{ and } (G, E)^c \text{ is finite soft set}\}$. It is easy to prove that η_d is a soft topology on Z. Now, let (F, E) be a finite soft set and $x_e \tilde{\notin} (F, E)$, then we have four cases:

Case(1): $x_e \neq 0_d$, $x \neq 0$ and $0_d \tilde{\notin} (F, E)$. In this case, we have $0_d \tilde{\in} (F, E)^c$. But, (F, E) is finite soft set, then $(F, E)^c$ is an open soft set. It follows that (F, E) is a closed soft set. Take $(M^{x_e}, E) = (F, E)$.

Case(2): $x_e \neq 0_d$, $x \neq 0$ and $0_d \tilde{\in} (F, E)$. In this case, we have $0_d \tilde{\in} (x_e, E)^c$. So, $(x_e, E)^c$ is an open soft set, $(F, E) \tilde{\subseteq} (x_e, E)^c$ and $x_e \tilde{\notin} (x_e, E)^c$. Take $(M^{x_e}, E) = (x_e, E)^c$.

Case(3): $x_e \neq 0_d$, $x = 0$. In this case, we have $e \neq d$. So, $0_d \tilde{\in} (x_e, E)^c$. Therefore, $(x_e, E)^c$ is an open soft set. Take $(M^{x_e}, E) = (x_e, E)^c$.

Case(4): $x_e = 0_d$. In this case, we have $e = d$ and $x = 0$ implies $0_d \tilde{\notin} (F, E)$. Therefore, $(F, E)^c$ is an open soft set. It follows that (F, E) is a closed soft set. Take $(M^{x_e}, E) = (F, E)$.

Consequently, (Z, η_d, E) is a $ST_{1/4}$.

Theorem 3.3 *Let (X, η, E) be a soft topological space. Then,*

1) If (X,η,E) is a $ST1_{1/4}$, then it is ST_0.
2) If (X,η,E) is a $ST1_{1/2}$, then it is $ST1_{1/4}$.

Proof. Straightforward.

Definition 3.2 *A sbts (X,η_1,η_2,E) is said to be a pairwise soft $T^*_{1/4}$ [briefly, $PST^*_{1/4}$] if for every finite soft set (G,E) over X and every $x_e \tilde{\notin} (G,E)$, there exists a soft set $(M^{x_e},E) \in \eta_{12} \bigcup \eta_{12}^c$ such that $x_e \tilde{\notin} (M^{x_e},E)$ and $(G,E) \tilde{\subseteq} (M^{x_e},E)$.*

Theorem 3.4 *Let (X,η_1,η_2,E) be a sbts. Then,*

(X,η_1,η_2,E) is a $PST^*_{1/4}$ iff every finite soft set is a $p\lambda$-closed soft set.

Proof. Let (G,E) be a finite soft set over X. Since, (X,η_1,η_2,E) is a $PST^*_{1/4}$, then for every $x_e \tilde{\notin} (G,E)$ there exist $(M^{x_e},E) \in \eta_{12} \bigcup \eta_{12}^c$ such that $x_e \tilde{\notin} (M^{x_e},E)$ and $(G,E) \tilde{\subseteq} (M^{x_e},E)$. Set $(H,E) = \bigcap_{x_e \tilde{\notin}(G,E)} \{(M^{x_e},E) \in \eta_{12} : x_e \tilde{\notin} (M^{x_e},E), (G,E) \tilde{\subseteq} (M^{x_e},E)\}$ and $(F,E) = \bigcap_{x_e \tilde{\notin}(G,E)} \{(M^{x_e},E) \in \eta_{12}^c : x_e \tilde{\notin} (M^{x_e},E), (G,E) \tilde{\subseteq} (M^{x_e},E)\}$. It is clear that (H,E) is a $p\Lambda$-soft set and (F,E) is a p-closed soft set. Moreover, $(G,E) = (H,E) \tilde{\cap} (F,E)$, Therefore, (G,E) is a $p\lambda$-closed soft set.

Conversely, let (G,E) be a finite soft set over X and let $x_e \tilde{\notin} (G,E)$. Since (G,E) is a $p\lambda$-closed soft set, then $(G,E) = (H,E) \tilde{\cap} (F,E)$, where (H,E) is a $p\Lambda$-soft set and (F,E) is a p-closed soft set. So, $x_e \tilde{\notin} (H,E)$ or $x_e \tilde{\notin} (F,E)$. If $x_e \tilde{\notin} (H,E)$, then $x_e \tilde{\notin} sker_{12}(H,E)$. It follows that there exists a p-open soft set (N,E) such that $(H,E) \tilde{\subseteq} (N,E)$ and $x_e \tilde{\notin} (N,E)$, in this case take $M^{x_e},E) = (N,E)$. If $x_e \tilde{\notin} (F,E)$, take $M^{x_e},E) = (F,E)$. Consequently, (X,η_1,η_2,E) is a $PST^*_{1/4}$.

