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Soft Pre-Open Sets In Soft Bitopological Spaces

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Abstract

In this work , I introduce the concept of soft bitopological space on a soft set and some definitions on soft pre-open set on soft bitopological space . Also introduce soft pre separation axioms , Spre-$T_\circ$, Spre-T_1 and Spre-T_2 , with study some properties in soft bitopological space.

Keywords: Soft set; Soft pre-open; Soft topology; Soft bitopological spaces.

1. Introduction

Many classical methods have been used to solve some complicated problems in engineering economics and environment. For instance, the interval mathematics, theory of fuzzy, theory of probability, and sets which can consider as mathematical tools for dealing with uncertainties since all these theories have their own problems and difficulties. In [2], the author in 1999 introduced the notion of soft set, which is free of difficulties in solving aforementioned problems, and it has been applied over many different fields. In 2011, Naim Cagman and his colleagues introduced a new concept of soft set called soft topology define by using the soft power set of soft set , and this first idea to soft mathematical concepts and structures that are based on the operations of theoretic soft set [6]. In 2011[7], the authors defined the concept of soft topology on the collection of soft sets over with some basic notations of soft topological spaces. In [6], the notion of soft topology was more general than that in [7]. Therefore, algebraists continue investigating the work of Cagman [6] and follow their notations and mathematical formalism. In 2013 J. Subhashini and C. Sekar defined soft pre-open sets [4] by following Cagman's theory of soft topology. Therefore, this paper has introduced soft bitopological space relying on [6] and defined the soft pre-open set of soft bitopological space. Also, I have discussed soft pre separation axioms , Spre-$T_\circ$, Spre-T_1 and Spre-T_2 .

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2. Preliminaries

Through this section , I give and introduction some important definitions and facts about soft topology and recall primary definition soft set that need in this work .

2.1. Definition [2]

The set of ordered pairs $F_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\}$ represents a soft set F_A on the universe U , where $f_A : A \to P(U)$ is mapping and f_A is called an approximate function of the soft set F_A . However , the set of all soft sets over U is denoted by $S_S(U)$.

2.2. Definition [5]

Let $F_A \in S_S(U)$. If $f_A(x) = \phi$ for all $x \in A$, then F_A is called an empty set and denoted by F_ϕ . Moreover , If $f_A = U$ for all $x \in A$, then F_A is called an A-universal soft set and denoted by $F_{\hat{A}}$. But , If $A = E$, then A-universal soft set is called universal soft set and denoted by $F_{\tilde{E}}$.

2.3. Definition [5]

Let $F_A, F_B \in S_S(U)$. If $f_A(x) \subseteq f_B(x)$ for all x , then , F_A is a soft subset of F_B and denoted by $F_A \tilde{\subseteq} F_B$.

2.4. Definition [5]

Let $F_A, F_B \in S_S(U)$. Then, the soft union is denoted by $F_A \tilde{\cup} F_B$, however , the soft intersection is denoted by $F_A \tilde{\cap} F_B$ Also , the soft difference of F_A and F_B is denoted by $F_A \tilde{\Delta} F_B$, are defined by the approximate functions $f_{A\tilde{\cup}B}(x) = f_A(x) \cup f_B(x)$, $f_{A\tilde{\cap}B}(x) = f_A(x) \cap f_B(x)$, $f_{A\tilde{\Delta}B}(x) = f_A(x) \Delta f_B(x)$, respectively, on the onther hand , the soft complement $F_A^{\tilde{c}}$ of F_A is defined by the approximate function $f_{A^c}(x) = f_A^c(x)$, where $f_A^c(x)$ is the complement of the set $f_A(x)$; that is, $f_A^c(x) = U - f_A(x)$ for all $x \in A$. It is easy to see that $(F_A^{\tilde{c}})^{\tilde{c}} = F_A$ and $F_\phi^{\tilde{c}} = F_{\tilde{E}}$.

2.5. Proposition [5]

Let $F_A, F_B, F_C \in S_S(U)$. Then ,

1-$F_A \tilde{\cup} F_A = F_A, F_A \tilde{\cap} F_A = F_A$.

2-$F_A \tilde{\cup} F_\phi = F_A, F_A \tilde{\cap} F_\phi = F_\phi$.

