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Thermoelastic Problem of an Infinite Plate Weakened by a Curvilinear Hole

R. T. Matoog*

Department of Mathematics, Faculty of Applied Sciences, University of Umm Al-Qura, KSA

Email: rmatoog_777@mail.com

Abstract

Mathematical model is considered, to discuss the analytic solution of the first and second boundary value problems (**BVPs**), for an infinite plate weakened by a curvilinear hole *C* having two poles. The elastic plate carries a steady uniformly distributed axial current of density J and is placed in an ambient medium of steady temperature T_e. Using a conformal mapping function, the curvilinear hole is conformally mapped on the domain outside (inside) a unit circle γ. Then, the Gaursat functions **(GFs)** are determined. Moreover, the three components of stresses, in the presence of temperature T distributed around the curvilinear hole are completely determined. Many special and new cases are derived from the work. In addition, many, applications for the first and second **BVPs** are discussed. Moreover, the three stresses components, in each application, are computed.

Keywords: Boundary value problems; thermoelastic plate; Gaursat functionsl; rational mapping; curvilinear hole ; **AMS (2010)**: 74B10; 30C20.

1. Introduction

Several authors have discussed the solutions of the **BVPs**, in two-dimensional problems, using some different methods; see [1-5]. In the two- dimensional problems, in the theory of elasticity, some authors have used complex variables method to obtain, in a closed form, the solution of the first and second fundamental problems, see [6-10]. Others authors have used Laurent's theorem to obtain the Gaursat functions for an infinite plate weakened by a curvilinear hole *C*, see [11-15].

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* Corresponding author.

In thermoelastic, problems for the first and second **BVPs** are equivalent to finding the thermoelastic potential Φ, in addition is finding the two analytic **GFs**, $\varphi(z)$ and $\psi(z)$ of one complex argument $z = x + iy$. These analytic functions must satisfy the boundary conditions,

$$K\,\varphi_1(t) - t\,\overline{\varphi_1'(t)} - \overline{\psi_1(t)} = f\,(t) \tag{1.1}$$

where t denotes the affix of a point on the boundary. In terms of $z = c\omega(\zeta), c > 0$, $\omega'(\zeta)$ does not vanish or become infinite for $|\zeta| > 1$, the infinite region outside a unit circle γ.

For the stress **BVPs**, $K = -1$ and $f\,(t)$ is a given function of stress. While for $K = k > 1$, $f\,(t)$ is a given function of displacement which called the thermal conductivity, we have the second fundamental **BVPs** or called the displacement **BVPs**. The **GFs** $\varphi_1(t)$ and $\psi_1(t)$ take the following form see [16]

$$\varphi_1(\zeta) = -\frac{S_x + iS_y}{2\pi(1+k)}\ln\zeta + c\Gamma\zeta + \varphi(\zeta) \tag{1.2}$$

and,

$$\psi_1(\zeta) = k\,\frac{(S_x - iS_y)}{2\pi(1+k)}\,\ln\zeta + c\Gamma^*\zeta + \psi(\zeta). \tag{1.3}$$

Where S_x , S_y are the components of the resultant vector of all external forces acting on the boundary and Γ, Γ^* are complex constants. Generally the two **GFs** $\varphi(\zeta)$ and $\psi(\zeta)$ are single value analytic functions within the region out side the unit circle γ and $\varphi(\infty) = 0$, $\psi(\infty) = 0$.

In this paper, we consider the **BVP** for isotropic homogeneous perforated infinite elastic media in presence of uniform flow of heat. Then, we use a general shape of conformal mapping to obtain the **GFs** for the problem in the form of integro-differential equation with singular kennel. Many special cases are obtained and several applications are discussed from the work.

2. Formulation of the problem

Consider a thin infinite plate of thickness h with a curvilinear hole C, where the origin lies inside the hole is conformally mapped on the domain outside a unit circle γ by means of a rational mapping function,

$$z = c\,\frac{\zeta + m_1\zeta^{-1} + m_2\zeta^{-2}}{(1 - n_1\zeta^{-1})(1 - n_2\zeta^{-1})}, \quad c > 0\,, n_1 \neq n_2 \tag{2.1}$$

where $z'(\zeta)$ does not vanish or become infinite outside the unit circle γ.

The uniform flow of heat is distributed by the presence of an insulated curvilinear hole *C*. Neglecting the variation of the strain and stress with respect to the thickness of the plate, and assuming the force of the plate are free of applied loads. In this case, the thermoelastic potential Φ satisfies the formula, see Parkus [16]

$$\nabla^2\Phi = (1+\upsilon)\alpha\,\Theta \tag{2.2}$$

where α is a scalar which present the coefficient of the thermal expansion and ν is poisson's ratio.

In the steady state, the temperature T as measured from a reference temperature T_r, satisfies Poisson's equation and the heat flow vector Q is given by Fourier law for heat conduction

$$\nabla^2 T = -\frac{\zeta(x,y)}{K}, \quad Q = -K\,\nabla T \tag{2.3}$$

Where, $\zeta(x,y)$ is the rate of heat supply per unit volume and K is the coefficient of heat conduction.

More extensive forms for Q may be found in [7, p. 101]. For our purposes, it is convenient to write

$$\zeta = \zeta_H + \zeta_T \tag{2.4}$$

Where, $\zeta_H = \dfrac{J^2}{\sigma}$, represents the contribution of the magnetic field (Joule heat supply), J represents a steady uniform distributed axial current of density, which carries by the plate and σ the coefficient of electrical conductivity.