Theorem 3.5 *Let (X,η_1,η_2,E) be a sbts. Then,*

1) If (X,η_1,E) or (X,η_1,E) is a $ST1_{1/4}$, then (X,η_1,η_2,E) is a $PST^*_{1/4}$.
2) If (X,η_1,η_2,E) is a $PST^*_{1/4}$, then $(X,\eta_{p\Lambda},E)$ is a $ST1_{1/4}$.

Proof. Straightforward.

Theorem 3.6 *Let (X,η_1,η_2,E) be a sbts. Then,*

1) If (X,η_1,η_2,E) is a $PST^*_{1/4}$, then it is PST^*_0.
2) If (X,η_1,η_2,E) is a $PST^*_{1/2}$, then it is $PST^*_{1/4}$.

Proof. (1): Let $x_\alpha, y_\beta \in \xi(X)_E$ such that $x_\alpha \neq y_\beta$. Since (x_α, E) is a finite soft set and $y_\beta \tilde{\notin} (x_\alpha, E)$, then by $PST^*_{1/4}$ property there exists $(G,E) \in \eta_{12} \cup \eta_{12}^c$ such that $(x_\alpha, E) \tilde{\subseteq} (G,E)$ and $y_\beta \tilde{\notin} (G,E)$. It follows that $x_\alpha \tilde{\in} (G,E)$ and $y_\beta \tilde{\notin} (G,E)$. Therefore, by Lemma 2.1, (X,η_1,η_2,E) is a $PST^*_{1/4}$.

(2): It follows from Theorems 3.2 and 3.4.

Remark 3.2 *The following examples shows that the converse of items (1) and (2) in Theorem 3.6 are not true in general.*

Example 3.3 *In Example 3.1, it is easy to prove that (X,η_1,η_2,E) is a PST^*_0. Let $(F,E) = \{(e_1,\{y\}),(e_2,X)\}$. We note that $x_{e_1} \tilde{\notin} (F,E)$ and (F,E) is finite soft set. But, there is no p-open or p-closed soft set (M,E) such that $(F,E) \tilde{\subseteq} (M,E)$ and $x_{e_1} \tilde{\notin} (M,E)$. Hence, (X,η_1,η_2,E) is not $PST^*_{1/4}$.*

Example 3.4 *From Example 3.2 and Theorem 3.3, we have (Z,η_d,η_d,E) is a $PST^*_{1/4}$, but it is not $PST^*_{1/2}$ because $(0_d,E)$ is a soft point in Z but it is neither p-open nor p-closed soft set.*

Corollary 3.1 *If X is a finite set, then (X,η_1,η_2,E) is a $PST^*_{1/2}$ iff it is a $PST^*_{1/4}$.*

4. Pairwise locally closed soft sets

In this section, we introduce the notion of pairwise locally closed soft sets. Some basic properties of them and their relationships with different types of soft sets are studied.

Definition 4.1 *A soft set (G,E) is said to be a pairwise locally closed soft set in a soft bitopological space (X,η_1,η_2,E) [briefly, p- locally -closed soft set] if $(G,E) = (F,E) \tilde{\cap} (H,E)$, where (F,E) is a p-closed soft set and (H,E) is a p-open soft set. The family of all p- locally closed soft sets we denoted by $PLCS(X,\eta_1,\eta_2)_E$.*

Theorem 4.1 *Let (X,η_1,η_2,E) be a sbts. Then,* every p -open (p -closed) soft set is p -locally closed soft set.

Proof. Immediate from definition.

Remark 4.1 *The union (intersection) of two p locally closed soft sets need not be a p -locally closed soft set as shown in the following example*

Example 4.1 *In Example 3.1, it is clear that (G_2, E) and (K, E) are p -locally closed soft sets [by Proposition 4.1] but $(G_2, E) \tilde{\cup} (K, E)$ is not p -locally closed soft set.*

Proposition 4.1 *Let (X, η_1, η_2, E) be a sbts. Then,*

every p -locally closed soft set is $p\lambda$ -closed soft set.

