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Performance Evaluation of Applying Fuzzy Multiple Regression Model to TLS in the Geodetic Coordinate Transformation

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Abstract

It is known that the classical technique for solving the linear regression problem of the geodetic transformation process is using least squares approach (LS). On the contrary, this research explores the application of total least squares (TLS) approach to solve linear regression with and without fuzzy multiple regression model in Bursa-Wolf similarity transformation process. In this research two groups of data sets are used; the first group is the solution points which are used to compute the values of the transformation parameters. The second is the check points that were used to assess the accuracy of the applied methods (in terms of mean and Root Mean Squares Errors RMSE). The applied four solutions show how the accuracy of TLS is relatively better than LS. The weight has a better effect on improving the accuracy of both cases, LS and TLS; however, its effects are greater on TLS. By using the fuzzy multiple regression models, the results improved further and the need for accurate weights/confidence is eliminated.

Keywords: Transformation; weighted least squares; weighted total least squares; singular value decomposition; Linear Regression; Fuzzy Multiple Regression Model.

1. Introduction

Coordinate transformation is one of the most common applications in geodesy, photogrammetry, mapping, surveying engineering, and computer algebra.

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The well-known technique for solving the transformation is the Least Squares Method (LS), where the coefficient matrix is assumed free of errors, and all errors come from the observation vector. However, both the coefficient matrix and observation vector may be contaminated by noise. Therefore; Total Least Squares (TLS) introduces the appropriate approach to solve this problem [1]. The authors in [2] are the first who formulated Weighted Total Least Squares (WTLS) adjustment by following the geodetic tradition using general non-diagonal variance covariance matrices and introduced it to the geodetic science community.

The author in [3] showed that the TLS solutions within an Error - In - Variables (EIV) model can also be identified as a special case of the method of least squares within an iteratively linearized Gauss- Helmart model where weight matrices can then be introduced without any limitations. The authors in [4] formulated WTLS as a nonlinear adjustment model without constraints and further extended it to a partial EIV model. Recently, the author in [5] presented an improved WTLS method derived from a more generic case in which there is no proportionality assumption for the cofactor matrix of the EIV model and an improved constrained WTLS with application in the linear fitting and coordinate transformation. As well as, [6] reformulates the EIV model when all the observations are linear and propose new alternative WTLS algorithm that considered all the errors of observations in a more reasonable and direct way. The results show that the relative bias is only 0.01% in 1000 simulations of 3D coordinate transformation by TLS, and the more redundant observations, the more approximated is the estimated variance. For more details about TLS see [7, 8, 9, 10,11].

Fuzzy regression (FR) analysis is a possibility type of classical regression analysis. It is used in evaluating the functional relationship between the dependent and independent variables in a fuzzy environment. The Fuzzy regression model is firstly introduced by [12]. Recent researches such as [13, 14 and 15] used FR in their analysis. The FR expected to improve the results of solving the linear regression model using both least squares and total least squares methods.

Herein, we are going to inspect the performance of total least squares on the geodetic coordinate transformation, and to assess the effect of applying fuzzy multiple regression model on both the least squares and total least squares adjustments.

2. Methodology

In this research, the Bursa-Wolf similarity transformation model is chosen to solve the 3D datum transformation. The mathematical equation of this model is given by [6]:

$$\begin{bmatrix} X_i^{`} \\ Y_i^{`} \\ Z_i^{`} \end{bmatrix}_{D2} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + (1+S).\, R_X(\propto) R_Y(\beta) R_Z(\gamma). \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix}_{D1} \tag{1}$$

Where S is the scale factor, ΔX, ΔY, ΔZ are the translation parameters, $[X \quad Y \quad Z]_{D1}^{T}$, $[X` \quad Y` \quad Z`]_{D2}^{T}$ are the coordinate vectors of the i^{th} station in $Datum_1$ and $Datum_2$ respectively, $R_X(\propto)R_Y(\beta)R_Z(\gamma)$ are the rotation matrices consisting of α,β,ϒ rotation angles around X, Y, Z axis respectively, that formulated as follows

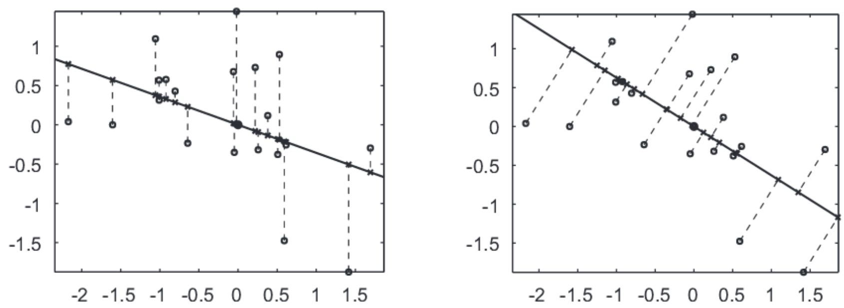
$$R_X(\propto) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\propto & \sin\propto \\ 0 & -\sin\propto & \cos\propto \end{bmatrix},$$

$$R_Y(\beta) = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}, \quad R_Z(\gamma) = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

By considering that the rotation parameters and scale are very small, Eq. (1) can be rewritten as Eq. (2)

$$\begin{bmatrix} X_i^` \\ Y_i^` \\ Z_i^` \end{bmatrix}_{D2} = \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix}_{D1} + \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + S.\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix}_{D1} + \begin{bmatrix} 0 & R_Z & -R_Y \\ -R_Z & 0 & R_X \\ R_Y & -R_X & 0 \end{bmatrix} . \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix}_{D1} \quad (2)$$

Solving the transformation parameters requires common points in both systems. Since the number of observations is larger than the number of unknowns; therefore, this redundancy obliges using the adjustment process. In order to estimate the best fitting of the transformation parameters, it is proposed both Least Squares and Total Least Squares techniques. The least squares theory supposed to minimize the sum of squares of the vertical distances of the data points to the regression line, while the total least squares assumes to minimize the sum of squares of the orthogonal distances, Fig. (1), [3].