In addition, ζ_T is the rate of heat supply from all other sources. In this work ζ_T will be neglected.

The general solution (2.3) is

$$T = T_h + T_p \tag{2.5}$$

Where T_h is the harmonic part of T i.e. is the solution of the boundary value problem

$$(i)\ \nabla^2 T_h = 0, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2},$$

$$(ii)\ \frac{\partial T_h}{\partial r} = 0 \qquad \text{On the boundary } r = r_o,\ r = x + iy \tag{2,6}$$

Moreover, T_p is any particular solution of Poisson's Eq. (2. 3). It easily to have

$$T = T_0\{\operatorname{Im} z + \frac{r_0}{\operatorname{Im} z}\} - \frac{J^2}{4K\sigma} z\,\overline{z}. \qquad (2.7)$$

The thermoelastic potential functionΦ, satisfies, see Parkus [16],

$$\nabla^2\Phi = (1+\upsilon)\alpha T. \qquad (2.8)$$

where z is defined by Eq. (2.1). Substituting from (2.8) into (2.2) and integrating the result, the thermoelastic potential becomes

$$\Phi(z) = (1+\upsilon)\alpha T_0\{r_0^2 \operatorname{Im} z\,[\ln(z - \overline{z}) - \frac{J^2}{4K\sigma} z^2\overline{z^2}\}. \qquad (2.9)$$

In this case, the formulas (1.1) for the first and second **BVPs** respectively, take the following forms,

$$\varphi_1(t) + t\,\overline{\varphi_1'(t)} + \overline{\psi_1(t)} = \frac{\partial\Phi}{\partial x} + i\,\frac{\partial\Phi}{\partial y} + \frac{1}{2G}\int_0^s [iX(s) - Y(s)]ds + c \qquad (2.10)$$

$$k\,\varphi_1(t) - t\,\overline{\varphi_1'(t)} - \overline{\psi_1(t)} = u + iv - \frac{\partial\Phi}{\partial x} - i\,\frac{\partial\Phi}{\partial y} \qquad (2.11)$$

where the applied stresses $X(s)$ and $Y(s)$ are prescribed on the boundary of the plane S is the length measured from an arbitrary point, u and v are the displacement components, G is the shear modulus and Φ represents the thermoelastic potential function. In addition, the applied stresses $X(s)$ and $Y(s)$ satisfy the following, see [16]

$$X(s) = \sigma_{xx}\frac{dy}{ds} - \sigma_{xy}\frac{dx}{ds}; \quad Y(s) = \sigma_{yx}\frac{dy}{ds} - \sigma_{yy}\frac{dx}{ds}. \qquad (2.12)$$

where σ_{xx} , σ_{xy} and σ_{yy} are the components of stresses given by the following relations,

$$\sigma_{xx} = 2G\,[-\frac{1}{2}(\frac{\partial^2\Phi}{\partial y^2} - \frac{\partial^2\Phi}{\partial x^2} + 2\lambda T) + \operatorname{Re}[2\varphi'(z) - \overline{z}\,\varphi''(z) - \psi''(z)]], \qquad (2.13)$$

$$\sigma_{yy} = 2G\,[\frac{1}{2}(\frac{\partial^2\Phi}{\partial y^2} - \frac{\partial^2\Phi}{\partial x^2} - 2\lambda T) + \operatorname{Re}[2\varphi'(z) + \overline{z}\,\varphi''(z) + \psi''(z)]], \qquad (2.14)$$

and,

$$\sigma_{xy} = 2G\left[\frac{\partial^2 \Phi}{\partial x\,\partial y} + \mathrm{Im}(\bar{z}\,\varphi''(z) + \psi''(z))\right] . \qquad (2.15)$$

Here, $\lambda = \frac{\alpha}{2}(1+\nu)$ is the coefficient of heat transfer and $T\,,\Phi$ are given respectively, by equations (2.7), and (2.9).

The rational mapping $z = c\,\omega(\zeta)$ maps the boundary C of the given region occupied by the middle plane of the plate in the $z-$plane onto the unit circle γ in the $\zeta-$plane. Curvilinear coordinates (ρ,θ) are thus introduce into the $Z-$plane which are the maps of the polar conditions in the $\zeta-$plane as given by $\zeta = \rho e^{i\theta}$, $0 \le \theta \le 2\pi$. By using the transformation $z = c\,\omega(\zeta)$, Eq. (1.1) reduce to,

$$K\,\varphi_1(c\,\omega(\zeta)) - \frac{\omega(\zeta)}{\overline{\omega'(\zeta)}}\,\overline{\varphi_1(c\,\omega(\zeta))} - \overline{\psi_1(c\,\omega(\zeta))} = f\,(c\,\omega(\zeta)) \qquad (2.16)$$

The last formulas represent the first and second **BVPs** in $\zeta-$ plane.

In this study, we use the rational mapping (2.1) to map the curvilinear hole C of the infinite plate outside a unit circle γ . Then, we use the properties of Cauchy integral to obtain the two **GFs**. After that, we can determine the components of stresses. Many applications and special cases are derived from the work.

3. The rational mapping

The mapping function (2.1) maps the curvilinear hole C in $Z-$plane onto the domain of outside unit circle in $\zeta-$plane under the condition $\omega'(\zeta)$ does not vanish or become infinite outside the unit circle γ . The following graphs give the different shapes of the rational mapping (2.1) for some different values of n's and m's.

