American Scientific Research Journal for Engineering, Technology, and Sciences (ASRJETS)
ISSN (Print) 2313-4410, ISSN (Online) 2313-4402

http://asrjetsjournal.org/

On the Use of Unit Root Test to Differentiate Between Deterministic and Stochastic Trend in Time Series Analysis

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Abstract

Deterministic and Stochastic trends in time series have different memory properties. Series with trend are non-stationary and must be transformed to be stabilized. The choice of appropriate de-trending procedure depends on the cause of non-stationarity. Mis-specifying the trend characteristics of the data are consequential and can result in biased test and false predictions. This study used the unit root test (ADF) to distinguish between stochastic and deterministic trend in time series analysis. The Nigeria All Share Index (1985-2013) and Nigeria Spot component price of oil (US Dollar per Barrel) data were considered. The results obtained reveals that the Nigeria All Share Index (1985-2013) has a stochastic trend while that of Nigeria Spot component price of oil (US Dollar per Barrel) between 1983-2013; has deterministic trend. Differencing was used to make the Nigeria All Share index data stationary while de-trending was used to remove the deterministic trend Nigeria Spot component price of oil (US Dollar per Barrel).

Keywords: *Unit Root; Deterministic trend; Stochastic trend; non-Stationary; Differencing.*

1. Introduction

A time-series is a collection of observations made sequentially through time. Examples include (i) sales of a particular product in successive months, (ii) the temperature at a particular location at noon on successive days, and (iii) electricity consumption in a particular area for successive one-hour periods [1].

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Before trying to model and forecast a given time series, it is important to have a preliminary look at the data so as to identify its main properties. The time plot is the most important tool, but other graphs and summary statistics may also help. The graph should show up important features of the data such as trend, seasonality, outliers, smooth changes in structure, turning points and/or sudden discontinuities, and is vital, both in describing the data, in helping to formulate a sensible model and in choosing an appropriate forecasting method. A time plot may also help to decide if any variables need to be transformed prior to the main analysis.

Simple descriptive techniques consist of plotting the data, looking for trend, seasonal and cyclical fluctuations, and so on. Generally, a time series (X_t) may usefully be considered as a mixture of four components, namely trend, seasonal, cyclical and irregular components. The task of the statisticians is to segregate each component in so far as it is possible. By isolating or removing individual components the impact of each may be assessed. It may happen that not all four components may be present [1].

Trend is defined as long-term changes in the mean and it refers to the general direction in which the graph of the time series appears to be going over a long interval of time. Trend may be upward (growth) or downwards (decline). A simple way of detecting trend in a seasonal data is to take the averages over a certain period. If this averages changes with time we can say there is evidence of a trend in the series. It can be helpful to model trend using straight lines, polynomials etc [2].

The seasonal component contains movements that are repeated in a more or less regular manner each year. In most series, a similar pattern is typically observed within a calendar year. It describes any regular fluctuations with a period of less than one year. We say that a series exhibits seasonal (periodic) behaviour with period s when similarities in the series occur after s basic time intervals.

The cyclical component describes any regular fluctuations in a series. It is a non-seasonal component which varies in a recognizable cycle. These cycles may or may not be periodic (they may or may not have exactly similar patterns after equal intervals of time). Only long period sets of data will show cyclical fluctuation of any appreciable magnitude. Usually for short series, the cyclical component is superimposed into the trend component [1]. After trend, seasonal and cyclical components have been removed from a set of data, we are left with a series of residual which may or may not be random. The residual component is the result of purely random and irregular once and for all events (e.g. flood, earthquakes, accident, strikes etc) which are completely unpredictable [1,2].

The classical time series analysis assumes that the systematic components, trend, seasonal and cyclical are not influenced by stochastic disturbances and can thus be represented by deterministic functions of time. Stochastic impact is restricted to the residuals, which, on the other do not contain any systematic movements. It is modeled as series of independent or uncorrelated random variable with expectation zero and constant variance, i.e. as a pure random process [3].

