On Extensions of the Optical Optimization

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Abstract

We offer an optimized image formation by considering the focal length of a lens/ mirror or their combination as the objective function. We characterize the stability and correlation properties of an image under fluctuations of the lateral magnification and object distance. The local stability of an image thus formed is determined by the positivity of the pure fluctuation components, while its global counterpart as that of the determinant of the fluctuation matrix. We find that the concave and convex systems render disjoint fluctuation surfaces about the line of the unit lateral magnification. Extended objective functions are constructed for optical systems with finitely many constrained and unconstrained components.

Keywords: Optics; Optimization; Instrument Designing; Stability Analysis; Fluctuation Model.

1. Introduction

Geometrical and physical optics are two rubrics of photonics under which the study of light is manifested [1- 3] towards the formation of the image of an object. The complex behavior of light results in the image formation of an object via lens/ mirror or their finite combination as a consequence of the ray model of the light [4]. This model is termed as the geometrical optics or ray optics because of the fact that the image is formed as an outcome of the beam or a collection of light rays propagating linearly [3].

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It is worth mentioning that physical processes with phase discontinuities which arise in realism of the geometrical optics are modelled by generalized principles of the reflection and refraction [5]. Further, metamaterial based generalizations, viz. optical cavities [6] offer perspective component designing by strongly modifying the spectrum of the light.

In this concern, the ray model characterizes solitonic features of light towards its pulse narrowing applications in optical instrumentations involving lenses, mirrors, optical fibers and prisms [7]. As per this consideration, the image is exhibited as an electromagnetic phenomenon of incoming light rays falling on the instrument. Hereby, we find that lenses, mirrors and their combinations arise as the central interest in optical designing and instrumentation engineering. This is characterized in two types. Namely, a convex lens brings all light rays incident on it to converge at its focal point, see for instance [4]. From this perspective, the distance between the optical center and its focal point termed as the focal length of the instrument offers an optimization function towards the stability and correlation analysis undermining the formation of an image [8].

Moreover, a concave lens likewise diverges the light rays passing through its boundary. Essentially, this forms a virtual image because of an apparent converging point of the refracted light rays from the lens. As far as mirrors are concerned, a concave type mirror converges the reflected rays to its single focal point, whereas the convex mirrors diverge the light rays falling across its boundary in order to form a virtual image. Such optical events happen because of an apparent converging of the reflected rays from the mirror [4], whereby an optimized image is obtained by considering its focal length as the objective function.

Concerning the physical dimensions of an image, it is worth noticing that an image can both be upright or inverted, and minimized or maximized than the corresponding dimensions of the object [9]. Optimized properties of an image essentially depend on a pair of physical quantities undermining the optical configuration. First of all, for an object situated at a finite distance from the optical center of a lens/ mirror or composite system consisting of multiple optical elements, the object distance u arises as an important factor in forming the corresponding image at a distance v, see [10] for cold neutron focusing with multiple biconcave lenses. Our consideration does not stop here, but it continues finding applications in real image formations by a single converging lens and concave mirror [9], and their confocal two lens extensions towards an efficient designing of laser diodes as a single-mode fiber [8]. Secondly, concerning this proposition, another notable quantity in rendering an optimized image is the lateral magnification m, which we consider as the ratio of the image distance to that of the object, or the image height to that of the object [1].

In this paper, we focus on the significance and extensions of the optical optimization procedure in order to examine the local and global stability criteria of an image formed by lenses, mirrors and their combinations both in constrained and unconstrained environments. Image formations involving a single lens/ mirror or optical system consisting of multiple elements comprising either converging and/ or diverging lenses/ mirrors are obtained by specific ray tracing model as the laws of reflection in the case of the mirrors and that of the refraction in the case of the lenses [2-4]. Perspective generalizations with phase discontinuities [5] are left open for a future research development.

The rest of the paper is organized as follows. In Section 2, we characterize the optical optimization problem by analyzing the local and global stabilities and correlations towards an image formation of a dynamical object. In Section 3, we offer physical significances undermining fluctuation analysis of image formed at a fixed lateral magnification and object distance. In Section 4, we provide perspective extensions for constrained and unconstrained optical systems with finitely many components. In Section 5, we conclude our paper with future research directions towards optoengineering and instrument designing.