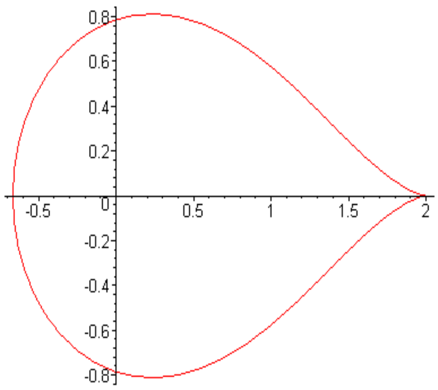

Figure 1: $(n_1 = n_2 = 0, m_1 = 0.987, m_2 = 0)$

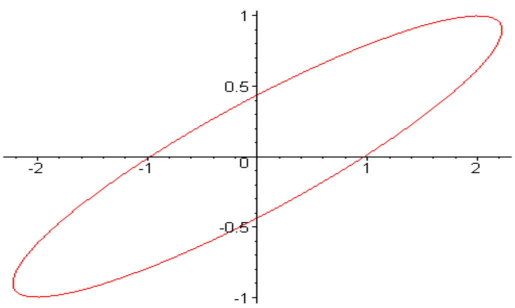

Figure 2: $(n_1 = 0.5, n_2 = m_1 = m_2 = 0)$

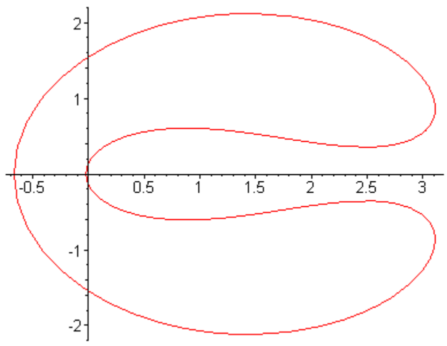

Figure 3: ($n_1 = 0, n_2 = 0.5, m_1 = 0, m_2 = 0.55$)

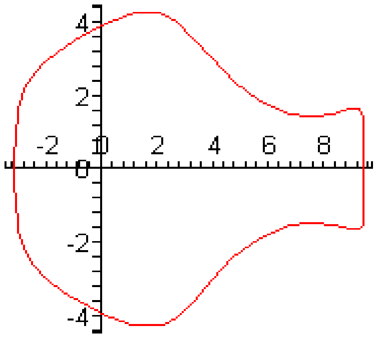

Figure 4: ($n_1 = 0.9, n_2 = 0.4, m_1 = -0.99, m_2 = -0.05$)

4. Gaursat functions

To obtain the two complex potential functions, **GFs** by using the conformal mapping (2.1) in the boundary conditions (1.1), we write the expression $\dfrac{\omega(\zeta)}{\omega'(\zeta^{-1})}$ in the form,

$$\frac{\omega(\zeta)}{\omega'(\zeta^{-1})} = \alpha(\zeta) + \beta(\zeta^{-1}) \tag{4.1}$$

where,

$$\alpha(\zeta) = \frac{h_1}{\zeta - n_1} + \frac{h_2}{\zeta - n_2}\,, \tag{4. 2}$$

and,

$$h_j = \frac{(n_j^3 + m_1 n_j + m_2)(1-n_j^2)^2(1-n_1 n_2)^2}{(n_j - n_{j\pm1})[1-2(n_1+n_2)n_j - (m_1 - 3n_1 n_2)n_j^2 - 2m_2 n_j^3 + (n_1 n_2 m_1 + n_1 m_2 + n_2 m_2)n_j^4]} \; (\; (j = 0,1) \quad (4.3)$$

$\beta(\zeta^{-1})$ is a regular function for $|\zeta| > 1$.

The term $\dfrac{\omega(\zeta)}{\omega'(\zeta^{-1})}$ has two singularities at $\zeta = n_1$ and $\zeta = n_2$.

Using equations (4.1) and (4.2) in equation (2.16), and integrating both sides, we get

$$-K\,\varphi(\zeta) = A(\zeta) - c\,\overline{\Gamma^*}\zeta^{-1} + \sum_{j=1}^{2} \frac{h_j}{n_j - \zeta}[N(n_j) + c\,b_j] \tag{4.4}$$

where, $A(\zeta)=\dfrac{1}{2\pi i}\displaystyle\int_{\gamma}\dfrac{F(\sigma)}{(\sigma-\zeta)}d\sigma;\ N(\zeta)=c\overline{\Gamma}-\dfrac{S_x-iS_y}{2\pi(1+k)}\zeta;\ F(\zeta)=f(c\omega(\zeta))=f(t)$ (4.5)

The function $F(\sigma)$ with its derivatives must satisfy Holder condition.

The complex constants b_j can be determined in the following form

$$b_j=\frac{KE_j-h_jd_j\overline{E}_j}{c(K^2-h_j^2d_j^2)},\quad E_j=-\overline{A'(n_j)}-c\Gamma^*n_j^2-h_jd_j\overline{N(n_j)},\quad d_j=\frac{n_j^2}{(1-n_j^2)^2},\quad j=1,2 \tag{4.6}$$

Using the results of (4.5) and (4.6), the function $\varphi(\zeta)$, is completely determined.