Researchers working with time series data are often faced with the problem of indecision when selecting between models with deterministic and stochastic trends. This is because differencing with deterministic model (trend) for stationarity, results in adding a moving average error while detrending a stochastic model will lead to still non-stationarity of the model. Therefore, it is important to understand their relative merits. Models with deterministic trends are widely used but stochastic trends are not so well known. Most works done usually use the unit test to identify stationarity of variables, but in this study we applied unit root test to differentiate when a series is deterministic or stochastic. The reason been that wrong transformation will lead to false or biased result. This study will therefore center on how to use the unit root test to differentiate between deterministic and stochastic trend in time series analysis.

2. Limitation of study

This study was carried out to ascertain the appropriate choice of transformation in Nigeria All Share Index and Nigeria Spot Component Price (US Dollar per Barrel)

3. Deterministic trend

A deterministic trend imposes that the level is not constant, but can be perfectly predicted if the underlying deterministic function is known. One can approximate the deterministic growth path by a function of time, [4]. The deterministic trend is used to measure the rate of technical progress. The simplest model that will generate a time series containing a deterministic trend is;

$$X_t = a_0 + \beta t + e_t \tag{1}$$

$where$ a_0 is constant increase at time t (or initial conditions), βt is the deterministic trend and e_t is a normally distributed random variable, with mean zero and variance σ^2. That is $e_t \sim iid\ N(0\ ,\ \sigma^2)$

The time point doesn't need to be linear, it can be also be polynomial. The presence of deterministic trend implies that the value of X_t increases in each period by a deterministic amount. When attempting to remove the deterministic trend, the appropriate transformations are polynomial de-trending. Differencing would not be the correct step when the time series contain deterministic time trend. Since a deterministic time trend is too restrictive, the obvious thing to do is to make it more flexible by letting the level and slope parameters change over time. In a structural time series model, these parameters are essentially assumed to follow random walks. This leads to a stochastic trend in which the level and slope are allowed to evolve over time [5].

4. Stochastic trend

Stochastic trend incorporate all random shocks that have permanent effect on the level of the series. Models in which at least one parameter or decision variable is a random variable are called stochastic model. A stochastic process is a sequence of random variable (X_1, X_2, …) that is define on a common probability space [6]. When a process is unit root nonstationary, it has a stochastic trend. The merit of stochastic trend model is that it will adapt to a break whenever it occurs and the forecast mean square error will reflect possibility of similar breaks in the future. Models with deterministic trend cannot be used to identify structural breaks.

However, stochastic model can be denoted as

$$\mathrm{X}_t = \beta t + \mathrm{X}_{t-1} + e_t \tag{2}$$

or

$$\nabla \mathrm{X}_t = \beta t + e_t \tag{3}$$

where β is constant increase at time t (or initial conditions) and e_t is a normally distributed random variable, with mean zero and variance σ^2. That is, $e_t \sim iid\ N(0\ ,\ \sigma^2)$. The properties of stochastic model "X_t" is given as

$$\mathrm{E}(\mathrm{X}_t) = \mathrm{E}(\beta t + \mathrm{X}_{t-1} + e_t) = \beta t \tag{4}$$

and

$$Var(\mathrm{X}_t) = Var(\beta t + \mathrm{X}_{t-1} + e_t) = Var(e_t) = t\sigma^2 \tag{5}$$

To derive the variance; we have

Let $Y_t = \nabla X_t = \beta t + e_t$

$$E(Y_t) = E(\beta t + e_t) = \beta t$$

$$E(Y_t^2) = E[(\beta t + e\ )(\beta t + e_t)] = E(\beta^2 t^2 + 2\beta t e_t + e_t^2) = \beta^2 t^2 + E(e_t^2)$$

$$Var(Y_t) = E(Y_t^2) - [E(Y_t)]^2$$

$$Var(Y_t) = \beta^2 t^2 + E(e_t^2) - [\beta t]^2 = E(e_t^2) = \sigma^2 + \sigma^2 + \ldots + \sigma^2 = t\sigma^2 \tag{6}$$