3-$F_A \tilde{\cup} F_{\tilde{E}} = F_{\tilde{E}}, F_A \tilde{\cap} F_{\tilde{E}} = F_A$.

4-$F_A \tilde{\cup} F_A^{\tilde{c}} = F_{\tilde{E}}, F_A \tilde{\cap} F_A^{\tilde{c}} = F_\phi$.

5-$F_A \tilde{\cup} F_B = F_B \tilde{\cup} F_A, F_A \tilde{\cap} F_B = F_B \tilde{\cap} F_A$.

6-$(F_A \tilde{\cup} F_B)^{\tilde{c}} = F_B^{\tilde{c}} \tilde{\cap} F_A^{\tilde{c}}, (F_A \tilde{\cap} F_B)^{\tilde{c}} = F_B^{\tilde{c}} \tilde{\cup} F_A^{\tilde{c}}$.

7-$(F_A \tilde{\cup} F_B) \tilde{\cup} F_C = F_A \tilde{\cup} (F_B \tilde{\cup} F_C), (F_A \tilde{\cap} F_B) \tilde{\cap} F_C = F_A \tilde{\cap} (F_B \tilde{\cap} F_C)$.

8- $F_A \tilde{\cup} (F_B \tilde{\cap} F_C) = (F_A \tilde{\cup} F_B) \tilde{\cap} (F_A \tilde{\cup} F_C)$ & $F_A \tilde{\cap} (F_B \tilde{\cup} F_C) = (F_A \tilde{\cap} F_B) \tilde{\cup} (F_A \tilde{\cap} F_C)$.

2.6. Definition [6]

Let $F_A \in S_S(U)$. The soft power set of F_A is defined by . $\tilde{P}(F_A) = \{F_{A_i} : F_{A_i} \tilde{\subseteq} F_A, i \in I \subseteq N\}$ and its cardinality is defined by $|\tilde{P}(F_A)| = 2^{\sum_{x \in E} |f_A(x)|}$, where $|f_A(x)|$ is the cardinality of $f_A(x)$.

2.7. Definition [6]

Let $F_A \in S_S(U)$. A soft topology on F_A, denoted by $\tilde{\tau}$, is a collection of soft subsets of F_A having the following properties:

- $F_\phi, F_A \in \tilde{\tau}$.
- $\{F_{A_i} \tilde{\subseteq} F_A, i \in I \subseteq N\} \subseteq \tilde{\tau} \Rightarrow \bigcup_{i \in I} F_{A_i} \in \tilde{\tau}\}$.
- $\{F_{A_i} \tilde{\subseteq} F_A, 1 \le i \le n, n \in N\} \subseteq \tilde{\tau} \Rightarrow \bigcap_{i=1}^{n} F_{A_i} \in \tilde{\tau}\}$.

2.8. Definition [6]

Let $(F_A, \tilde{\tau})$ be a soft space on F_A . Every element of $\tilde{\tau}$ is called a soft open sets .

2.9. Definition [6]

Let $(F_A, \tilde{\tau})$ be a soft space on F_A and $F_B \tilde{\subseteq} F_A$. Then , the collection

$\tilde{\tau}_{F_B} = \{F_{A_i} \tilde{\cap} F_B : F_{A_i} \in \tilde{\tau}, i \in I \subseteq N\}$ is called a soft subspace topology on F_B. Hence, $(F_B, \tilde{\tau}_{F_B})$ is called a soft topological subspace of $(F_A, \tilde{\tau})$.

2.10. Definition [6]

Let $(F_A, \tilde{\tau})$ be a soft space on F_A and $F_B \tilde{\subseteq} F_A$. The soft interior of F_B , denoted F_B° , is defined as the union of all soft open subsets of F_B. Note that F_B° is the biggest soft open set that is contained by F_B .

2.11. Theorem [6]

Let $(F_A, \tilde{\tau})$ be a soft space on F_A and $F_B, F_C \tilde{\subseteq} F_A$. Then ,

1- $(F_B^{\circ})^{\circ} = F_B^{\circ}$

2- $F_B \tilde{\subseteq} F_C$. Then , $F_B^{\circ} \tilde{\subseteq} F_C^{\circ}$

3- $F_B^{\circ} \tilde{\cap} F_C^{\circ} = (F_B \tilde{\cap} F_C)^{\circ}$

4- $F_B^{\circ} \tilde{\cup} F_C^{\circ} \tilde{\subseteq} (F_B \tilde{\cup} F_C)^{\circ}$

2.12. Definition [6]

Let $(F_A, \tilde{\tau})$ be a soft space on F_A and $F_B \tilde{\subseteq} F_A$. Then, the soft closure of F_B , denoted $\overline{F}_B$ is defined as the soft intersection of all soft closed superset of F_B. Note that $\overline{F}_B$ is the smallest soft closed set that containing F_B.