The value of the function $\psi(\zeta)$ takes the form

$$\psi(\zeta)=cK\Gamma\zeta^{-1}-\frac{\omega(\zeta^{-1})}{\omega'(\zeta)}\varphi_*(\zeta)+\sum_{j=1}^{2}\frac{h_j\zeta}{1-n_j\zeta}\varphi_*(n_j^{-1})+B(\zeta)-B, \tag{4.7}$$

Where

$$\varphi_*(\sigma)=\varphi'(\sigma)+\overline{N(\sigma)};\ B(\zeta)=\frac{1}{2\pi i}\int_{\gamma}\frac{\overline{F(\sigma)}}{(\sigma-\zeta)}d\sigma;\ B=\frac{1}{2\pi i}\int_{\gamma}\frac{\overline{F(\sigma)}}{\sigma}d\sigma\ .$$

5. Some applications

In this section, we assume different values of the given functions in the first or second **BVPs**. Then, we obtain the expression of **GFs**. Hence, the stresses components can be calculated directly.

5.1. Application 1: Curvilinear hole for an infinite plate subjected to uniform tensile stress in the presence of heat

For $K=-1,\quad \Gamma=\dfrac{P}{4},\quad \Gamma^*=\dfrac{-P}{2}e^{-2i\theta},\quad 0\le\theta\le 2\pi,\quad S_x=S_y=f=0,$ we have

$$\varphi(\zeta)=\frac{cP}{2}\zeta^{-1}e^{2i\theta}+\sum_{i=1}^{2}\frac{L_j}{n_j-\zeta}\quad , \tag{5.1}$$

$$\psi(\zeta)=-\frac{cP}{4}\zeta^{-1}-\frac{\omega(\zeta^{-1})}{\omega'(\zeta)}\varphi_*(\zeta)+\sum_{j=1}^{2}\frac{h_j\zeta}{1-n_j\zeta}\varphi_*(n_j^{-1}) \tag{5.2}$$

where,

$$L_j = \frac{cP}{4} h_j \left[\frac{1-2n_j^2 \cos 2\theta}{1-h_j d_j} + i\,\frac{2n_j^2 \sin 2\theta}{1+h_j d_j}\right]; \quad \varphi_*(\zeta) = \varphi'(\zeta) + \frac{cP}{4}. \tag{5.3}$$

The application discusses the first **BVP** of an infinite plate stretched at infinity by the application of a uniform tensile stress of intensity Γ. The plate is weakened by a curvilinear hole C which is free from stress and the intensity temperature T, in the all edge of the curvilinear hole C.

Figure(5) describes the following:

1) σ_{xx} has a positive values at $0.127\pi \prec \theta \prec 0.7\pi$, $1.27\pi \prec 1.4\pi$ and $1.6\pi \prec \theta \prec 1.75\pi$.

2) σ_{yy} has a positive values at $0 \prec \theta \prec 0.127\pi$ *and* $1.37\pi \prec \theta \prec 2\pi$.

3) σ_{xy} has a positive values at $\frac{\pi}{3} \prec \theta \prec 1.5\pi$ and $1.656\pi \prec \theta \prec 1.94\pi$.

4) $\max \frac{\sigma_{xx}}{\sigma_{yy}}$ at $\theta \approx 1.146\pi$. $\max \frac{\sigma_{yy}}{\sigma_{xx}}$ at $\theta \approx 1.56\pi$.

For $n_1 = 0.3$, $n_2 = 0.25$, $m_1 = 1$, $m_2 = -0.01$, and $p = 0.25$ the stress components σ_{xx} , σ_{yy} and σ_{xy} are obtained in large forms calculated by computer and illustrated in Fig.(5) and Fig. (6), respectively.