5. Unit Root Test

It is a common practice in Econometrics that testing for stationarity requires that we test for the existence or the inexistence of a unit root. The unit root is a feature of processes that evolve through time that can cause problems in statistical inference involving time series models [7]. In statistics, a unit root tests whether a time series variable is non-stationary using an autoregressive model [8]. A non-stationary time series is said to be integrated to order one or I(1) if the series of its first differences $\nabla X_t = X_t - X_{t-1}$ is I(0). More generally, a series is integrated to order *d*, or I(*d*), if it must be differenced *d* times before an I(0) series results. A series is I (1) if it contains what is called a unit root [9]. Many time series of economic data contain a time trend. Other time series can even grow exponentially. Exponentially growing time series are typically transformed by taking a natural logarithm, which generate a time series containing a linear trend. Any trending time series is not stationary. Therefore, we must first remove the trend from the analysed data before we can proceed with estimating the irregular pattern [10]. There exist numerous unit root test but one of the most popular among them is Augmented Dickey-Fuller (ADF) test.

There are three main version of Augumented Dickey-Fuller for unit root test:

Random walk: This process is also known in time series literatures as unit root process. The term unit root refers to the root of the polynomial in the lag variable. Random walk predicts that the value at time lag will be equal to the last period value plus a stochastic component that is a white noise, which means ε_t is independent and identically distributed with mean "0" and variance "σ^2". One characteristic of a random walk is that the variance evolves over time and goes to infinity as time goes to infinity; therefore, a random walk cannot be predicted [11]. A simple equation of random walk is given below:

$$X_t = X_{t-1} + \varepsilon_t \tag{7}$$

Random walk with drift: When a random walk model predicts that the value at time lag will equal the last period's value plus a constant, or drift (b_0), and a white noise term (ε_t), then the process is random walk with drift. It does not revert to a long-run mean and has variance dependent on time [12]. Depending on b_0 being negative or positive, X_t exhibit a negative or positive stochastic trend. A simple equation is given below;

$$X_t = b_0 + \alpha X_{t-1} + \varepsilon_t \tag{8}$$

Random walk with drift and deterministic trend: It specifies the value at time lag by the last period's value, a drift, a trend and a stochastic component

$$X_t = b_0 + b_1 t + \alpha X_{t-1} + \varepsilon_t \tag{9}$$

From Equation (9), if $b_0 = 0$, $b_1 = 0$ and $\alpha = 1$, we will have a purely random walk. The process is non stationary as we will get $X_t = X_{t-1} + \varepsilon_t$. If we difference, we get $\nabla X_t = \varepsilon_t$. Note that difference series is stationary because $E(\nabla X_t) = E(\varepsilon_t) = 0$ and Var(∇X_t) = $Var(\varepsilon_t) = \sigma^2$. Hence a random walk without a drift is difference- stationary. If $a_0 \neq 0$, $a_1 = 0$ and $\alpha = 1$, we will have a random walk with a drift and the model $X_t = a_0 + X_{t-1} + \varepsilon_t$ is nonstationary. If we difference, we get $\nabla X_t = a_0 + \varepsilon_t$. Note that difference series is stationary because $E(\nabla X_t) = E(a_0 + \varepsilon_t) = a_0$ and $Var(\nabla X_t) = Var(\varepsilon_t) = \sigma^2$. Hence a random walk with a drift is also difference- stationary. Also X_t is trending upward or downward depending on the sign of the drift (a_0). This is called a stochastic trend rather than deterministic trend. If $a_0 \neq 0$, $a_1 \neq 0$ *and* $\alpha = 0$, we get $X_t = a_0 + a_1 t + \varepsilon_t$, then the mean of the series $E(X_t) = E(a_0 + a_1 t) = a_0 + a_1 t$ is time varying but it variance; Var(X_t) = $Var(a_0 + a_1 t + \varepsilon_t) = \sigma^2$ is time invariant. Hence, the series with deterministic trend is also non-stationary. Once the value of a_0 *and* a_1 is known, (we can estimate them by regressing the series on t). We can estimate the mean and then subtract it from the series (de-trending) and create a de-trended series which are stationary. If $a_0 \neq 0$, $a_1 \neq 0$ *and* $\alpha = 1$, we get $X_t = a_0 + a_1 t + X_{t-1} + \varepsilon_t$. The model is random walk with drift and deterministic trend. The difference series $\nabla X_t = a_0 + a_1 t + \varepsilon_t$ is time-varying and hence, the mean of the differenced series is non-stationary. De-trending is necessary on the differenced series to make it stationary [13].