2. Optical Optimization

In this section, we present the fundamentals behind the optical optimization. In the thin approximation of a given lens/ mirror, we recall [1-4] that the expression of the focal length f reads as

$$f(u,m) = \frac{um}{1-m},\tag{1}$$

where u denotes the object distance and m the lateral magnification. When the optimization variables $\{u,m\}$ are varied simultaneously, the stationary points as the zeros of the first order partial derivatives $\{\frac{\partial f}{\partial u}, \frac{\partial f}{\partial m}\}$ of the objective function f, it follows that the optical center of the optical instrument, viz. a lens and mirror is the only stationary point. This is because of the fact that m and u vanish identically as the simultaneous root of the flow equations $\frac{\partial f}{\partial u} = 0$ and $\frac{\partial f}{\partial m} = 0$. Furthermore, by invoking the second order partial derivatives, we determine the stability characterization of the optical objective function f. Namely, via the second order derivatives, we notice that the pure capacity $\frac{\partial^2 f}{\partial u^2}$ vanishes identically for all values of the optimization variables $\{u,m\}$. On the other hand, it follows from Eqn. (1) that we have the following non-vanishing pure magnification capacity

$$\frac{\partial^2 f}{\partial m^2} = \frac{2u}{(1-m)^3}\tag{2}$$

Furthermore, we notice that the cross components $\frac{\partial^2 f}{\partial u \partial m}$ and $\frac{\partial^2 f}{\partial m \partial u}$ remain identical because of the smooth nature of the objective function $f(u,m)$ as a real function of $\{u,m\}$. Namely, from Eqn. (1), it follows that we have the following absolutely positive mixed capacity or the optical correlation

$$\frac{\partial^2 f}{\partial u \partial m} = \frac{\partial^2 f}{\partial m \partial u} = \frac{1}{(1-m)^2}\tag{3}$$

for all values of the optimization variables $\{u,m\}$. Thus, by substituting the corresponding pure and mixed fluctuation components as in Eqns. (2, 3) and the fact that $\frac{\partial^2 f}{\partial u^2}$ vanishes identically for all values of $\{u,m\}$, we find from the definition of fluctuation theory and its embedding perspectives [11, 12] that the objective function f as in Eqn. (1) yields the following Hessian matrix

$$H = \begin{pmatrix} 0 & \frac{1}{(1-m)^2} \\ \frac{1}{(1-m)^2} & \frac{2u}{(1-m)^3} \end{pmatrix}\tag{4}$$

As per the above critical point evaluation, the analysis of the concave and convex nature of the lenses/ mirrors with the corresponding objective function f at its stationary point $(0, 0)$ is performed by computing the eigenvalues λ of the Hessian matrix H. Considering the Hessian matrix H as above in Eqn. (4) having a nonzero column vector x corresponding to a given eigenvalue λ, the eigenvalue equation $(H - \lambda I)x = 0$ gives a pair of eigenvalues $\{\lambda_1, \lambda_2\}$ as the solutions to the characteristic equation $|H - \lambda I| = 0$, where I is a 2×2 identity matrix and $|A|$ signifies the determinant of the matrix $A := H - \lambda I$. Herewith, it follows from Eqn. (4) that we have the following pair of eigenvalues

$$\lambda_1(u, m) = \frac{u + \sqrt{u^2 + (1-m)^2}}{(1-m)^3} \tag{5}$$

$$\lambda_2(u, m) = \frac{u - \sqrt{u^2 + (1-m)^2}}{(1-m)^3} \tag{6}$$

At the stationary point $(0,0)$, we observe that $\lambda_1(0,0) > 0$ and $\lambda_2(0,0) < 0$. From an independent signature of the eigenvalues $\{\lambda_1, \ \lambda_2\}$, we derive in this case that the determinant Δ of the fluctuation matrix H reads as the product of the eigenvalues λ_1 and λ_2, that's we have $\Delta = \lambda_1 \lambda_2$. From Eqns. (5, 6), it follows for all values of $\{u, m\}$ that we have the following unconditionally negative determinant

$$\Delta(u, m) = -\frac{1}{(1-m)^4} \tag{7}$$

This implies that Δ remains negative for all values of the lateral magnification m, implying a universal existence of global instabilities in optical image formation. For a real image formation, we require a positive determinant satisfying the local inequality $u^2 + m^2 + 1 > 2m$ in optimization variables $\{u, m\}$, whereby the image formed by the optical system under consideration remains globally unstable. By invoking the eigenvalue equation $Hx = \lambda x$ with $x \in \mathbb{R}^2$ as the real valued column vector with its two entries as the object distance u as the first component and lateral magnification m as the second, it follows from Eqns. (5, 6) that the normalized eigenvectors $\{\hat{x}_1, \hat{x}_2\}$ corresponding to $\lambda = \lambda_1$ and $\lambda = \lambda_2$ read as