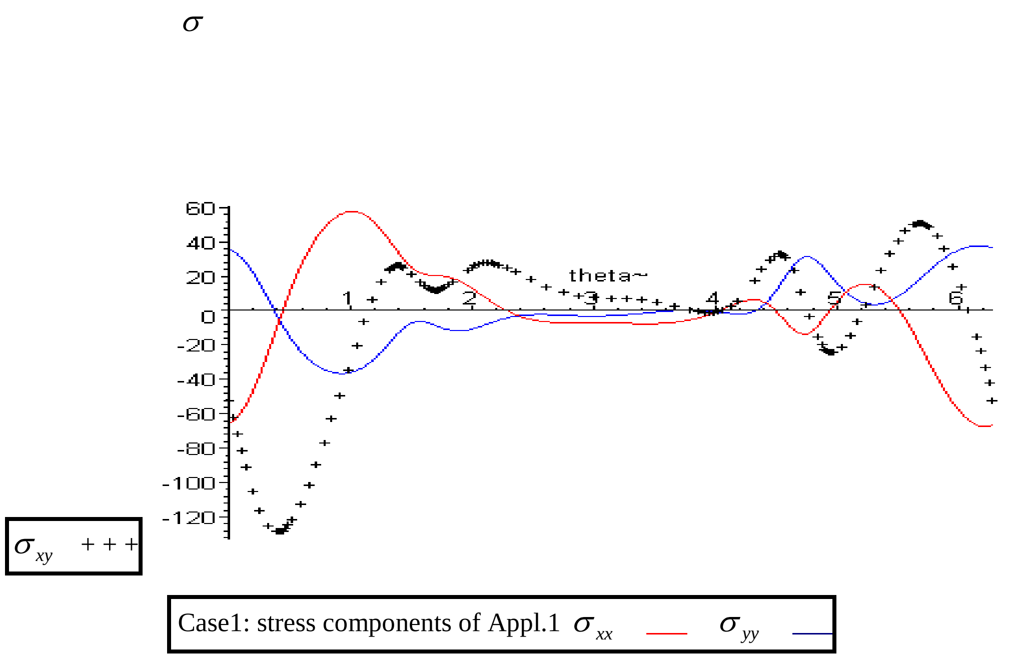

Figure 5

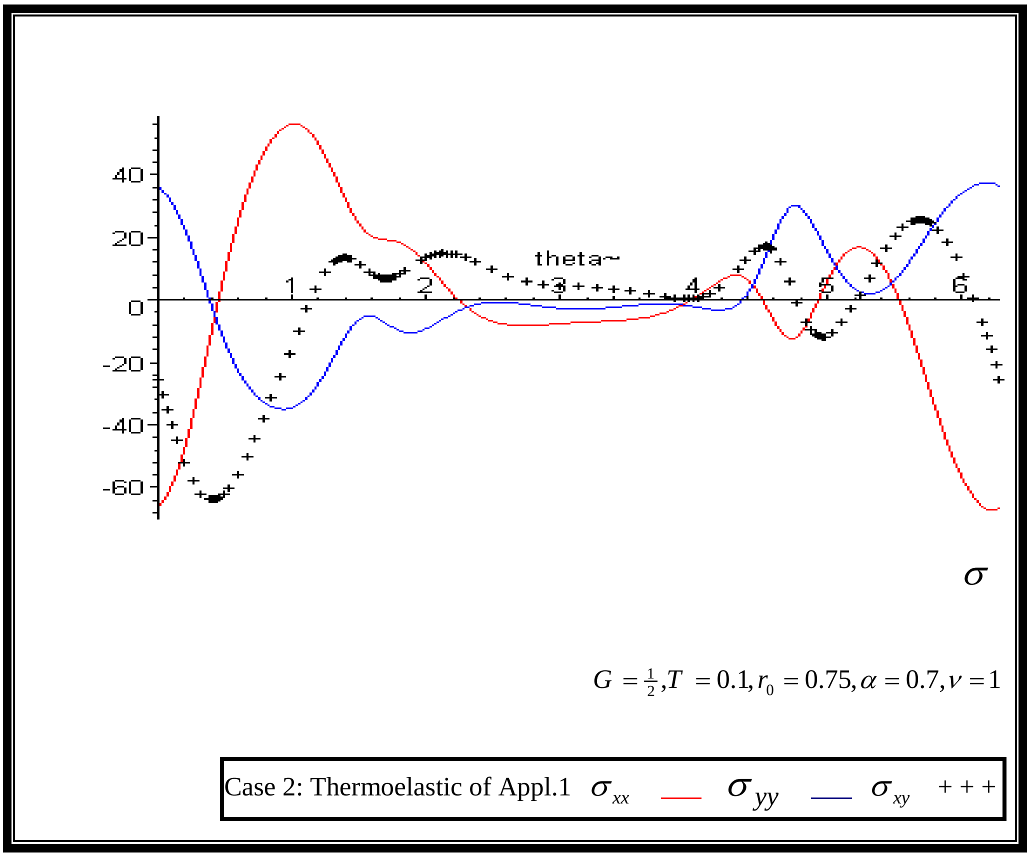

Figure 6

From Figure (6), we deduce

1*) σ_{xx} has a positive values at $0.127\pi \prec \theta \prec 0.7\pi$, $1.27\pi \prec 1.4\pi$ and $1.6\pi \prec \theta \prec 1.75\pi$

2*) σ_{yy} has a positive values at $0 \prec \theta \prec 0.127\pi$ *and* $1.37\pi \prec \theta \prec 2\pi$.

3*) σ_{xy} has a positive values at $\frac{\pi}{3} \prec \theta \prec 1.5\pi$ and $1.656\pi \prec \theta \prec 1.94\pi$.

4*) $\max \frac{\sigma_{xx}}{\sigma_{yy}}$ at $\theta \approx 1.178\pi$. $\max \frac{\sigma_{yy}}{\sigma_{xx}}$ at $\theta \approx 0.732\pi$.

Application 2: ***Curvilinear hole having two poles and the edge is subject to a uniform pressure*** P **.**

If $K = -1,\ S_x = S_y = \Gamma = \Gamma^* = 0,\quad f(t) = P\,t$. Then, we have

$$\varphi(\zeta) = cP\sum_{j=1}^{2}\frac{n_j^3 + m_1 n_j + m_2}{(n_j - n_{j\pm 1})(n_j - \zeta)} - \sum_{j=1}^{2}\frac{h_j E_j}{(n_j - \zeta)(1 - h_j d_j)}, \quad . \tag{5.4}$$

$$\psi(\zeta) = -\frac{\omega(\zeta^{-1})}{\omega'(\zeta)}\varphi'(\zeta) - cP\zeta^{-1} - (n_1+n_2)cP + \sum_{j=1}^{2}\frac{h_j\zeta}{1-n_j\zeta}\varphi'(n_j^{-1}) \quad . \qquad (5.4)$$

The previous discussing give the solutions of first boundary value problem when the edge of hole is subject to a uniform pressure P . For $n_1 = 0.3$, $n_2 = 0.25$, $m_1 = 1$, $m_2 = -0.01$, $p = 0.25$.