2.13. Theorem [6]

Let $(F_A, \tilde{\tau})$ be a soft space on F_A and $F_B, F_C \tilde{\subseteq} F_A$. Then ,

1- $(\overline{\overline{F}}_B) = \overline{F}_B$.

2- $(\overline{F}_B)^{\tilde{c}} = (F_B^{\tilde{c}})^{\circ}$.

3- $F_B \tilde{\subseteq} F_C$. Then , $\overline{F}_B \tilde{\subseteq} \overline{F}_C$.

4-$\overline{(F_B \tilde{\cap} F_C)} \tilde{\subseteq} \overline{F}_B \tilde{\cap} \overline{F}_C$.

5-$\overline{F}_B \tilde{\cup} \overline{F}_C = \overline{(F_B \tilde{\cup} F_C)}$.

2.14. Theorem [6]

Let $(F_A, \tilde{\tau})$ be a soft space on F_A and $F_B \tilde{\subseteq} F_A$ Then $F_B^{\circ} \tilde{\subseteq} F_B \tilde{\subseteq} \overline{F_B}$.

2.15. Definition

Let $(F_A, \tilde{\tau}_1)$ and $(F_A, \tilde{\tau}_2)$ be the two different soft topologies on F_A. Then $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is called a soft bitopological space .

2.16. Example

Let $U = \{u_1, u_2, u_3, u_4\}$, $E = \{w_1, w_2, w_3\}$, $A = \{w_1, w_2\}$ such that $A \tilde{\subseteq} E$ and $F_A = \{(w_1, \{u_1, u_2\}), (w_2, \{u_3, u_4\})\}$ then

$F_{A_1} = \{(w_1, \{u_1\})\}$

$F_{A_2} = \{(w_1, \{u_2\})\}$

$F_{A_3} = \{(w_1, \{u_1, u_2\})\}$

$F_{A_4} = \{(w_2, \{u_3\})\}$

$F_{A_5} = \{(w_2, \{u_4\})\}$

$F_{A_6} = \{(w_2, \{u_3, u_4\})\}$

$F_{A_7} = \{(w_1, \{u_1\}), (w_2, \{u_3\})\}$

$F_{A_8} = \{(w_1, \{u_1\}), (w_2, \{u_4\})\}$

$F_{A_9} = \{(w_1, \{u_1\}), (w_2, \{u_3, u_4\})\}$

$F_{A_{10}} = \{(w_1, \{u_2\}), ((w_2, \{u_3\})\}$

$F_{A_{11}} = \{(w_1, \{u_2\}), (w_2, \{u_4\})\}$

$F_{A_{12}} = \{(w_1, \{u_2\}), (w_2, \{u_3, u_4\})\}$

$F_{A_{13}} = \{(w_1, \{u_1, u_2\}), (w_2, \{u_3\})\}$

$F_{A_{14}} = \{(w_1, \{u_1, u_2\}), (w_2, \{u_4\})\}$

$F_{A_{15}} = F_A$

$F_{A_{16}} = F_\phi$

Then $\tilde{\tau}_1 = \{F_\phi, F_A\}$ and $\tilde{\tau}_2 = \{F_\phi, F_A, F_{A_2}, F_{A_{11}}\}$ are a soft topology of F_A then $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$, is a soft bitopological space .

2.17. Definition

Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space over F_A and $F_B \tilde{\subseteq} F_A$. Then $\tilde{\tau}_1 F_B = \{F_{A_i} \tilde{\cap} F_B : F_{A_i} \in \tilde{\tau}_1, i \in I \subseteq N\}$ and $\tilde{\tau}_2 F_B = \{F_{A_i} \tilde{\cap} F_B : F_{A_i} \in \tilde{\tau}_2, i \in I \subseteq N\}$ are said to be the relative topologies on F_B . Then $(F_B, \tilde{\tau}_1 F_B, \tilde{\tau}_2 F_B)$ is called a relative soft bitopological space of $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$

2.18. Theorem

If $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is a soft bitopological space then $\tilde{\tau}_1 \tilde{\cap} \tilde{\tau}_2$ is a soft topological space over F_A.