(a) Standard (Vertical) Regression (b) Orthogonal Regression

Figure 1: Least Squares Vs. Total Least Squares

In case of Least Squares (LS), the LS error equation which derived from Eq. (2) can be rewritten as follows [1]

$$\begin{bmatrix} v_{X_i} \\ v_{Y_i} \\ v_{Z_i} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & -Z_i & Y_i & X_i \\ 0 & 1 & 0 & Z_i & 0 & -X_i & Y_i \\ 0 & 0 & 1 & -Y_i & X_i & 0 & Z_i \end{bmatrix} . [\Delta X \quad \Delta Y \quad \Delta Z \quad R_X \quad R_Y \quad R_Z \quad S]^T - \begin{bmatrix} W_{X_i} \\ W_{Y_i} \\ W_{Z_i} \end{bmatrix} \quad (3)$$

Where $\begin{bmatrix} W_{X_i} \\ W_{Y_i} \\ W_{Z_i} \end{bmatrix} = \begin{bmatrix} X_i^` \\ Y_i^` \\ Z_i^` \end{bmatrix}_{D2} - \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix}_{D1}$ (4)

Eq. (3) can be expressed shortly as Eq. (5)

$$v = AX - W \quad (5)$$

In case of weighted Least Square (WLS), the least square estimator of the parameters vector will be calculated according to Eq.(6), [16]

$$X = (A^T PA)^{-1} * (A^T PW) \quad (6)$$

Where P is the weight matrix, since the correlated weight matrix (variance-covariance matrix) is complicated, we designed a diagonal matrix that determined by the variance of W_{X_i} , W_{Y_i}, W_{Z_i}.

Also, the variance component of the Least squares D is computed as Eq. (6), [17]

$$D = (A^T PA)^{-1} \quad (7)$$

However, in the case of Total Least Squares (TLS) which assumes that all the elements of the data are erroneous, the equation will be as follows: [18]

$$(A + \Delta E)X = W + \Delta W \qquad (Rank(A) = m < n) \quad (8)$$

$$\| [A; W] - [\widehat{A}; \widehat{W}] \|_F \qquad [\hat{A} ; \widehat{W}] \in R\, n(m + 1) \quad (9)$$

Where ΔW is the error vector of observations, ΔE is the error matrix of design matrix A, m is the number of unknowns and n is the number of observations. From Eq.(9), $\| \; \|_F$ denotes the Frobenius norm. The basic TLS problem can be solved using The Singular Value Decomposition (SVD). The SVD of the augmented matrix [A; W] can be computed according to Eq.(10), [19]

$$[A; W] = U\Sigma V^T \quad (10)$$

Similarly to Least Squares, the Weighted Total Least Squares (WTLS) SVD of the augmented matrix is defined as follows, [1]

$$[A^T P^{-1} A; A^T P\, W] = U\Sigma V^T \quad (11)$$

Where,

U = [u1,1 , …,u1,n , ….un,1,….un,n] ϵ Rnxn ,

V = [v1,1 ,….,v1,m+1, …. Vm,m+1 ,…., vm+1,m+1] ϵ R (m+1)(m+1) and

Σ = [σ1,1 , …,σ1,m+1, ….σm+1,m+1, …., σn,1,…., σn,m+1] ϵ R n(m+1) matrix with diagonal elements. So, the matrix Σ will be as

Σ = diag (σ1, … σm, σm+1)

The solution of TLS is obtained after the rank reduced from (m+1) to (m) for Eq. (9), [20]

$$X = \frac{-1}{V_{m+1,m+1}} V_{m+1} \tag{12}$$

The biased-corrected variance component estimator is computed as in Eq.(13)

$$\sigma_0^2 = \frac{(W-AX)^T P (W-AX)}{r} \tag{13}$$

Where r denotes the redundancy and is equal to (m-n). The formula of approximately computing the mean standard error (D) that represents the unbiased variance-covariance matrix of TLS derived as in Eq. (14), [17]

$$D = \sigma_0^2 (A^T P^{-1} A)^{-1} \tag{14}$$

WTLS algorithm is a linear regression where the target is to determine the best fitting line to 23 and 7 observed and check points. But the coordinates of points have not been measured with the same precision, [2].

Theoretically, the TLS technique exerts higher reliability as it considers both sides of the noise contamination problem – in terms of design matrix and observation vector – whereas the LS method considers only the observation vector (neglecting design matrix).

One more important issue that affects the accuracy of the results is the data precision, that a general multiple linear regression with fuzzy technique is proposed based on each of the Least Squares and Total Least Squares in both weighted and equal weight approaches.