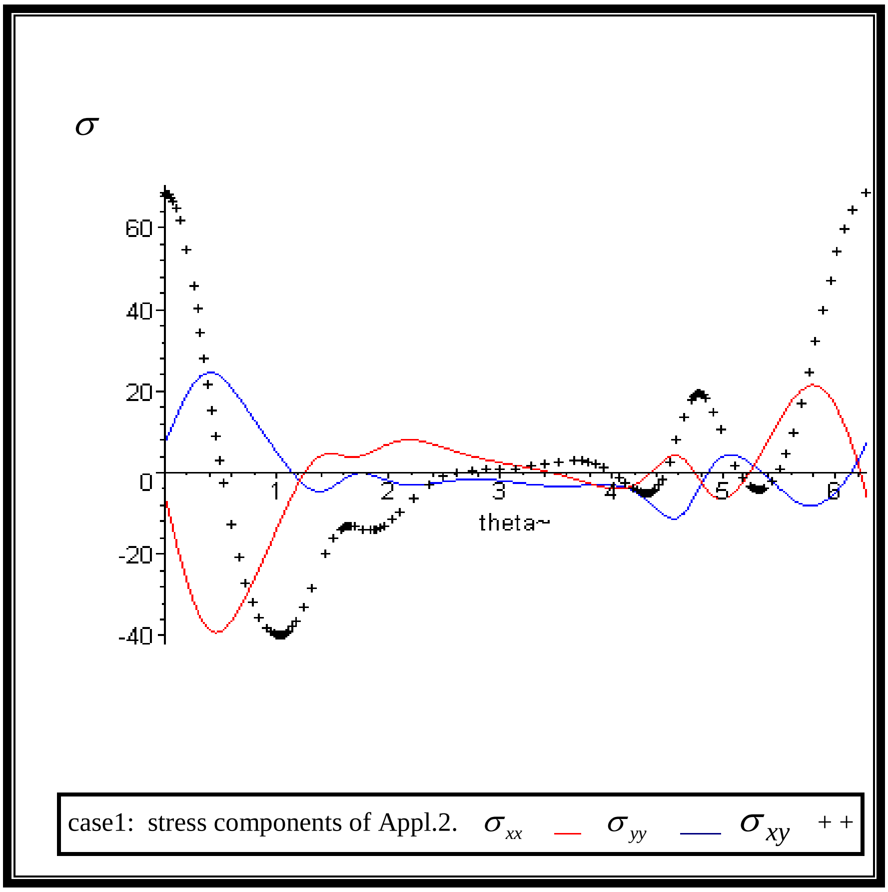

Figure 7

From figure 7, we have $\sigma_{xx} > 0$ at $0.38\pi \prec \theta \prec 1.1\pi$, $1.4\pi \prec \theta \prec 1.53\pi$ and $1.656\pi \prec \theta \prec 2\pi$. $\sigma_{yy} > 0$ at $0 \prec \theta \prec 0.38\pi$ *and* $1.53\pi \prec \theta \prec 1.72\pi$. $\sigma_{xy} > 0$ at $0 \prec \theta \prec 0.16\pi$, $0.828\pi \prec \theta \prec 1.274\pi$, $1.4\pi \prec \theta \prec 1.656\pi$ and $1.75\pi \prec \theta \prec 2\pi$. $\max \frac{\sigma_{xx}}{\sigma_{yy}}$ at $\theta \approx 1.17\pi$. $\max \frac{\sigma_{yy}}{\sigma_{xx}}$ at $\theta \approx 1.40127\pi$.

In figure (8), we deduce that: $\sigma_{xx} > 0$ at $0.38\pi \prec \theta \prec 1.1\pi$, $1.4\pi \prec \theta \prec 1.53\pi$ and $1.656\pi \prec \theta \prec 2\pi$. Also, $\sigma_{yy} .> 0$ at $0 \prec \theta \prec 0.38\pi$ *and* $1.53\pi \prec \theta \prec 1.72\pi$. Finally, $\sigma_{xy} > 0$ at $0.828\pi \prec \theta \prec 1.274\pi$; $1.4\pi \prec \theta \prec 1.656\pi$; $1.4\pi \prec \theta \prec 1.656\pi$ and $1.75\pi \prec \theta \prec 2\pi$. $\max \frac{\sigma_{xx}}{\sigma_{yy}}$ at $\theta \approx 1.52866\pi$. $\max \frac{\sigma_{yy}}{\sigma_{xx}}$ at $\theta \approx 1.0987\pi$.