Augmented Dickey-Fuller Test

For each of the above named cases

$H_0 : \alpha = 0;$ (There is a unit root and the series is non-stationary, or it has a stochastic trend)

$H_1 : \alpha < 0;$ (There is no unit root and the series is stationary possibly around a deterministic trend).

If the null hypothesis is rejected, it means in the first case (Random walk) that X_t is stationary with a zero mean. In the second case (Random walk with drift), X_t is stationary with nonzero mean and in the third case (Random walk with drift and deterministic trend) X_t is stationary around a deterministic trend. The simultaneous existence of a unit root and a deterministic trend is thought to be unrealistic [11]. They also propose beginning with the hypothesis of a unit root test with the third most general hypothesis from the ADF family (a unit root with drift and a time trend) and then continue with the more restricted case of a unit root with drift $\alpha = 0, (\rho = 1)$, $a_0 \neq 0$. If this null is not rejected, the trend is stochastic. In case of rejection, the series under study is probably stationary around a deterministic time trend because two other possibilities $(\alpha \neq 0, a_1 = 0,$ *and* $\alpha = 0, a_1 \neq 0)$ are unreasonable and can be ruled out. Also rejection of the null hypothesis in the first test can be treated as strong evidence of deterministic trend. If the null is not rejected in either test, the growth in the observe series is probably due to a stochastic trend.

We have seen that the decision as to whether to difference or to de-trend a time series before proceeding with further analysis depends upon whether the series is DSP or TSP. This in turn depends, as we know, upon whether the root of the series $\alpha = 1$ or $|\alpha| < 1$. Hence the significance of unit root tests [14].

6. Material and Methods

To estimate the unit root tests, we have used E-views 9.1. The data used for illustration in both approaches are from Nigeria All Share Index (1985-2013) and Nigeria Spot component price of oil (US Dollar per Barrel)(1983-2013) from Nigeria Bureau of Statistics (NBS). The summary of the E-view 9.1 results and graph are shown below. For unit roots tests the simple ADF test is used, but the results are not reported to conserve space. We have allowed for shifts in the deterministic trend based on the plots of the coefficients from the rolling least squares estimates in E-views. The results of the analysis are shown in appendix while the summary of result is given below.

Result

Figure 1 below shows the original time plot for Nigeria all Share index. The series shows exponential trend curve, hence the need for logarithm transformation.