Proof. Straightforward

Theorem 4.2 *A soft set (G, E) in a sbts (X, η_1, η_2, E) is a p -locally closed soft set iff $(G, E) = scl_{12}(G, E) \tilde{\cap} (H, E)$ for some $(H, E) \in \eta_{12}$.*

Proof. Let (L, E) be a p -locally closed soft set. Then, there exist p -closed soft set (F, E) and p -open soft set (H, E) such that $(L, E) = (F, E) \tilde{\cap} (G, E)$. It follows that $scl_{12}(L, E) \tilde{\subseteq} (F, E) \tilde{\cap} scl_{12}(G, E)$ which implies that $scl_{12}(L, E) \tilde{\subseteq} (F, E)$ and so

$scl_{12}(L, E) \tilde{\cap} (G, E) \tilde{\subseteq} (F, E) \tilde{\cap} (G, E)$. Therefore, $scl_{12}(L, E) \tilde{\cap} (G, E) \tilde{\subseteq} (L, E)$. On the other hand, since $(L, E) \tilde{\subseteq} scl_{12}(L, E)$, then $(L, E) \tilde{\cap} (G, E) \tilde{\subseteq} scl_{12}(L, E) \tilde{\cap} (G, E)$. It follows that $(L, E) \tilde{\subseteq} scl_{12}(L, E) \tilde{\cap} (G, E)$. Consequently, $(L, E) = scl_{12}(L, E) \tilde{\cap} (G, E)$.

The sufficiency of the theorem is clear.

Definition 4.2 *A sbts (X, η_1, η_2, E) is said to be a pairwise soft T_L^* [briefly, PST_L^*] if every soft point in X is a p -locally closed soft set.*

Theorem 4.3 *Let (X, η_1, η_2, E) be a sbts. Then*

1) If (X, η_1, η_2, E) is a $PST^*_{1/2}$, then it is PST_L^*.
2) If (X, η_1, η_2, E) is a PST_L^*, then it is PST_0^*.

Proof. (1): Since (X,η_1,η_2,E) is a $PST^*_{1/2}$, then every soft point is a p-open soft set or a p-closed soft set. It follows that, by Proposition 4.1, every soft point is a p-locally closed soft set. Consequently, (X,η_1,η_2,E) is a PST^*_L.

(2): Since (X,η_1,η_2,E) is a PST^*_L, then every soft point is a p-locally closed soft set. It follows that, by Proposition 4.1, every soft point is a $p\lambda$-closed soft set. Therefore, (X,η_1,η_2,E) is a PST^*_0 [by Theorem 2.8].

Remark 4.2 *The converse of items (1) and (2) in the above theorem is not true in general which is shown in the following example.*

Example 4.2 *Let (X,η_1,η_2,E) be the same in Example 3.1. Since $(P_1,E)^c=(y_{e_1},E)$, then (y_{e_1},E) is a p-closed soft set. It follows that (y_{e_1},E) is a p-locally closed soft set. Also, $(P_2,E)^c=(y_{e_2},E)$ is a p-locally closed soft set. Now, since $(x_{e_1},E)=(H,E)\tilde{\cap}(G_2,E)^c$, then (x_{e_1},E) is a p-locally closed soft set. Also, $(x_{e_2},E)=(H,E)\tilde{\cap}(G_1,E)^c$. It follows that (x_{e_2},E) is a a p-locally closed soft set. Therefore, every soft point in X is a p-locally closed soft set. Hence, (X,η_1,η_2,E) is a PST^*_L. On the other hand, (X,η_1,η_2,E) is not $PST^*_{1/2}$ because (x_{e_1},E) neither p-open nor p-closed soft set.*

Remark 4.3 *The concepts of PST^*_L and $PST^*_{1/4}$ are independent as shown in the following example.*

Example 4.3 *In Example 4.2, we have (X,η_1,η_2,E) is a PST^*_L but it is not $PST^*_{1/2}$ which implies that (X,η_1,η_2,E) is not $PST^*_{1/4}$ [by Corollary 3.1].* From Examples 3.2 and 3.4, we have (X,η_d,η_d,E) is a $PST^*_{1/4}$ but it is not PST^*_L for $(0_d,E)$ is not p-locally closed soft set.

5. On $p\lambda$ - open soft sets

In this section, we introduce the notion of pairwise λ-open soft sets. Some basic properties of them and their relationships with different types of soft sets are studied.