$$\hat{x}_1(u, m) = \frac{1}{\sqrt{1 + (\frac{1}{(1-m)^2 \lambda_1})^2}} \begin{pmatrix} \frac{1}{(1-m)^2 \lambda_1} \\ 1 \end{pmatrix} \tag{8}$$

$$\hat{x}_2(u, m) = \frac{1}{\sqrt{1 + (\frac{1}{(1-m)^2 \lambda_2})^2}} \begin{pmatrix} \frac{1}{(1-m)^2 \lambda_2} \\ 1 \end{pmatrix} \tag{9}$$

Herewith, the physical implication of the above eigenvectors as in Eqns. (8, 9) illustrates an undermining optimal zooming qualification in the two dimensional fluctuation space of optical optimization variables $\{u, m\}$.

3. Physical Implications

In this section, we discuss the stability and correlation structures of the image formed by a given lens/ mirror

under specific fluctuations of the object distance u and lateral magnification m. In the sequel, in the first part of this section, we examine the stability components of an image formed by the optical instrument at a given lateral magnification. In the second part of this section, we focus on the image stability properties evaluated at a given object distance in the unit of the focal length as a characteristic property of the lens/ mirror.

3.1. Fixed Lateral Magnification

From the perspective of the stability analysis as introduced in Section 2, we offer the functional qualification of the optical designing components, viz. the focal length f, flow components $\{f_u, f_m\}$, fluctuation capacities $\{f_{mm}, f_{mu}, f_{uu}\}$ and eigenvalues $\{\lambda_1, \lambda_2\}$ at a fixed lateral magnification m of the system. As a function of the object distance u from the optical center of an instrument, the corresponding expressions are hereby enlisted for the lateral magnification $m \in \{0, 1/2, 1, 3/2, 2, 5/2, 3\}$ in Table 1 below.

Table 1: Stability components at a fixed lateral magnification

SN m	f	f_u	f_m	f_{mm}	f_{mu}	f_{uu}	λ_1	λ_2
0	0	0	u	$2u$	1	0	$u + \sqrt{u^2 + 1}$	$u - \sqrt{u^2 + 1}$
$\frac{1}{2}$	u	1	$4u$	$16u$	4	0	$8u + 4\sqrt{4u^2 + 1}$	$8u - 4\sqrt{4u^2 + 1}$
1	∞	∞	∞	∞	∞	0	∞	∞
$\frac{3}{2}$	$-3u$	-3	$4u$	$-16u$	4	0	$-8u - 4\sqrt{4u^2 + 1}$	$-8u + 4\sqrt{4u^2 + 1}$
2	$-2u$	-2	u	$-2u$	1	0	$-u - \sqrt{u^2 + 1}$	$-u + \sqrt{u^2 + 1}$
$\frac{5}{2}$	$-\frac{5}{2}u$	$-\frac{5}{3}$	$\frac{10}{9}$	$-\frac{16}{27}u$	$\frac{4}{9}$	0	$\dfrac{-8u - 4\sqrt{4u^2 + 9}}{27}$	$\dfrac{-8u + 4\sqrt{4u^2 + 9}}{27}$
3	$-\frac{3}{2}u$	$-\frac{3}{2}$	$4u$	$-\frac{1}{4}u$	$\frac{1}{4}$	0	$\dfrac{-u - \sqrt{u^2 + 4}}{8}$	$\dfrac{-u + \sqrt{u^2 + 4}}{8}$

For a given lateral magnification m, namely, at its discrete values $m \in \{0, 1/2, 1, 3/2, 2, 5/2, 3\}$, we analyze the nature of the fluctuation quantities, viz. the focal length f, flow components $\{f_u, f_m\}$, fluctuation capacities $\{f_{mm}, f_{mu}, f_{uu}\}$ and eigenvalues $\{\lambda_1, \lambda_2\}$ of the fluctuation matrix H.