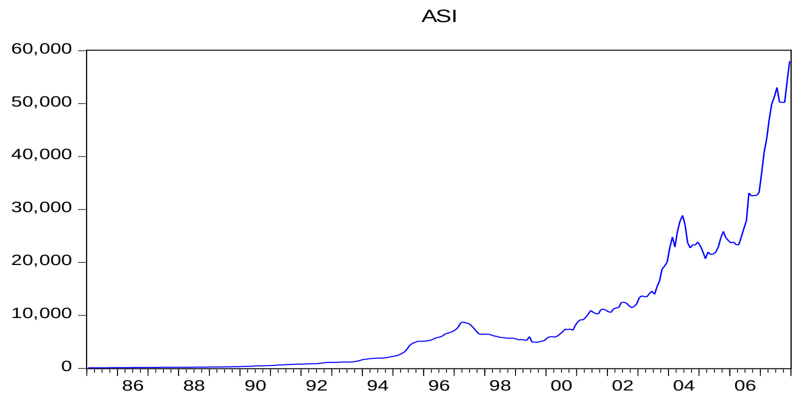

Figure 1: Time plot of All Share Index

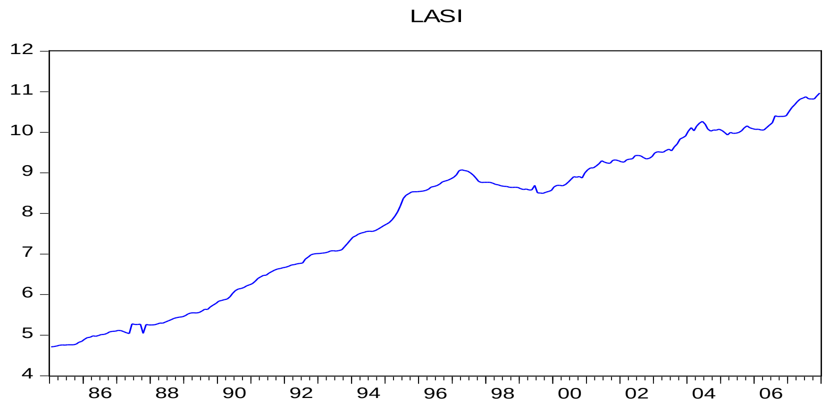

Figure 2: Logarithm Transformation of Nigeria All Share Index

Figure 2 shows that the series has a clear upward trend direction. We therefore include a constant and a deterministic trend in the ADF test equation.

Table 1: Summary of Log Transformation of Original All Share Index

	Log Transformation of Original All Share Index				
Regressor	Coefficient	T-test Critical values	ADF T-test and Decision	P-Value	Decision
LASI	-0.0007		-0.4140	0.6792	
Model with Constant	0.0238	1%: -3.4541 5%: -2.8719 10%: -2.5724	|ADF|< |t-crit|	0.0682	Non-stationary
Model with Constant and @Trend	0.0781 0.0003	1%: -3.9919 5%: -3.4263 10%: -3.1364	|ADF|< |t-crit|	0.0583 0.1646	Stochastic Trend
LASI	-0.0125		-1.4417		

From table 1 above, the ADF t-test has value t = -1.4417 which is smaller in absolute value than the 1% and 5% critical value of -3.9919 and -3.1364 respectively.

Therefore, the ADF t-test will accept the null hypothesis of a stochastic trend (at the 5% significance level, and also at the 1% significance level).

However, if the ADF t-test is larger in absolute value than the 1% and 5% critical value, then the ADF t-test will reject the null hypothesis of a stochastic trend and conclude that the trend in the series is deterministic. When a process is unit root nonstationary, it has a stochastic trend.

From Table 2 above, the ADF t-test has value t = -13.60337 which is larger in absolute value than the 1% and 5% critical value of -3.9919 and -3.4263 respectively.

The p-value for the model with constant and trend was significant which indicates that the null hypothesis of a unit root is rejected. Hence, the series is stationary. The residual plot is shown in figure 3.