Definition 5.1 *A soft set (G,E) in a sbts (X,η_1,η_2,E) is said to be a pairwise λ-open soft set[briefly, $p\lambda$-open soft set] if its complement is a $p\lambda$-closed soft set. Clearly, (G,E) is a $p\lambda$-open soft set iff $(G,E)=(H,E)\tilde{\cup}(M,E)$, where $(H,E)\in\eta_{12}$ and $(M,E)\in\eta^c_{p\wedge}=\eta_{p\vee}$. We denoted the family of all $p\lambda$-open soft sets by $P\lambda OS(X,\eta_1,\eta_2)_E$.*

Theorem 5.1 *Let* (X,η_1,η_2,E) *be a sbts and* $(G,E)\in SS(X)_E$ *. Then:*

1) Every p -open(closed) soft set is a $p\lambda$ -open soft set.
2) Every $p\vee$ -soft set is a $p\lambda$ -open soft set.
3) An arbitrary union of $p\lambda$ -open soft sets is a $p\lambda$ -open soft set.

Proof. Straightforward.

Corollary 5.1 *The family of all* $p\lambda$ *-open soft sets is a supra soft topology.*

Proof. The proof is direct from Theorem 5.1.

Remark 5.1 *The family of all* $p\lambda$ *-open soft sets may not be a soft topology as shown by the following example.*

Example 5.1 In Example 3.1, we have $SS(X)_E=\eta_{12}\bigcup\eta_{12}^c\bigcup\{(M_i,E):i=1,...,6\}$

where

$(M_1,E)=\{(e_1,\{x\}),(e_2,\phi)\}$, $(M_2,E)=\{(e_1,\phi),(e_2,\{x\})\}$,

$(M_3,E)=\{(e_1,\phi),(e_2,X)\}$, $(M_4,E)=\{(e_1,X),(e_2,\phi)\}$,

$(M_5,E)=\{(e_1,\{y\}),(e_2,X)\}$, $(M_6,E)=\{(e_1,X),(e_2,\{y\})\}$.

It is easy to show that $P\lambda OS(X,\eta_1,\eta_2)_E=\eta_{12}\bigcup\eta_{12}^c\bigcup\{(M_5,E),(M_6,E)\}$.

It is clear that $(P_2,E),(G_1,E)\in P\lambda OS(X,\eta_1,\eta_2)_E$ but $(P_2,E)\tilde{\cap}(G_1,E)=(M_1,E)$ which is not $p\lambda$ -open soft set.

Theorem 5.2 *Let* (X,η_1,η_2,E) *be a sbts and* $(G,E)\in SS(X)_E$ *. The following statements are equivalent.*

1) (G,E) is a $p\lambda$ -open soft set.
2) $(G,E)=(H,E)\tilde{\cup}(M,E)$, where $(H,E)\in\eta_{12}$, $(M,E)\in\eta_{p\vee}$.
3) $(G,E)=sint_{12}(G,E)\tilde{\cup}(M,E)$, where $(M,E)\in\eta_{p\vee}$.
4) $(G,E)=sint_{12}(G,E)\tilde{\cup}\,sker_{12}^*(G,E)$.

Proof. $(1) \Rightarrow (2)$: Let (G,E) be a $p\lambda$ -open soft set. Then, $(G,E) = (H,E) \tilde{\cup} (M,E)$, where $(H,E) \in \eta_{12}$, $(M,E) \in \eta^c_{p\Lambda}$, but $\eta^c_{p\Lambda} = \eta_{p\vee}$ [by Theorem 2.7], then (2) holds.

$(2) \Rightarrow (3)$: Let $(G,E) = (H,E) \tilde{\cup} (M,E)$, where $(H,E) \in \eta_{12}$, $(M,E) \in \eta_{p\vee}$. Since $(H,E) \tilde{\subseteq} (G,E)$, then $sint_{12}(H,E) \tilde{\subseteq} sint_{12}(G,E)$. Therefore, $(H,E) \tilde{\subseteq} sint_{12}(G,E)$. It follows that $(H,E) \tilde{\cup} (M,E) \tilde{\subseteq} sint_{12}(G,E) \tilde{\cup} (M,E)$. So, $(G,E) \tilde{\subseteq} sint_{12}(G,E) \tilde{\cup} (M,E)$, but

$sint_{12}(G,E) \tilde{\subseteq} (G,E)$ and $(M,E) \tilde{\subseteq} (G,E)$ then $sint_{12}(G,E) \tilde{\cup} (M,E) \tilde{\subseteq} (G,E)$. Therefore, $(G,E) = sint_{12}(G,E) \tilde{\cup} (M,E)$. Hence, (3) holds.

$(3) \Rightarrow (4)$: Let $(G,E) = sint_{12}(G,E) \tilde{\cup} (M,E)$, where $(M,E) \in \eta_{p\vee}$. Then $(G,E) = sint_{12}(G,E) \tilde{\cup}$ $sker^*_{12}(M,E)$. Since $(M,E) \tilde{\subseteq} (G,E)$, then $sker^*_{12}(M,E) \tilde{\subseteq} sker^*_{12}(G,E)$ which implies that $sint_{12}(G,E) \tilde{\cup} sker^*_{12}(M,E) \tilde{\subseteq} sint_{12}(G,E) \tilde{\cup} sker^*_{12}(G,E)$.