The advantage of this technique is to detect the outlier points that are outside the usual range, and reduce their effects. On the other hand, there are some limitations that must be addressed when applying fuzzy linear regression such as using small data, which is inadequate, or when the outliers' data set is large and misleads the trend of the fuzzy regression, or if there is imprecision between the dependent and independent variables, or if there is distortion introduced by linearization [21].

Fuzzy linear regression takes the following form: [22]

$$Y_i = B_0 + B_1 Z_{i1} + B_2 Z_{i2} + \cdots + B_m Z_{in} \tag{15}$$

Where $Z_{ij} \in R$, Y_i is the fuzzy response, B_o is a fuzzy intercept, $B_1, B_2, \ldots\ldots\ldots B_m$ are fuzzy coefficients.

The rule of determining regression parameters is to calculate the root mean square errors until it reach the minimum, where the error is the resultant of residuals in X, Y, and Z coordinates. This will be applied to both the least squares and total least squares methods [23].

Accordingly, an iterative algorithm that uses the initial weight for the above mentioned approaches is applied. This iteration method is applied for (T) times, at every time the weight is re-evaluated based on multiple fuzzy linear regression and stopped when Root Mean Squares Errors (RMSE) is minimized.

In this paper, the transformation parameters using Bursa-Wolf model are calculated based on Least Squares adjustments. The solutions are designed to examine the performance of least squares and total least squares for both weighted and equal weight as well as the efficiency of fuzzy multiple linear regression on the different solutions. The modeling of these solutions is implemented using MATLAB. Fig. (2) Represents the conceptual applied methodology.

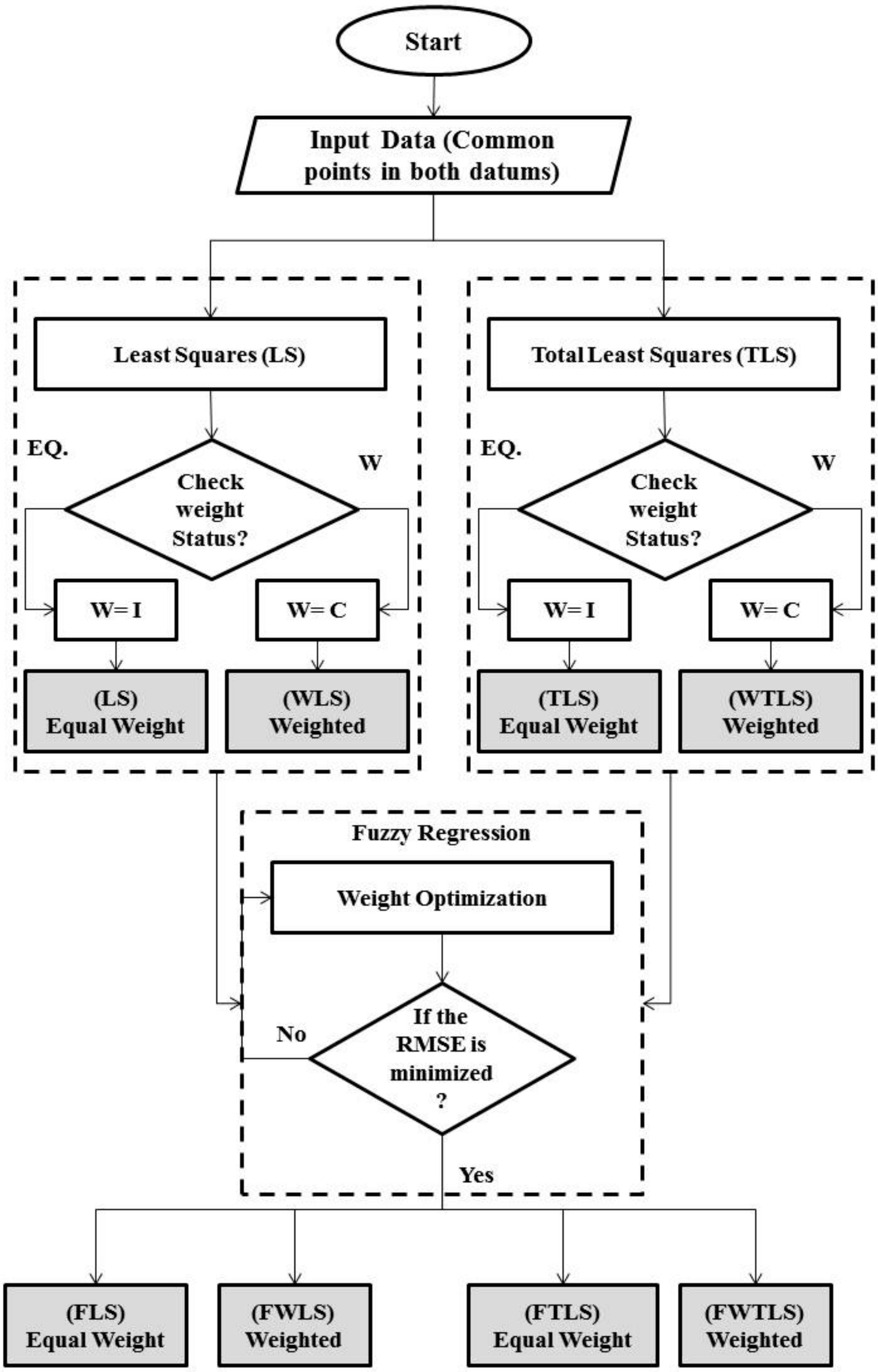


Figure 2: Conceptual Methodology

The data used in this research are the local geodetic coordinates of first order triangulation stations known in the