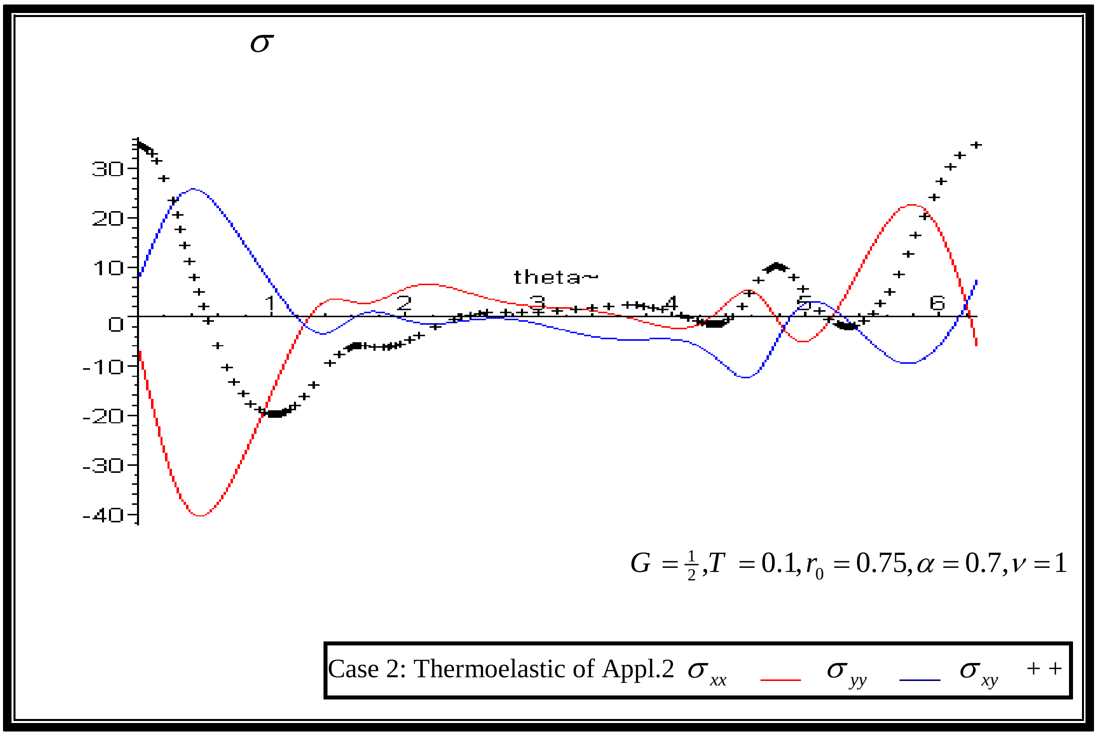

Figure 8

Application 3: ***The external force acts on the center of the curvilinear***. When, $K = k, \Gamma = \Gamma^* = f = 0$, we have

$$-k\,\varphi(\zeta) = \frac{1}{2\pi(1+k)} \sum_{j=1}^{2} \frac{h_j n_j}{(n_j - \zeta)} \left[\frac{k h_j d_j (S_x + iS_y)}{(k^2 - h_j^2 d_j^2)} - (1 + \frac{h_j^2 d_j^2}{(k^2 - h_j^2 d_j^2)}(S_x - iS_y)) \right], \quad (5.5)$$

$$\psi(\zeta) = -\frac{\omega(\zeta^{-1})}{\omega'(\zeta)} \varphi_*(\zeta) + \sum_{j=1}^{2} \frac{h_j \zeta}{1 - n_j \zeta} \varphi_*(n_j^{-1}), \quad (5.6)$$

where,

$$\varphi_*(\zeta) = \varphi'(\zeta) - \frac{S_x + iS_y}{2\pi(1+k)} \zeta^{-1}. \quad (5.7)$$

The last application gives the solution of the second fundamental **BVP** in the presence of temperature around the edge of the hole. The force acts on the center of the curvilinear hole. For $n_1 = 0.3$, $n_2 = 0.25$, $m_1 = 1$, $m_2 = -0.01$, $p = 0.25$, the stress components σ_{xx}, σ_{yy} and σ_{xy} are obtained in large forms calculated by computer and illustrated in figures (9) and (10). In figure (9): σ_{xx} positive at $0 \prec \theta \prec 1.7197\pi$. σ_{yy} at $0.828 \prec \theta \prec 0.9554\pi$ and $1.7197\pi \prec \theta \prec 1.91\pi$. σ_{xy} at $0 \prec \theta \prec 0.1274\pi$, $\theta \prec 1.91\pi$. max $\frac{\sigma_{xx}}{\sigma_{yy}}$ at

$\theta \approx 0.98726\pi$. $\max \frac{\sigma_{yy}}{\sigma_{xx}}$ at $\theta \approx 1.9745\pi$ .In the thermoelastic plate, we have the shapes of figure(10), for the stress components by using the substitutions $G = \frac{1}{2}$, $q = 0.1$, $r_0 = 0.75$, $\alpha = 0.7$, $\nu = 1$. In figure (10) $\sigma_{xx} > 0$ at $0 \prec \theta \prec 1.7197\pi$. $\sigma_{yy} > 0$ at $0.828 \prec \theta \prec 0.9554\pi$ and $1.7197\pi \prec \theta \prec 1.91\pi$. $\sigma_{xy} > 0$ at $0 \prec \theta \prec 0.1274\pi$ and $\theta \prec 1.91\pi$. $\max \frac{\sigma_{xx}}{\sigma_{yy}}$ at $\theta \approx 0.9809\pi$. $\max \frac{\sigma_{yy}}{\sigma_{xx}}$ at $\theta \approx 1.9586\pi$.