From the viewpoint of the eigenvalue $\lambda_1(u)$ as a function of the object distance u considered at a fixed lateral magnification $m \in \{0, 1/2\}$, we find from Eqn. (5) that the image stability monotonically increases in the interval $u \in (0, 2)$. Namely, the formation of an image locally exhibits a nonlinear behavior in the vicinity of the optical center of the instrument. However, we find that it fluctuates approximately linearly for a distant object from the optical center of the instrument. Moreover, for a higher value of the lateral magnification $m \in \{3/2, 2, 5/2, 3\}$, we notice that $\lambda_1(u)$ decreases continuously with same characterization as the foregoing values of the lateral magnification, viz. $m \in \{0, 1/2\}$. Furthermore, it is worth remarking that the amplitude

variations of the above mentioned fluctuation capacities differ from the one another for different values of the lateral magnification of the optical instrument.

Similarly, we find from Eqn. (6) that the eigenvalue $\lambda_2(u)$ of the fluctuation matrix H at the lateral magnification $m \in \{0, 1/2\}$ produces an approximate semi-parabola while the object distance u varies in the interval $(0, 2)$.

We notice further that the parabola thus formed has its vertex at the optical center of the instrument. On the other hand, we find that the eigenvalue $\lambda_2(u)$ at the lateral magnifications $m \in \{3/2, 2, 5/2, 3\}$ describes an inverse proportional decrease in its amplitude as the object moves away from the optical center of the instrument. Hereby, we notice from Eqn. (6) that $\lambda_2(u)$ has a clear-cut distinction in its amplitude under variations of the lateral magnification of the lens/ mirror.

3.2. Fixed Object Distance

When the object distance u is measured in the unit of the focal length f of the instrument with the scaling expression $u = kf$, we find from Eqn. (1) that the objective function f leads to the lateral magnification

$$m(k) = \frac{1}{k+1} \tag{10}$$

Herewith, we provide specific values of the focal length, flow components, fluctuation capacities and eigenvalues for varying values of the object distance in Table 2 below.

With the scaling $u = kf$, it follows from Eqn. (2) that the correlation component f_{mm} simplifies as

$$f_{mm} = \frac{2kf}{(1-m)^3} \tag{11}$$

In general, we see from Eqns. (5, 6) that λ_1 and λ_2 reduce as per the following expressions

$$\lambda_1 = \frac{kf + \sqrt{(kf)^2 + (1-m)^2}}{(1-m)^3} \tag{12}$$

$$\lambda_2 = \frac{kf - \sqrt{(kf)^2 + (1-m)^2}}{(1-m)^3} \tag{13}$$

Therefore, for the scalar quantity $k \in \{1/2, 1, 3/2, 2, 5/2, 3\}$, the corresponding values of the eigenvalues $\{\lambda_1, \lambda_2\}$ are enlisted in Table 2. It is worth noticing that the Hessian fluctuation components, viz. f_{mu} and f_{uu} as mentioned before remain independent of the object distance u.

In other words, we see from the Table 2 below that there is no change in the behavior of the fluctuation components $\{f_{mu}, f_{uu}\}$ irrespective of variations of the object distance $u = kf$ from the optical center of the instrument.

Table 2: Stability components as a function of the lateral magnification and focal length