Egyptian datum (Helmert 1906), where 23 stations belong to Network I and the other 7 stations belong to Network II. The precision of the stations of Network I is taken 1:100,000 and that the stations of Network II is taken 1: 50,000 based on an error analysis applied in [24]. The global geodetic coordinates of the above mentioned 30 stations defined in WGS-84, where 16 stations belong to the Egyptian High Accurate Reference Network (HARN) with precision 1:10,000,000 [25]. Another 3 stations belong to the Egyptian Aviation project with precision 1: 7000,000, [26]. The other 11 stations belong to National Agricultural Cadastral Network (NACN) with precision 1:1000, 000, [27]. To achieve the target of the research, the coordinates of the above mentioned 30 stations defined in the local Egyptian datum (D1) and global WGS-84 (D2) are used in the solutions, where twenty three (23) represents the solution points and the rest seven (7) points are the check points as shown in Fig.(3).

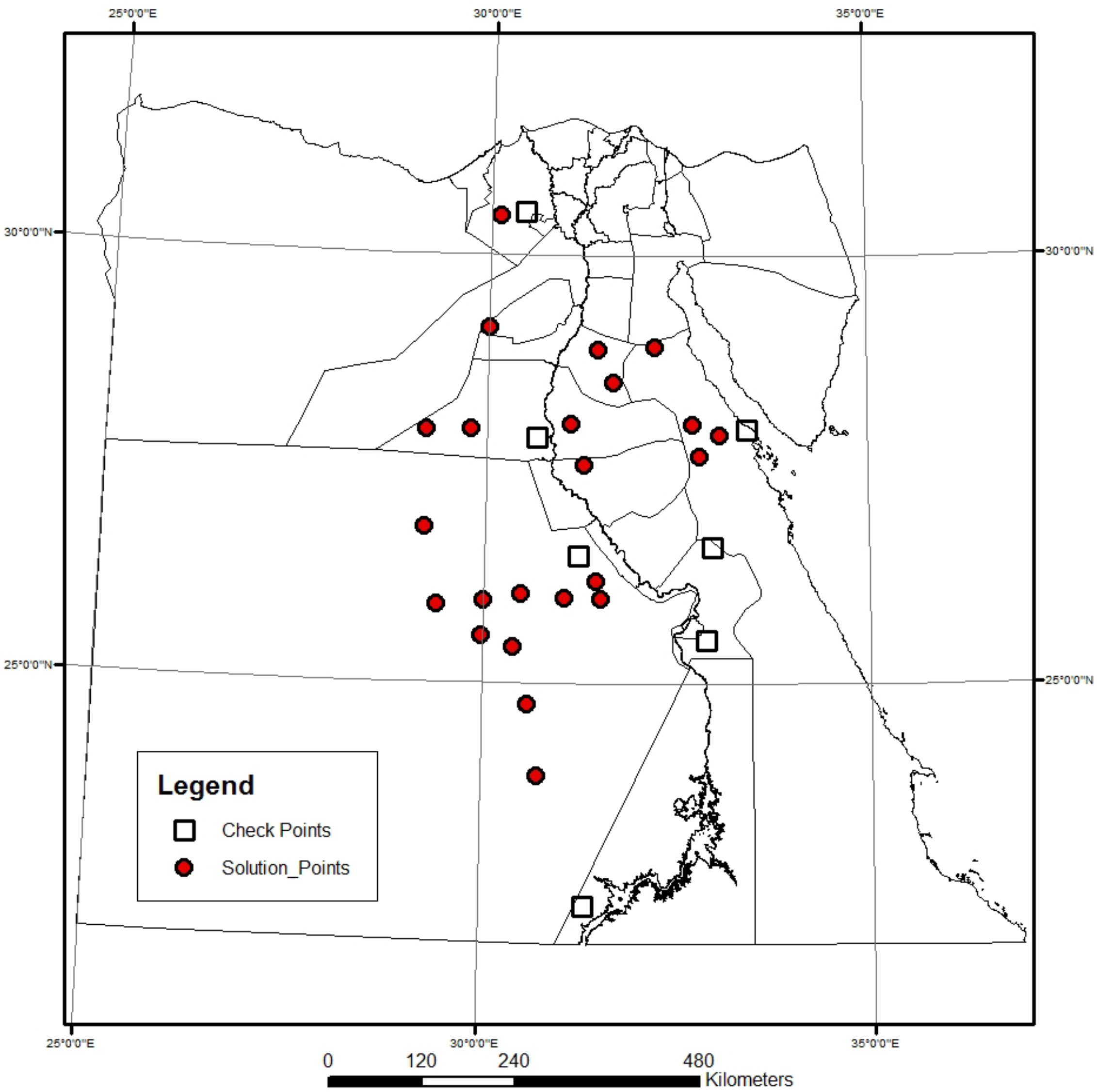


Figure 3: Map of Egypt representing the solution points and check points in the study area

The comparison between eight hybrids scenarios were applied with different solutions, where the seven transformation parameters are computed using 23 stations, and by using these parameters the residuals of the solution stations and the 7 check points are calculated

These scenarios are dividing into 4 solutions that described as follows:

- Solution 1: Assessment of Least Squares (LS) versus Total Least Square (TLS), this is applied for equal weight mode and linear regression modeling.
- Solution 2: Checking the performance of Weighted Least Squares (WLS) versus Weighted Total Least Square (WTLS), here in the weight influence exist and similar to solution 1; the linear regression modeling is applied.
- Solution 3: Same as solution 1; however, the fuzzy multiple linear regression is applied instead of linear regression.
- Solution 4: Similar to solution 2, the fuzzy multiple linear regression is applied instead of linear regression.