It follows that $(G,E) \tilde{\subseteq} sint_{12}(G,E) \tilde{\cup} sker^*_{12}(G,E)$, but $sint_{12}(G,E) \tilde{\subseteq} (G,E)$ and $sker^*_{12}(G,E) \tilde{\subseteq} (G,E)$, then $sint_{12}(G,E) \tilde{\cup} sker^*_{12}(G,E) \tilde{\subseteq} (G,E)$. Therefore, $(G,E) = sint_{12}(G,E) \tilde{\cup} sker^*_{12}(G,E)$. Hence, (4) holds.

$(4) \Rightarrow (1)$: Since $sint_{12}(G,E) \in \eta_{12}$, $sker^*_{12}(G,E) \in \eta_{p\vee}$, then (G,E) is a $p\lambda$ -open soft set. Hence, (1) holds.

Definition 5.2 *Let (X,η_1,η_2,E) be a sbts, and let $(G,E) \in SS(X)_E$. The $p\lambda$ -soft closure of (G,E), denoted by $scl_{p\lambda}(G,E)$, is defined by*

$$scl_{p\lambda}(G,E) = \bigcap \{(F,E) \in P\lambda CS(X,\eta_1,\eta_2)_E : (G,E) \tilde{\subseteq} (F,E)\}.$$

Lemma 5.1 *Let (X,η_1,η_2,E) be a sbts and $(G,E) \in SS(X)_E$. The $p\lambda$ -soft closure of (G,E) is the smallest $p\lambda$ -closed soft superset of (G,E).*

Proof. Immediate from Theorem 3.1.

Theorem 5.3 *Let (X,η_1,η_2,E) be a sbts and let $(G,E),(H,E) \in SS(X)_E$. Then*

1) $scl_{p\lambda}(\tilde{X},E) = (\tilde{X},E)$; $scl_{p\lambda}(\tilde{\phi},E) = (\tilde{\phi},E)$.

2) $(G,E)\tilde{\subseteq} scl_{p\lambda}(G,E)$.

3) $(G,E)\tilde{\subseteq}(H,E)\Rightarrow scl_{p\lambda}(G,E)\tilde{\subseteq} scl_{p\lambda}(H,E)$.

4) (G,E) is a $p\lambda$ -closed soft set $\Leftrightarrow scl_{p\lambda}(G,E)=(G,E)$.

5) $scl_{p\lambda}(G,E)\tilde{\cup} scl_{p\lambda}(H,E)\tilde{\subseteq} scl_{p\lambda}[(G,E)\tilde{\cup}(H,E)]$.

6) $scl_{p\lambda}[scl_{p\lambda}(G,E)]=scl_{p\lambda}(G,E)$.

Proof. Straightforward.

Corollary 5.2 $scl_{p\lambda}$ *is a supra soft closure operator and it is induced a supra soft topology given by*

$\eta_{p\lambda}=\{(G,E)\in SS(X)_E : scl_{p\lambda}(G,E)^c=(G,E)^c\}$ which is precisely $P\lambda OS(X,\eta_1,\eta_2)_E$.

Remark 5.2 *The equality in Theorem 5.3* (5) *may not be satisfied as shown in the following example.*

Example 5.2 Consider the Example 3.1, since $(H,E)^c$ $(=\{(e_1,\{y\}),(e_2,\{y\})\})$ is p -closed soft set, then $(H,E)^c$ is a $p\lambda$ -closed soft set. Therefore, $scl_{p\lambda}(H,E)^c=(H,E)^c$ [by Theorem 5.3 (4)]. Similarly, $scl_{p\lambda}(G_1,E)^c=(G_1,E)^c$. It follows that $scl_{p\lambda}(H,E)^c\tilde{\cup} scl_{p\lambda}(G_1,E)^c=\{(e_1,\{y\}),(e_2,\{X\})\}$. On the other hand, $scl_{p\lambda}[(H,E)^c\tilde{\cup}(G_1,E)^c]=scl_{p\lambda}\{(e_1,\{y\}),(e_2,\{X\})\}=(\tilde{X},E)$. Hence, $scl_{p\lambda}[(H,E)^c\tilde{\cup}(G_1,E)^c]\neq scl_{p\lambda}(H,E)^c\tilde{\cup} scl_{p\lambda}(G_1,E)^c$.