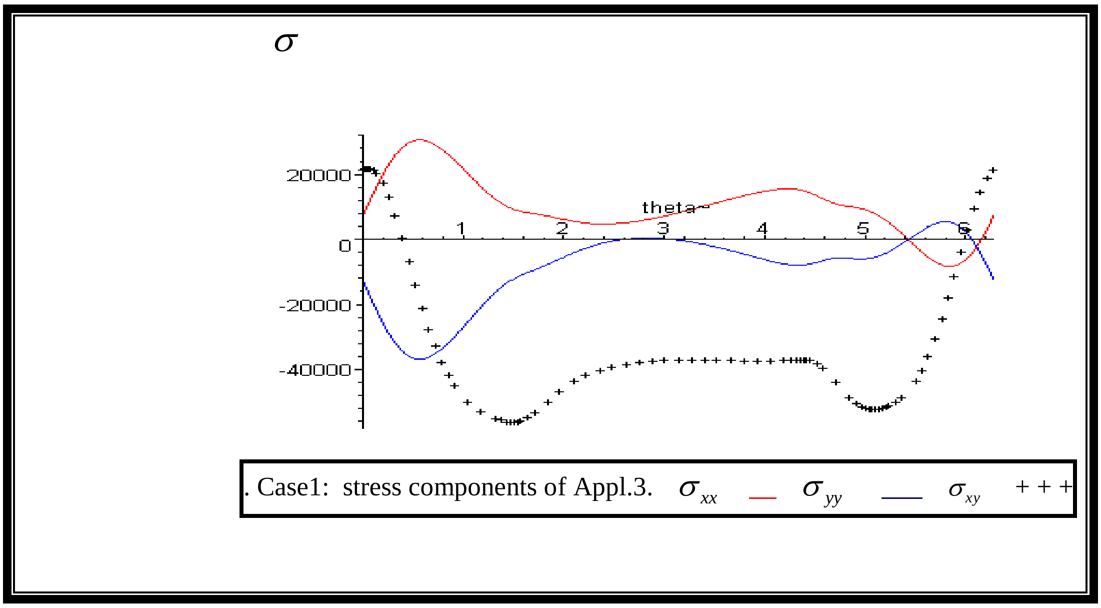

Figure 9

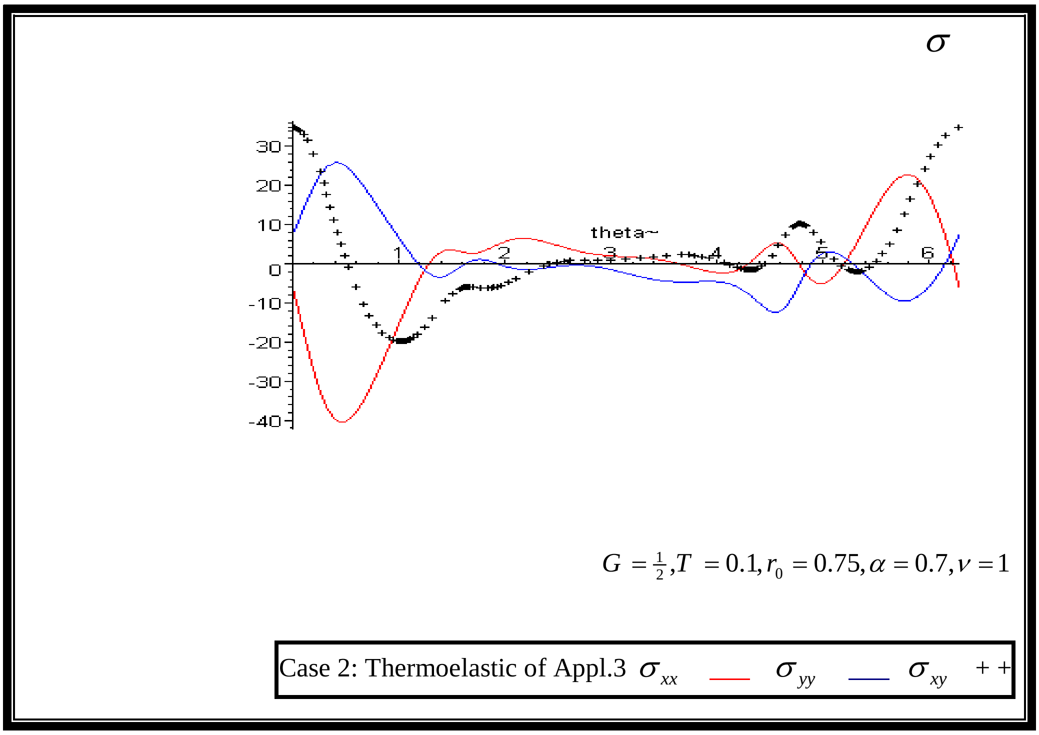

Figure 10

5. Conclusion

From the previous discussions we have the following results

(i) The solution of the **BVPs** for isotropic homogeneous infinite elastic media in $Z-$plane reduce to obtain the two complex potential functions, Gaursat functions, in $\zeta-$plane by using conformal mapping. $z = c\,\omega(\zeta)\,, c > 0$, where $\omega'(\zeta) \neq 0, \infty$, for $|\zeta| > 1$ mapped infinite region to out side a unit circle γ.

(ii) After applying the conformal mapping, $z = c\,\omega(\zeta)\,, c > 0$ the **BVPs** reduce to an integro- differential equation with discontinuous kernel.

(iii) Cauchy method is the best method to solving the integro-differential equation with Cauchy kernel and obtaining the two complex functions $\varphi(z)$ and $\psi(z)$ directly.

(iv) The components of stress $\sigma_{xx}\,, \sigma_{yy}$ and σ_{xy} is completely determine and plotting after obtaining the two complex functions.

(v) As a main result of inserting the effect of temperature T around the edge of the curvilinear hole we have, $\max \sigma_{xy}^{N} > \max \sigma_{xy}^{H}$ and $\min \sigma_{xy}^{N} < \min \sigma_{xy}^{H}$. Where, σ_{xy}^{N} represent the shear components of stress at normal state, while σ_{xy}^{H} represent the shear components of stress after inserting the effect of heating.

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