The assessment of the accuracy is based on calculating Errors (E), Mean, and root mean square error (RMSE) of the results using Eq. (15), Eq. (16), and Eq. (17).

$$E_i = \sqrt{\Delta X_i{}^2 + \Delta Y_i{}^2 + \Delta Z_i{}^2} \quad (15)$$

Where $\Delta X_i, \Delta Y_i, \Delta Z$ are the residuals (the difference between the given coordinates and the computed one) in X, Y and Z coordinates respectively.

$$\text{Mean} = \sum_{i=1}^{N} \frac{E_i}{N} \quad (16)$$

Where N is the number of stations.

$$\text{RMSE} = \sqrt{\sum_{i=1}^{N} \frac{E_i^2}{N}} \quad (17)$$

3. Results and Analysis

The proposed methodology is applied on the above mentioned solutions. The seven transformation parameters are calculated in each case. The residuals are computed at the solution points as well as the check points.

The mean of absolute residuals and root mean square error are computed for the different solutions. The results of the solutions are tabulated and discussed below. The obtained seven transformation parameters in each case of the 4 solutions are presented in Table (1).

Table 1: Seven transformation parameters

	Solution 1		Solution 2		Solution 3		Solution 4	
	LS	**TLS**	**WLS**	**WTLS**	**FLS**	**FTLS**	**FWLS**	**FWTLS**
ΔX (m)	-104.81 ± 8.65	-178.17 ± 8.65	-89.22 ± 11.03	-105.88 ± 11.03	-99.89 ± 16.28	-92.61 ± 16.69	-95.17 ± 29.32	-94.51 ±19.67
ΔY (m)	-11.90 ± 9.81	120.45 ± 9.81	-38.77 ± 12.00	-1.79 ± 12.00	-14.08 ± 17.67	-44.51 ±18.07	-22.09 ± 36.14	-16.84 ±26.29
ΔZ (m)	-6.20 ± 6.47	61.25 ± 6.47	-4.52 ± 8.54	-11.95 ± 8.54	-14.30 ± 12.41	16.79 ±13.19	-14.67 ± 17.87	18.73 ±11.22
R_X **(sec)**	1.26 ± 0.17	0.06 ± 0.17	1.22 ± 0.22	1.19 ± 0.22	1.28 ± 0.31	1.11 ±0.34	1.28 ± 0.49	0.68 ±0.33
R_Y **(sec)**	-3.90 ± 0.35	0.26 ± 0.35	-4.60 ± 0.43	-3.44 ± 0.43	-4.06 ± 0.64	-5.08 ±0.65	-4.30 ± 1.28	-4.30 ±0.91
R_Z**(sec)**	-0.34 ± 0.25	-3.29 ± 0.25	0.03 ± 0.33	-0.04 ± 0.33	-0.07 ± 0.49	-0.91 ±0.51	0.04 ± 0.77	-0.89 ±0.47
S (ppm)	5.31 ± 0.94	0.94 ±0.61	8.83 ± 1.15	5.65 ± 1.15	4.86 ± 1.67	7.91 ±1.79	5.61 ± 2.98	6.72 ±2.20

It is obvious that the three translation parameters of solutions 1, 2 are totally different in both cases of LS and TLS; however, the other four parameters for the same solutions are not much different. On the other hand, all the seven parameters in solutions 3, 4 are close to each other in both cases.

The residuals of the solution points of the resultant (E) in meter for each case of the 4 solutions are computed. Moreover, the mean error and RMSE are also calculated. Table (2) illustrates the values of the results.

Within the different applied solutions techniques, it is noticeable that the values of the residuals in total least squares are less than their corresponding values in least squares techniques.

Regarding the linear regression (Solution 1, 2), the WTLS gives relatively the best results. On the other hand, the fuzzy multiple linear regression improved the results of total least squares cases.

FTLS gives the minimum RMSE (4.59 m) and this indicates that fuzzy multiple linear regressions can substitute the weight in both least squares and total least squares solutions. However, it is slightly more suitable in case of TLS as it minimizes the overall residuals of the solution points. Fig. (4), (5) show the residuals of the solution points for linear regression and fuzzy multiple linear regression respectively that confirms the above mentioned analysis.