Theorem 5.4 *Let* (X,η_1,η_2,E) *be a sbts and* $(G,E)\in SS(X)_E$. *Then,*

$$x_e\tilde{\in} scl_{p\lambda}(G,E)\Leftrightarrow (O_{x_e},E)\tilde{\cap}(G,E)\neq(\tilde{\phi},E),\ \forall\,(O_{x_e},E),$$

where (O_{x_e},E) is a $p\lambda$ -open soft set containing x_e.

Proof. Let $x_e\tilde{\in} scl_{p\lambda}(G,E)$ and assume that there exists $(O_{x_e},E)\in\eta_{12}$ such that $(O_{x_e},E)\tilde{\cap}(G,E)=(\tilde{\phi},E)$. Then $(G,E)\tilde{\subseteq}(O_{x_e},E)^c$ which implies that $scl_{p\lambda}(G,E)\tilde{\subseteq} scl_{p\lambda}(O_{x_e},E)^c=(O_{x_e},E)^c$, therefor $scl_{p\lambda}(G,E)\tilde{\cap}(O_{x_e},E)=(\tilde{\phi},E)$, a contradiction.

Conversely, assume that $x_e\tilde{\notin} scl_{p\lambda}(G,E)$. Then $x_e\tilde{\in}[scl_{p\lambda}(G,E)]^c$ it follows tha $[scl_{12}(G,E)]^c$ is a $p\lambda$ -open soft set containing x_e. Thus, by hypothesis $[scl_{12}(G,E)]^c\tilde{\cap}(G,E)\neq(\tilde{\phi},E)$ which contradicts with $(G,E)\tilde{\subseteq} scl_{p\lambda}(G,E)$.

Corollary 5.3 *Let (X,η_1,η_2,E) be a sbts. Then,*

$\eta_{12}^c \subseteq \eta_{p\vee} \subseteq \eta_{p\lambda} \subseteq SS(X)_E$.

Proof. Straightforward.

Remark 5.3 *The equality in Corollary 5.3 may not be satisfied as shown in the following example.*

Example 5.3 *In Example 5.1, clear that $\eta_{p\lambda} \neq SS(X)_E$. Also, we have $(P_1,E)\in\eta_{p\lambda}$ but $(P_1,E)\notin\eta_{p\vee}$ because $sker_{12}^*(P_1,E)=\{(e_1,\{x\}),(e_1,\{y\})\}\neq(P_1,E)$. Hence, $\eta_{p\vee}\neq\eta_{p\lambda}$.*

Lemma 5.2 *Let (X,η_1,η_2,E) be a sbts. Then, the supra soft topology $\eta_{p\lambda}$ is a soft topology on X iff the finite intersection of $p\lambda$ -open soft sets is a $p\lambda$ -open soft set or equivalently, the finite union of $p\lambda$ -closed soft sets is a $p\lambda$ -closed soft set.*

Proof. Suppose that $\eta_{p\lambda}$ is a soft topology on X. Let (G,E) and (H,E) be two $p\lambda$ -closed soft sets. Then, $(G,E)^c$ and $(H,E)^c$ are $p\lambda$ -open soft sets. Therefore, $(G,E)_c \tilde{\cap} (H,E)_c$ is a $p\lambda$ -open soft set[for $\eta_{p\lambda}$ is a soft topology]. Consequently, $(G,E)\tilde{\cup}(H,E)$ is a $p\lambda$ -closed soft set.

Conversely, it is obvious.

Definition 5.3 *A sbts (X,η_1,η_2,E) is called a pairwise soft T_λ^* [briefly, PST_λ^*] if $\eta_{p\lambda}$ is a soft topology on X.*

Theorem 5.5 *Every $PST_{1/2}^*$ is a PST_λ^*.*

Proof. Let (X,η_1,η_2,E) be a $PST_{1/2}^*$ space. Then, by Theorem 3.2, every soft set is a $p\lambda$ -closed soft set. Consequently, the union of any two $p\lambda$ -closed soft sets is a $p\lambda$ -closed soft set. Hence, by Lemma 5.2, $\eta_{p\lambda}$ is a soft topology on X. Hence, (X,η_1,η_2,E) is a PST_λ^*.