SN ($u = kf$)	m	f	f_u	f_m	f_{mm}	f_{mu}	f_{uu}	λ_1	λ_2
$\dfrac{f}{2}$	$\dfrac{2}{3}$	$\dfrac{fm}{2(1-m)}$	$\dfrac{m}{(1-m)}$	$\dfrac{f}{2(1-m)^2}$	$\dfrac{f}{(1-m)^3}$	$\dfrac{1}{(1-m)^2}$	0	$\dfrac{\frac{f}{2}+\sqrt{\frac{f^2}{4}+(1-m)^2}}{(1-m)^3}$	$\dfrac{\frac{f}{2}-\sqrt{\frac{f^2}{4}+(1-m)^2}}{(1-m)^3}$
f	$\dfrac{1}{2}$	$\dfrac{fm}{(1-m)}$	$\dfrac{m}{(1-m)}$	$\dfrac{f}{(1-m)^2}$	$\dfrac{2f}{(1-m)^3}$	$\dfrac{1}{(1-m)^2}$	0	$\dfrac{f+\sqrt{f^2+(1-m)^2}}{(1-m)^3}$	$\dfrac{f-\sqrt{f^2+(1-m)^2}}{(1-m)^3}$
$\dfrac{3}{2}f$	$\dfrac{2}{5}$	$\dfrac{3fm}{2(1-m)}$	$\dfrac{m}{(1-m)}$	$\dfrac{3f}{2(1-m)^2}$	$\dfrac{3f}{(1-m)^3}$	$\dfrac{1}{(1-m)^2}$	0	$\dfrac{\frac{3f}{2}+\sqrt{\frac{9f^2}{4}+(1-m)^2}}{(1-m)^3}$	$\dfrac{\frac{3f}{2}-\sqrt{\frac{9f^2}{4}+(1-m)^2}}{(1-m)^3}$
$2f$	$\dfrac{1}{3}$	$\dfrac{2fm}{(1-m)}$	$\dfrac{m}{(1-m)}$	$\dfrac{2f}{(1-m)^2}$	$\dfrac{4f}{(1-m)^3}$	$\dfrac{1}{(1-m)^2}$	0	$\dfrac{2f+\sqrt{4f^2+(1-m)^2}}{(1-m)^3}$	$\dfrac{2f-\sqrt{4f^2+(1-m)^2}}{(1-m)^3}$
$\dfrac{5}{2}f$	$\dfrac{2}{7}$	$\dfrac{5fm}{2(1-m)}$	$\dfrac{m}{(1-m)}$	$\dfrac{5f}{2(1-m)^2}$	$\dfrac{5f}{(1-m)^3}$	$\dfrac{1}{(1-m)^2}$	0	$\dfrac{\frac{5f}{2}+\sqrt{\frac{25f^2}{4}+(1-m)^2}}{(1-m)^3}$	$\dfrac{\frac{5f}{2}-\sqrt{\frac{25f^2}{4}+(1-m)}}{(1-m)^3}$
$3f$	$\dfrac{1}{4}$	$\dfrac{3fm}{(1-m)}$	$\dfrac{m}{(1-m)}$	$\dfrac{3f}{(1-m)^2}$	$\dfrac{6f}{(1-m)^3}$	$\dfrac{1}{(1-m)^2}$	0	$\dfrac{3f+\sqrt{9f^2+(1-m)^2}}{(1-m)^3}$	$\dfrac{3f-\sqrt{9f^2+(1-m)^2}}{(1-m)^3}$

For the above values of the ratio k, we find in the magnification interval $(0.9, 1.1)$ that the pure component f_{mm} as in Eqn. (3) and eigenvalue $\lambda_1(f, m)$ as in Eqn. (5) generate a separate pair of negatively and positively curved surfaces under simultaneous fluctuations of the focal length f and lateral magnification m. This pair corresponds to the fluctuation surfaces of concave and convex lenses/ mirrors respectively. Such a pair of surfaces turns out to be asymptotically disjoint about the line of the unit lateral magnification $m = 1$ with large fluctuation amplitudes of the pure magnification capacity f_{mm}. Moreover, at a given object distance u, we find that a finite behavior of f_{mm} and λ_1 - - as observed in their amplitude modulations under the fluctuations of the lateral magnification $m \in (0.9, 1.1)$ - - offers an optical fine-tuning towards lens designing.

Under the aforementioned values of k, we further anticipate from Eqn. (6) that the eigenvalue $\lambda_2(f, m)$ of the fluctuation matrix H forms a pair of positively and negatively curved surfaces corresponding to the convex and concave lenses/ mirrors, as well. Moreover, it follows that the point of discontinuity arises in fluctuations of the eigenvalue $\lambda_2(f, m)$ as above in Eqn. (6) about the line of the unit lateral magnification $m = 1$. Herewith, as far as the stability of an image is concerned, we find that similar conclusions hold for higher values of k in fine-tuning the image for a given pair of varying lateral magnification and focal length of the instrument.

4. Extended Optical Optimizations

In this section, we offer perspective directions of the foregoing optical optimization model for finite combinations of lenses and/ or mirrors with and without their extensions to constrained settings. With reference to a constrained optimization setting, we formulate an extended model with possible involvements of additional optimization variables, apart from the object distance and lateral magnification.