Table 2: Residuals of the resultant at solution points (m)

No.	**Solution 1**		**Solution 2**		**Solution 3**		**Solution 4**	
	LS	**TLS**	**WLS**	**WTLS**	**FLS**	**FTLS**	**FWLS**	**FWTLS**
1	7.72	2.74	7.73	5.45	8.51	5.50	7.78	5.61
2	5.55	4.30	5.42	2.36	5.24	1.14	5.26	1.41
3	7.93	4.79	7.88	4.52	6.92	1.85	7.62	2.84
4	6.86	4.10	6.77	3.56	6.00	1.65	6.52	2.36
5	2.69	1.43	2.56	2.06	1.41	0.96	2.30	1.71
6	13.31	9.68	13.44	9.93	12.02	7.78	13.26	9.64
7	5.11	3.97	4.96	2.42	4.73	1.09	4.79	1.39
8	14.17	14.07	14.49	10.54	14.15	9.49	14.55	11.78
9	2.42	1.46	2.40	2.35	1.59	1.45	2.29	2.16
10	11.33	12.71	11.44	3.84	10.56	7.41	11.20	5.20
11	9.41	2.65	9.57	9.31	10.47	7.48	9.70	9.00
12	8.14	4.60	8.06	2.42	9.14	5.34	8.29	2.88
13	3.11	1.92	3.14	3.90	4.48	3.72	3.33	4.04
14	2.62	2.79	2.67	1.56	1.73	1.47	2.50	2.08
15	3.97	3.09	4.09	4.88	5.48	4.04	4.36	4.87
16	6.74	7.55	6.74	4.70	8.10	5.44	7.02	5.29
17	5.41	4.26	5.27	2.72	4.99	1.08	5.09	1.66
18	8.34	4.67	8.26	2.47	9.34	5.45	8.49	2.93
19	6.43	2.44	6.32	1.56	7.30	4.80	6.54	1.42
20	2.37	1.62	2.47	1.10	2.81	1.24	2.61	1.33
21	6.22	8.33	6.02	1.93	7.61	4.64	6.30	1.31
22	3.30	1.90	3.14	1.30	2.77	0.73	2.92	0.37
23	4.42	2.35	4.35	1.17	5.27	3.50	4.56	0.41
Mean	**6.42**	**4.67**	**6.40**	**3.74**	**6.55**	**3.79**	**6.40**	**3.55**
RMSE	**7.20**	**5.79**	**7.22**	**4.61**	**7.36**	**4.59**	**7.22**	**4.64**

Similarly, Table (3) illustrates the values of the resultant residuals of the check points in meter for each case of the 4 solutions. Moreover, the mean error and RMSE are also presented.

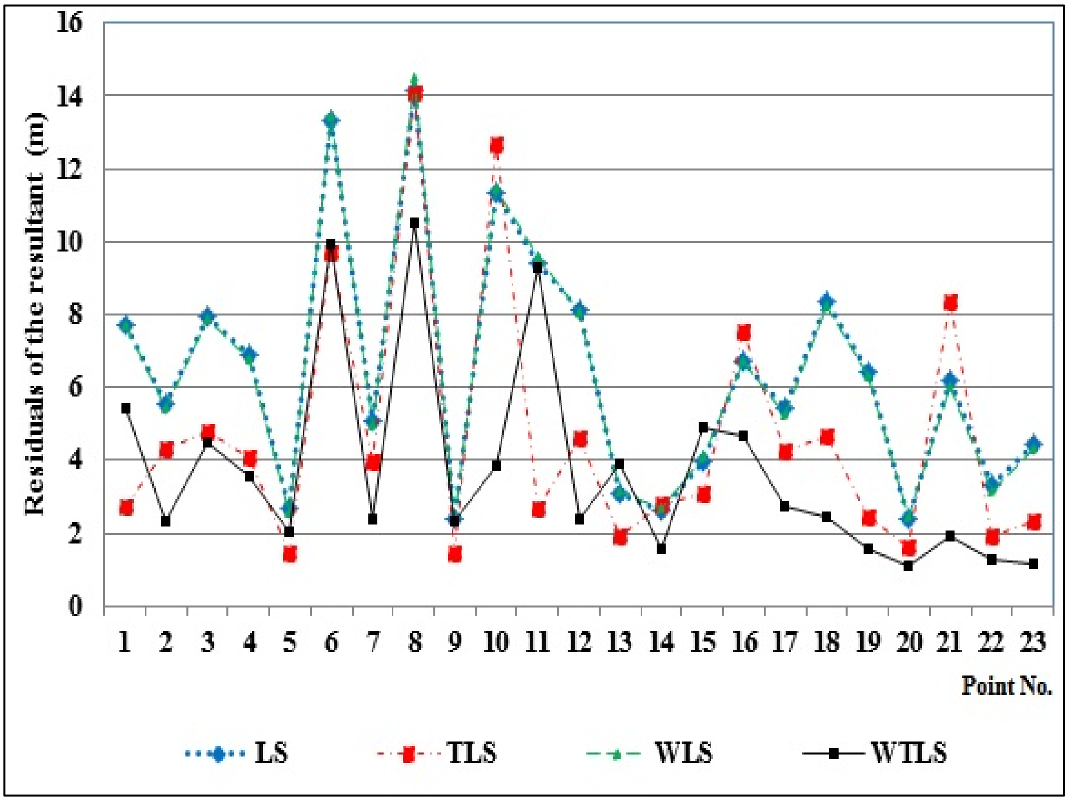


Figure 4: Computational accuracy comparison of LS, TLS, WLS, and WTLS solutions at solution points