Remark 5.4 *A PST_λ^* need not be a $PST_{1/2}^*$ or PST_0^* as shown by the following example.*

Example 5.4 *Let $X=\{x,y,z\}$, $E=\{e_1,e_2\}$ and let*

$$\eta_1 = \{(\tilde{\phi}, E), (\tilde{X}, E), (G_1, E), (G_2, E), (G_3, E)\},$$

$$\eta_2 = \{(\tilde{\phi}, E), (\tilde{X}, E), (H_1, E), (H_2, E), (H_3, E)\},$$

such that

$$(G_1, E) = \{(e_1, \{x, z\}), (e_2, \{x, y\})\},$$

$$(G_2, E) = \{(e_1, \{y, z\}), (e_2, \{y, z\})\},$$

$$(G_3, E) = \{(e_1, \{z\}), (e_2, \{y\})\},$$

$$(H_1, E) = \{(e_1, \{x, y\}), (e_2, \{x, z\})\},$$

$$(H_2, E) = \{(e_1, \{z\}), (e_2, \{x, y\})\},$$

$$(H_3, E) = \{(e_1, \phi), (e_2, \{x\})\}.$$

It is easily seen that (X, η_1, η_2, E) is a sbts and

$$\eta_{12} = \{(\tilde{\phi}, E), (\tilde{X}, E), (G_1, E), (G_2, E), (G_3, E), (H_1, E), (H_2, E), (H_3, E), (P, E)\},$$

where

$(P, E) = \{(e_1, \{y, z\}), (e_2, X)\}$. Consequently,

$$\eta_{p\Lambda} = \eta_{12} \bigcup \{(G_1, E)^c, (G_2, E)^c\}.$$

By studying all soft sets such that $SS(X)_E = \eta_{12} \bigcup \eta_{12}^c \bigcup \{(M_i, E) : i = 1,2,3,...,50\}$ we found that:

$P\lambda CS(X, \eta_1, \eta_2)_E = \eta_{12} \bigcup \eta_{12}^c$. Moreover, $P\lambda CS(X, \eta_1, \eta_2)_E = \eta_{p\lambda}$. Now, let $(N, E), (K, E) \in \eta_{p\lambda}$, then $(N, E), (K, E) \in P\lambda CS(X, \eta_1, \eta_2)_E$. Therefore, $(N, E) \tilde{\cap} (K, E) \in P\lambda CS(X, \eta_1, \eta_2)_E$ [by Theorem 3.1]. Hence, $(N, E) \tilde{\cap} (K, E) \in \eta_{p\lambda}$. Therefore, $\eta_{p\lambda}$ is a soft topology. Consequently, (X, η_1, η_2, E) is a PST_λ^* but it is not $PST_{1/2}^*$ because (y_{e_1}, E) neither p-open soft set nor p-closed soft set. Also, (X, η_1, η_2, E) is not PST_0^* for $y_{e_1} \neq z_{e_2}$ but there is no p-open soft set contains one of them but not contains the other.

Theorem 5.6 *A sbts* (X,η_1,η_2,E) *is a* $PST^*_{1/4}$ *if it is a* PST^*_0 *and* PST^*_λ.

Proof. Suppose that (X,η_1,η_2,E) is a PST^*_0 and PST^*_λ. Let (F,E) be a finite soft set. Then $(F,E)=\bigcup\{(x_e,E):x_e\,\tilde{\in}\,(F,E)\}$ [by Proposition 2.1]. Since (X,η_1,η_2,E) is a PST^*_0, then (x_e,E) is a $p\lambda$-closed soft set for all $x_e\,\tilde{\in}\,(F,E)$. But, (X,η_1,η_2,E) is a PST^*_λ, then $\eta_{p\lambda}$ is a soft topology on X. It follows that, by Lemma 5.2,the finite union of $p\lambda$-closed soft sets is a $p\lambda$-closed soft set. Consequently, $\bigcup\{(x_e,E):x_e\,\tilde{\in}\,(F,E)\}$ is a $p\lambda$-closed soft set. Therefore, (F,E) is a $p\lambda$-closed soft set. Hence, by Theorem 3.4 we conclude that (X,η_1,η_2,E) is a $PST^*_{1/4}$.

Remark 5.5 A $\mathbf{PST}^*_\lambda$ space need not be $\boldsymbol{PST}^*_{1/4}$. In Example 5.4, we proved that (X,η_1,η_2,E) is a $\boldsymbol{PST}^*_\lambda$. Since X is a finite set and (X,η_1,η_2,E) is not $\boldsymbol{PST}^*_{1/2}$, then by Corollary 3.1 we conclude that (X,η_1,η_2,E) is not $\boldsymbol{PST}^*_{1/4}$.