Table 2: Summary of Differenced Log Transformation of All Share Index

Regressor	Coefficient	T-test Critical values	ADF T-test and Decision	P-Value	Decision
			-13.60337	0.0000	Significant
DASI(-1)	-0.8129			0.0000	Stationary
Constant	0.0194	1%: -3.9919 5%: -3.4263 10%: -3.1364	|ADF| > |t-crit|	0.0017	Significant
@Trend	-5.58E-06	1%: -3.9919 5%: -3.4263 10%: -3.1364	|ADF| > |t-crit|	0.8806	Not Significant

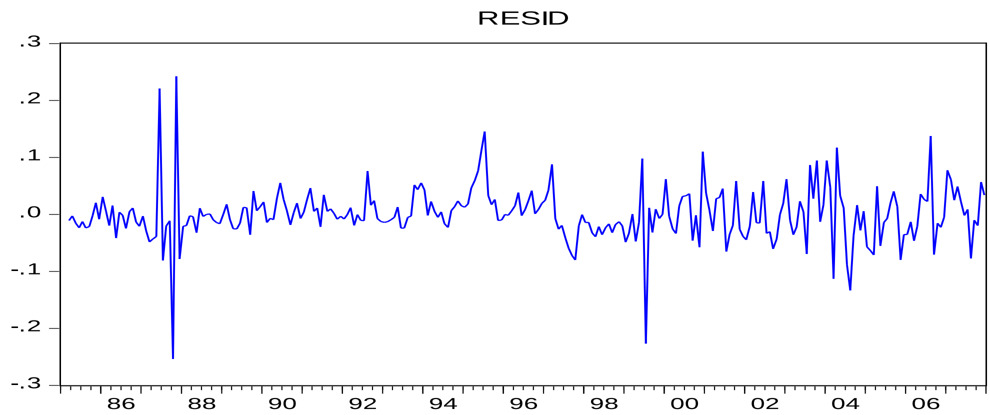

Figure 3: Residual Plot

The data from Nigeria Spot price (1983-2013) were also considered and the unit root test was used to ascertain if the series has a deterministic or stochastic trend. The summary of the E-view results and graph are shown below.

From Table 3 above, the ADF t-test has t-value of -2.848774 which is smaller in absolute value than the 1% and 5% critical value of -3.98306 and -3.42202 respectively. The p-value for the model with constant and trend is not significant which indicates that the null hypothesis of unit root is accepted and thus the series is non-stationary. Also the p-value for the trend (0.0061) is significant which indicate the presence of deterministic trend in the series. In other to make the series stationary, the series was de-trended so as to remove the deterministic trend (quadratic) present in the series.