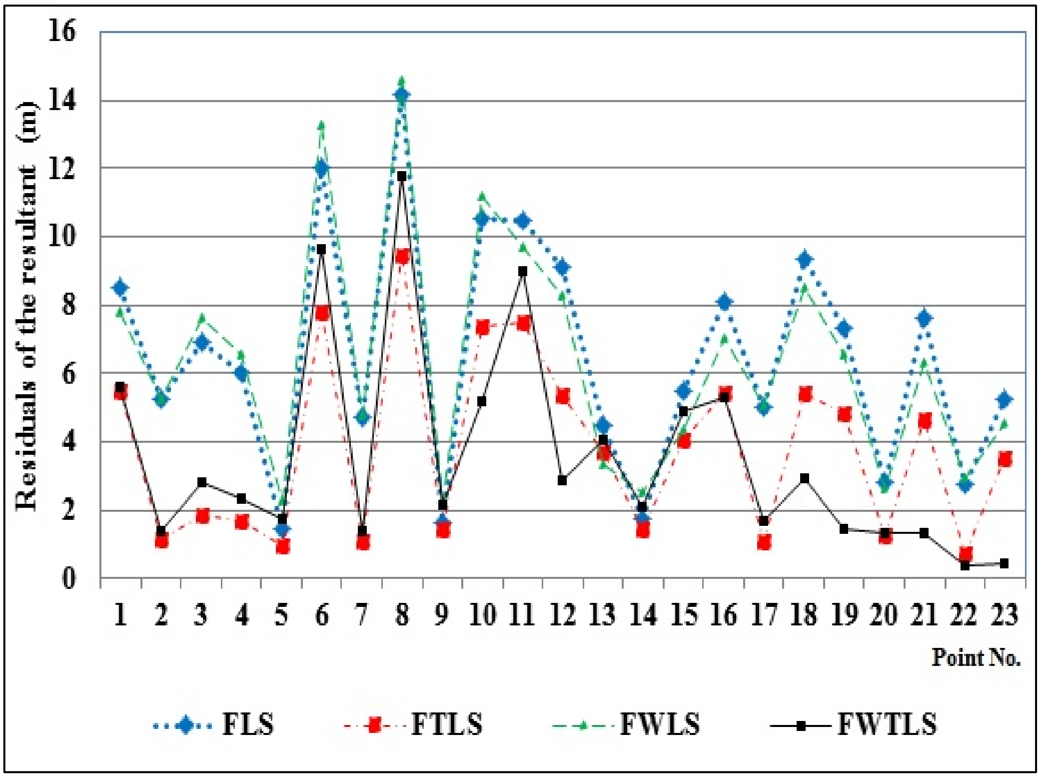


Figure 5: Computational accuracy comparison of FLS, FTLS, FWLS, and FWTLS solutions at solution points

Table 3: Residuals at check points (m)

	Solution 1		Solution 2		Solution 3		Solution 4	
	LS	**TLS**	**WLS**	**WTLS**	**FLS**	**FTLS**	**FWLS**	**FWTLS**
1	3.15	8.25	3.08	3.32	3.86	3.32	3.26	3.19
2	6.94	3.87	6.88	2.32	6.02	4.71	7.13	2.79
3	5.54	8.95	5.27	4.02	5.19	2.61	5.12	3.46
4	5.44	3.25	5.30	2.72	4.71	0.90	5.06	1.58
5	4.10	11.90	3.91	8.44	3.38	9.38	3.77	7.56
6	2.43	2.02	2.48	1.72	1.59	1.22	2.34	1.99
7	2.66	2.68	2.61	0.85	3.31	2.14	2.79	0.60
Mean	**4.32**	**5.85**	**4.22**	**3.34**	**4.01**	**3.47**	**4.21**	**3.02**
RMSE	**4.60**	**6.83**	**4.49**	**4.05**	**4.23**	**4.39**	**4.49**	**3.66**

On the contrary to the trend of solution points, TLS is worse than LS. The best results comes from the FWTLS; however, FTLS results still in range and accepted that confirm the same trend of solution points that fuzzy multiple linear regression can be an alternative to the weight influence. Figure (6), (7) represent the check points, the residuals for linear regression and fuzzy multiple linear regression respectively.