6. Conclusion

The notions of pairwise λ-closed soft sets, pairwise λ-open soft sets and pairwise locally closed soft sets are turn out to be useful in the study of some soft bitopologies which are not PST_1^*. In this paper, we introduce new classes of soft sets called $p\lambda$-closed soft sets, $p\lambda$-open soft sets and p-locally closed soft sets in soft bitopological spaces. It turn out that $p\lambda$-closed soft sets, $p\lambda$-open soft sets and p-locally closed soft sets are weaker forms of p-open(closed) soft sets. We also conclude several important properties of such soft sets. Moreover, we introduce and study a related pairwise soft separation axioms, namely, $\boldsymbol{PST}^*_{1/4}$, $\boldsymbol{PST}^*_L$ and $\boldsymbol{PST}^*_\lambda$. We studied the relationships between these types of separation axioms.

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References

[1] A. Aygunoglu and H. Aygun, Some notes on soft topological spaces, Neural Comput. Applic., vol. 21 (Suppl 1), pp. S113 − S119, 2012.

[2] Basavaraj Ittanagi, Soft bitopological spaces, International journal of Computer Applications, vol. 107

no. 7, pp. 1 − 4, 2011.

[3] N. Cagman, S. Karatas and S. Enginoglu, Soft topology, Comput. Math. Appl. vol. 62, pp. 351 − 358, 2011.

[4] S. A. El-Sheikh and A. M. Abd El-latif, Decompositions of some types of supra soft sets and soft continuity, International Journal of Mathematics Trends and Technology, vol. 9, no. 1, pp. 37 − 56, 2014.

[5] D. N. Georgiou a and A. C. Megaritis, Soft set theory and topology, Appl. Gen. Topol., vol. 15, no. 1, pp. 93 − 109, 2014.

[6] H. Hazra, P. Majumdar and S. K.Samanta, Soft topology, Fuzzy Inform. Eng., vol. 4, no. 1, pp.105 − 115, 2012.

[7] S. Hussain and B. Ahmad, Some properties of soft topological spaces, Comput. Math. Appl., vol. 62, pp. 4058 − 4067, 2011.

[8] M. Irfan Ali, M. Shabir and M. Naz, Algebraic structures of soft sets associated with new operations, Comput. Math. Appl., vol. 61, pp. 2647 − 2654, 2011.

[9] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and Shawqi A. Hazza, Generalized pairwise closed soft sets and the associate pairwise soft separation axioms, South Asian J. Math., vol. 6, no. 2, pp., 43 − 57, 2016.

[10] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and Shawqi A. Hazza, Pairwise open (closed) soft sets in a soft bitopological spaces, Ann. Fuzzy Math. Inform., vol. 11, no. 4, pp. 571 − 588, 2016.

[11] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and Shawqi A. Hazza, Pairwise soft separation axioms in soft bitopological spaces, Ann. Fuzzy Math. Inform., vol. xx, no. xx, pp. 1-20, 2017.

[12] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and Shawqi A. Hazza, Some types of pairwise soft sets and the associated soft topologies, Journal of Intelligent and Fuzzy Syst., vol. 32, no. 2, pp. 1007 − 1018, 2017.

[13] K. Kannan, Soft generalized closed sets in soft topological spaces, Journal of Theoretical and Appl. Inform. Technology, vol. 37, no. 1, pp. 17 − 21, 2012.

[14] D. Molodtsov, Soft set theory-First results, Comput. Math. Appl., vol. 37, pp. 19 − 31, 1999.

[15] Sk. Nazmul and S. K. Samanta, Neighbourhood properties of soft topological spaces, Ann. Fuzzy Math. Inform. , vol. 6, no. 1, pp. 1 − 15, 2013.

[16] Sk. Nazmul and S. K. Samanta, Some properties of soft topologies and group soft topologies, Ann. Fuzzy Math. Inform.m vol. 8, no. 4, pp. 645−661, 2014.

[17] Ningxin Xie, Soft points and the structure of soft topological spaces, Ann. Fuzzy. Math. inform., vol. 10, no. 2, pp. 309−322, 2015.

[18] E. Peyghan, B. Samadi and A. Tayebi, About soft topological spaces, Journal of New Results in Science, vol. 2, pp. 60−75, 2013.

[19] D. Pie and D. Miao, From soft sets to information systems, Granular computing, IEEE Inter. Conf., vol. 2, pp. 617−621, 2005.

[20] M. Shabir and M. Naz, On soft toplogical spaces, Comput. Math. Appl., vol. 61, no. 7, pp. 1786−1799, 2011.

[21] Sujoy Das and S. K. Samanta, Soft metric, Ann. Fuzzy Math. Inform., vol.6, pp. 77−94, 2013.

[22] I. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, Remarks on soft topological spaces, Ann. Fuzzy Math. Inform., vol. 3, no. 2, pp. 171−185, 2012.