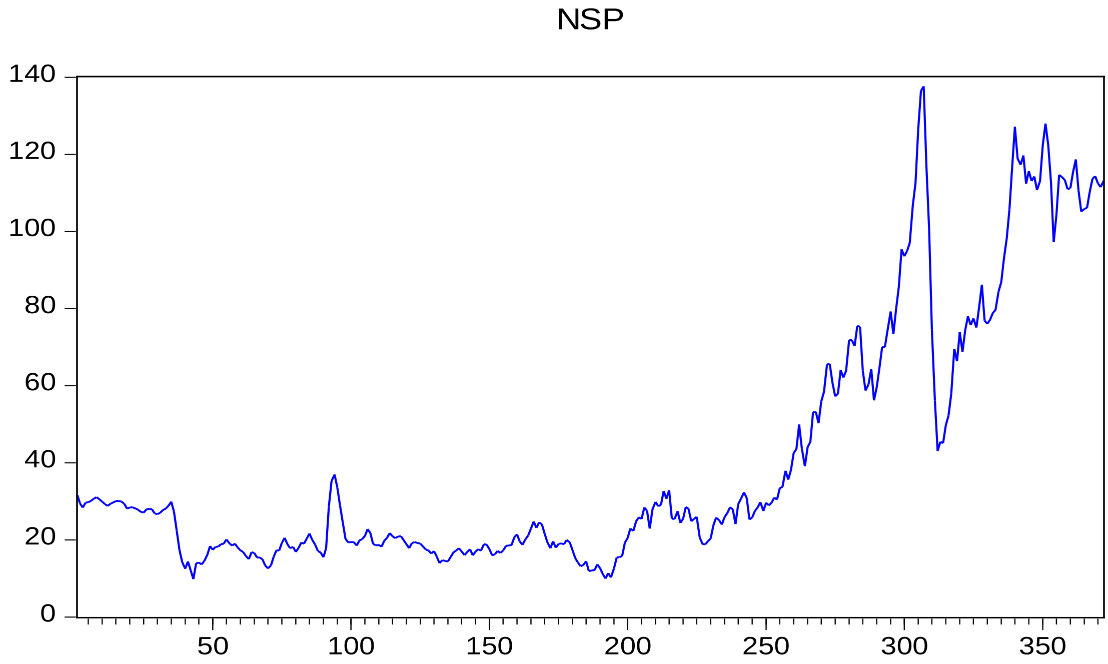

Figure 4: Nigeria Spot Price

Table 3: Summary of Nigeria Spot price (1983-2013)

	Nigeria Spot price				
Regressor	Coefficient	T-test Critical values	ADF T-test and Decision	P-Value	Decision
			-2.848774	0.1808	Non- Stationary
NSP(-1)	-0.026558			0.0046	
Constant	-0.242431	1%: -3.98306 5%: -3.42202 10%:-3.13384	|ADF|< |t-crit|	0.5365	Non-significant
@Trend	0.00785	1%: -3.98306 5%: -3.42202 10%:-3.13384	|ADF|< |t-crit|	**0.0061**	Deterministic Trend present

Figure 4 shows that the trend in the original series is trending upward

Table 4: Summary of De-trended Nigeria Spot price(1983-2013)

	De-trended Nigeria Spot price				
Regressor	Coefficient	T-test Critical values	ADF T-test and Decision	P-Value	Decision
			-5.437000	0.0000	Stationary
DNSP(-1)	-0.092603			0.0000	
Constant	0.081463	1%: -3.983055 5%: -3.422016 10%: -3.13384	|ADF|> |t-crit|	0.8303	Non-Significant
@Trend	-0.00043	1%: -3.983055 5%: -3.422016 10%: -3.13384	|ADF|> |t-crit|	0.8069	Non- Significant

Table 4 above shows the summary of the de-trended Nigeria Spot Price series from (1983-2015). The ADF t-test has t-value of -5.437000 which is greater in absolute value than the 1% and 5% critical value of -3.983055 and -3.42202 respectively. The p-value for the model with constant and trend is significant which indicates that the null hypothesis of unit root is rejected and thus the series is stationary. Also the p-value for the trend (0.8069) is not significant which indicate the presence of deterministic trend in the series has being removed. The coefficient of DNSP(-1) has a negative sign (-0.092603) which indicate that the model is viable.

7. Conclusion

Most economic time series data are non-stationary. Increasing or decreasing behavior of the observed series can be due to a deterministic or stochastic trend. The ADF test allows determining the cause of non-stationarity in the data. Trend-stationary and Difference-stationary series are both trending over time, however the correct approach needs to be used in each case. If a trend-stationary series is differenced, the non-stationarity in the series will be remove at the expense of introducing an MA(1) structure into the errors. Conversely if a difference stationary series is de-trended, the series will still remain non-stationary. Therefore, testing for unit root always requires strategies and the first strategy is to plot data against time so as to rule out unreasonable hypotheses.

The test regression for a growing time series should include a constant and time trend (case III from the ADF family). From the above illustration, the unit root test has been used to identify that the Nigeria All Share Index series for the period considered has a stochastic trend while the Nigeria Spot price has a deterministic trend. In other to obtain stationary, the log transformation of Nigeria All Share Index was difference once and the Nigeria Spot price was de-trended (quadratic trend) so as to make the series trend stationary. The result of the first example reveals that the difference series is random walk with drift while the second is random walk.

8. Recommendation

This paper has shown that the Unit Root test can be used to ascertain when a series is trend stationary or difference stationary in other to avoid wrong transformation. The authors of this paper recommend the test for other micro economic variables that are non-stationary so as to avoid the problem of wrong transformation and bias result.

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