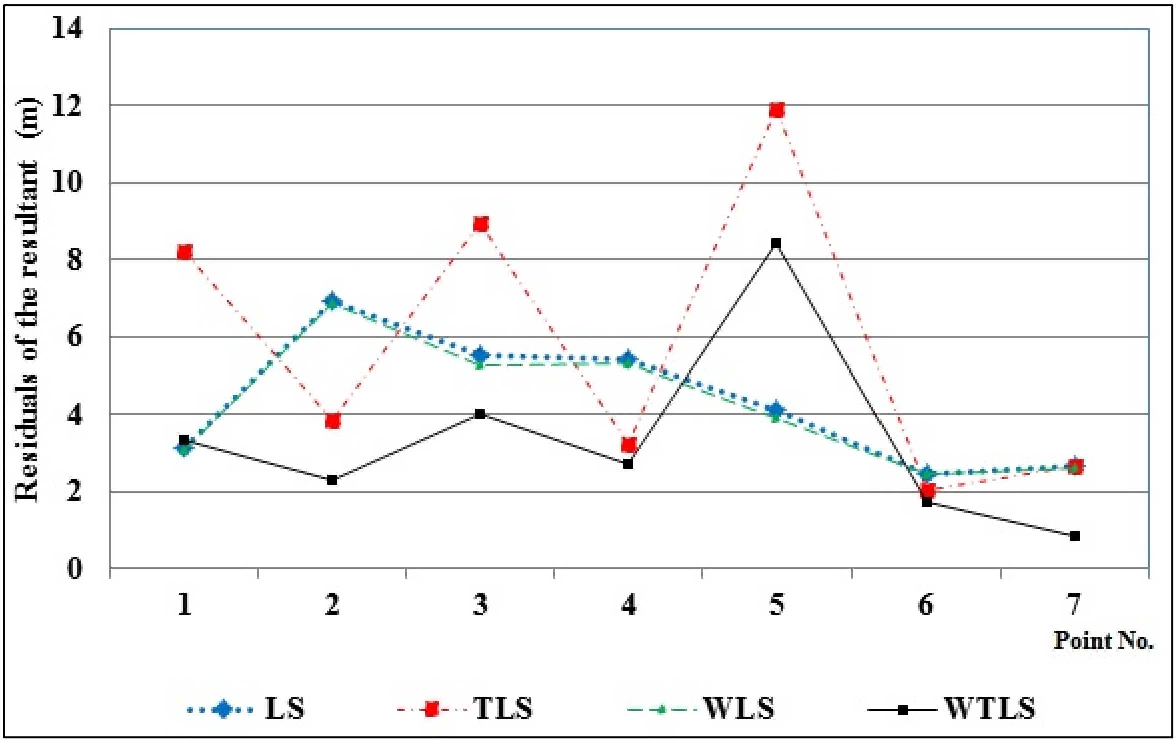


Figure 6: Computational accuracy comparison of LS, TLS, WLS, and WTLS solutions at check points

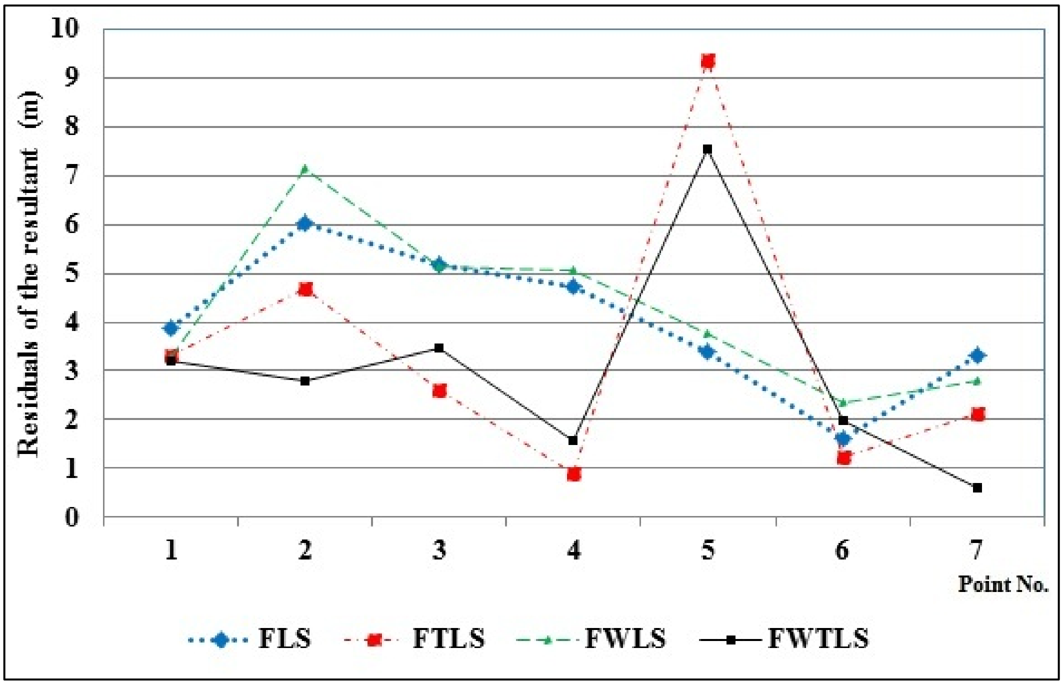


Figure 7: Computational accuracy comparison of FLS, FTLS, FWLS, and FWTLS solutions at check points

4. Conclusions and Recommendations

This research applied different regression models on estimating the geodetic transformation parameters. Four different solutions are designed to examine the performance of least squares and total least squares for both weighted and equal weight as well as the efficiency of fuzzy linear regression. Through the results of the applied solutions, the following remarks can be concluded:

- The total least squares approach is a suitable approach for solving many surveying engineering problems where it assumes the presence of the noise in both coefficient matrix and observations vector.
- The weighted total least squares gives results better than weighted least squares; however, applying the total least squares without weight is not usually better than its corresponding of least squares.
- The fuzzy linear regression improved the accuracy of the TLS results and it can eliminate the need for accurate weights/confidence; however, using large outliers' data set can mislead the precision of the fuzzy linear regression.

Finally, it is recommended to apply further studies for the data density and distribution, to test multiple methods of solving total least squares instead of SVD, to assess other methods for solving geodetic transformation such as Moldensky, and to evaluate the supplementary investigation on other surveying problems.

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