4.1. Power Optimization

From the perspective of the thin lens approximation as discussed above for a given single lens/ mirror or an optical instrument involving their finite combination, we may express the net focal length f_{net} of the resulting system [1-4] as

$$\frac{1}{f_{net}} = \frac{1}{f_1(u_1, v_1)} + \frac{1}{f_2(u_2, v_2)}, \tag{14}$$

where f_1 as the focal length of the first component emerges as a function of the object distance u_1 and image distance v_1 from its optical center, see Eqn. (1). Similarly, it follows that f_2 is solely a function of the associated object and image distances $\{u_2, v_2\}$ from the optical center of the second component. Notice further that f_2 satisfies Eqn. (1) in its thin lens approximation, as well. Thus, the final object distance u and image distance v of the combination with a common reference point can be obtained as per the transformations

$$u_2 \to u_2' = u_2 + v_1, \tag{15}$$

$$v_2 \to v_2' = v_2 + v_1 + u_1 \tag{16}$$

In the presence of an interaction term f_i which arises by going beyond the thin lens approximation [1-4], the net focal length f_{net} of the combined optical system can be represented as

$$\frac{1}{f_{net}(u_2', v_2')} = \frac{1}{f_1(u_1, v_1)} + \frac{1}{f_2(u_2, v_2)} - \frac{1}{f_i(u_1, v_1, u_2, v_2)} \tag{17}$$

In the light of the above definition, we find that the net optical power D of the combined optical system arises as the inverse of the total focal length f, viz. we have $D = 1/f_{net}$. Notice further that the optical power of an instrument is measured in the units of Diopter [1]. Similarly, we may define the corresponding component powers as $D_i = 1/f_i, i = 1, 2$ and the combined interactive power of the system as $d(\{D_i\}) = 1/f_i$. Therefore, under small fluctuation approximation, it follows that the net optical power $D(\{D_i\})$ of the combined system as a function of the component optical powers $\{ D_i \mid i = 1, 2 \}$ can be expressed as

$$D(\{D_i\}) = D_1 + D_2 - d(\{D_i\}) \tag{18}$$

Hereby, the interaction quantity f_i of the considered optical components, viz**.** the constituent lenses and/ or mirrors is determined via their respective curvatures. With the initial conditions $D(0,0) = 0$, $\partial D/\partial D_1 = 1$

and $\partial D / \partial D_2 = 1$, in the linear approximation of the Taylor series expansion of the net power $D(\{D_i\})$, we see that it reads as per the following summation

$$D(\{D_i\}) \simeq D_1 + D_2 \qquad (19)$$

Beyond the thin lens approximation, we incorporate terms above the linear orders in the series expansion of $D(\{D_i\})$ of the combined optical system. Specifically, as a function of the component optical powers $\{D_i \mid i = 1, 2\}$ forming an optimization basis, the effective optical power $D(\{D_i\})$ of the combined system in its quadratic approximation reads as per the expansion

$$D(D_1, D_2) \simeq \sum_{\substack{i, j=0 \\ i+j=2}}^{2} D_{ij} \, \Delta D_i \Delta D_j \,, \qquad (20)$$

where $H := (D_{ij})$ is the fluctuation matrix having entries as the fluctuation components

$$D_{ij} := \frac{\partial^{i+j} D}{\partial D_1{}^i \, \partial D_2{}^j} \qquad (21)$$

with respect to the component optical powers $\{D_i \mid i, \ j = 1, 2\}$ of the combined system.

4.2 Constrained Optimization

In the presence of a constraint function $c(u, m)$ acting on the optical system, the corresponding objective function f_c of the constrained optimization system takes the form

$$f_c(u, m) = \frac{um}{1 - m} + \lambda c(u, m), \qquad (22)$$

where λ is the Lagrange multiplier defining the constraint optical surface $c(u, m) = 0$. Under fluctuations of the optimization variables $\{u, m\}$, a constantly constraint system is defined by $c(u, m) = c_0$, where c_0 is a fixed real constant independent of the optimization variables $\{u, m\}$. In this case, we find hereby that the same optimization conclusions hold as given in Section 2 and Section 3 for the associated unconstraint optical system. Moreover, as long as the partial derivatives c_u and c_m of $c(u, m)$ with respect to the variables $\{u, m\}$ exist, it follows that the linear truncation of the constraint function $c(u, m)$ redefines the lens designing problem as the constrained optimization on the surface

$$c(u, m) \simeq c_0 + c_u u + c_m m \qquad (23)$$

Towards an extension with multiple constraints $c_i(u, m)$ to the above optimization of a single component as in Eqn. (22), viz. lens/ mirror or their combinations, the corresponding objective function can be generalized as

$$f_c(u, m) = \frac{um}{1-m} + \sum_{i=1}^{k} \lambda_i c_i(u, m) \qquad (24)$$

Here, the parameters $\{\lambda_i \mid i = 1, 2, \dots, k\}$ represent the Lagrange multipliers of the respective constraining

surfaces $c_i(u, m) = 0$ acting on i^{th} component of the optical system, where $i = 1, 2, \ldots, k$. Furthermore, considering the foregoing k constraining optical surfaces acting on a finite combination of n optical components, we find from Eqn. (24) that the net objective function extends as per the following expression

$$f_{net}(\{u_i, m_i\}) = \sum_{i=1}^{n} \left(\frac{u_i\, m_i}{1 - m_i} + \sum_{j=1}^{k} \lambda_{ij}\, c_{ij} \right), \qquad (25)$$

where the first term in the summation denotes i^{th} objective function of the free optical component. The second term sums over the respective constraints given by the constraining hypersurfaces $c_{ij}(\{u_i, m_i\}) = 0$, where we have $j = 1, 2, \ldots, k$ for each component $i = 1, 2, \ldots, n$ of the optical system. Herewith, an optimization involving k number of constraints to the net objective function $f_{net}(\{u_i, m_i\})$ as in Eqn. (25) of the combined optical system is anticipated to offer potential industrial applications.

It is worth remarking that nonlinear extensions of an optical system with the constraining hypersurfaces $c_{ij}(\{u_i, m_i\}) = 0$ involve an optimization of the net objective function $f_{net}(\{u_i, m_i\})$ as mentioned in Eqn. (25) in order to form a fine-tuned image. In general, such an optical optimization may have different conclusions than its unconstraint counterpart as outlined in the foregoing Sections (2, 3). Hereby, we find that similar conclusions hold for both the constantly and linearly constraint optical systems as an extension of the associated unconstrained system, except the fact that there may exist shifts in the fixed points for the linearly constrained case. Beyond the linear approximation, it is worth mentioning that the stability analysis of a constrained optical system with finitely many components could depend in principle upon the nature of constraints concerning the experimental setup, as well. Such nonlinear extensions are the subject matter of our current research.

5. Discussion and Conclusion

Geometrical optics plays a vital role in understanding image formation via optical instruments, viz. lenses, mirrors and their arbitrary combinations. An optimal optical designing is likewise reinforced under motion of the object and variations over the lateral magnification of the instrument. Further, we discuss possible extensions with nonlinear constraints by invoking extended optical optimization functions, as well. Having discussed the construction of optical objective functions, we concentrate towards its role in optoengineering and instrument designing. In this concern, we focus on the stability and correlation analysis of the image of a dynamical object formed by an optical instrument, viz. lens and/ or mirror. From the perspective of the fluctuation theory, we concentrate on the optimal designing of optical systems at a given lateral magnification and object distance. In the sequel, we prolong our analysis towards stable image formation by an optical system having finitely many components, in the light of the constrained Lagrangian systems.

Herewith, in the thin approximation of components undermining the optical instrument, we characterize the objective function of an arbitrary optical system as the net focal length by considering it as a function of the optimization variables, viz. the object distance, lateral magnification and others, if any. Under fluctuations of the optimization variables, we offer stability and correlation analysis towards an apt optical instrument designing. As per this consideration, we find that an image as formed by the concave and convex lens/ mirror appears disjoint, whose fluctuation surfaces are separated by the line of the unit lateral magnification. By

invoking multivariable calculus based saddle point analysis, the local stability is characterized by the positivity of the pure components of the fluctuation matrix of the optical optimization function, whilst the global stability as the positivity of the product of its eigenvalues. It is worth mentioning that the corresponding eigenvectors qualify optimal zooming in the space of the optical optimization variables.

We anticipate that the outcomes emerging from an unconstraint optical system hold in extended optimization settings with finitely many specific constraints, as well. In this regard, we find that a linearly constraint optical system has the same fluctuation qualifications as that of the free system, except the fact that the corresponding optical extrema are translated. It is worth anticipating that constrained optimizations of nonlinear systems offer key industrial significances towards the modern optical instrument designing. Such innovations include rear-view mirrors [13], small concave metal-replica mirrors [14] and small-angle neutron scattering phenomenon in focusing cold neutrons involving multiple biconcave lenses [15]. The quality reinforcement of such an optimized instrument designing we leave open for a future research investigation.

Acknowledgements

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