Proof :-

- $F_\phi, F_A \in \tilde{\tau}_1 \tilde{\cap} \tilde{\tau}_2$.

- Let $\{F_{A_i}, i \in I\}$ be a family of soft sets in $\tilde{\tau}_1 \tilde{\cap} \tilde{\tau}_2 \Rightarrow F_{A_i} \in \tilde{\tau}_1$ and $F_{A_i} \in \tilde{\tau}_2$ for all $i \in I$. Therefore $\bigcup_{i \in I} F_{A_i} \in \tilde{\tau}_1$ and $\bigcup_{i \in I} F_{A_i} \in \tilde{\tau}_2$. Thus $\bigcup_{i \in I} F_{A_i} \in \tilde{\tau}_1 \tilde{\cap} \tilde{\tau}_2$.

- Let $F_{A_i} \in \tilde{\tau}_1 \tilde{\cap} \tilde{\tau}_2, 1 \le i \le n', n' \in N$. Then $F_{A_i} \in \tilde{\tau}_1$ and $F_{A_i} \in \tilde{\tau}_2, 1 \le i \le n', n' \in N$. Since $\bigcap_{i=1}^{n'} F_{A_i} \in \tilde{\tau}_1$ and $\bigcap_{i=1}^{n'} F_{A_i} \in \tilde{\tau}_2$. Therefore $\bigcap_{i=1}^{n'} F_{A_i} \in \tilde{\tau}_1 \tilde{\cap} \tilde{\tau}_2, 1 \le i \le n', n' \in N$.

2.19. Remark

If $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is a soft bitopological space then $\tilde{\tau}_1 \tilde{\cup} \tilde{\tau}_2$ is not a soft topological space over F_A.

2.20. Example

Let us consider 2.16 and let $\tilde{\tau}_1 = \{F_\phi, F_A, F_{A_1}\}$ and $\tilde{\tau}_2 = \{F_\phi, F_A, F_{A_2}\}$ are soft topology of F_A then $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is soft bitopological space . Now $\tilde{\tau}_1 \tilde{\cup} \tilde{\tau}_2 = \{F_\phi, F_A, F_{A_1}, F_{A_2}\}$. If take F_{A_1}, F_{A_2} , $F_{A_1} \tilde{\cup} F_{A_2} = \{(x_1, \{u_1\}), (x_1, \{u_2\})\} \notin \tilde{\tau}_1 \tilde{\cup} \tilde{\tau}_2$. Thus $\tilde{\tau}_1 \tilde{\cup} \tilde{\tau}_2$ is not soft topology on F_A.

3. Some Definition of Soft Pre-open set in soft bitopological space

In this section introduce some definitions of soft pre-open set , soft pre-closed , soft pre-neighborhood , soft pre-closure and soft pre-interior on soft bitopological space .

3.1. Definition

Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ be soft bitopological space and let $F_B \tilde{\subseteq} F_A$, F_B is called soft pre-open set with respect to the two soft topological spaces $\tilde{\tau}_1$ and $\tilde{\tau}_2$ if $F_B \tilde{\subseteq} (\overline{F}_B)^\circ$.

3.2. Notes

1- The set of all soft pre-open set with respect to the two soft topologies is denoted by Pre(F_A).

2- The relative soft bitopological space for F_B with respect to Soft pre-open sets is the collection Pre $(F_A)_{F_B}$ given by Pre $(F_A)_{F_B} = \{F_C \tilde{\cap} F_B : F_C \in Pre(F_A)\}$

3- Any $\tilde{\tau}_2$ -open soft set is not necessarily to be soft pre-open set .

4- Any soft pre-open set is not necessarily to be of $\tilde{\tau}_1$ -open ($\tilde{\tau}_2$ -open) soft set .

3.3. Example

Let us consider example 2.20 , $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is soft bitopological space . Take $F_{A_1} \tilde{\subseteq} F_A$ then $F_{A_1} \tilde{\subseteq} (\overline{F}_{A_1})^\circ \Rightarrow F_{A_1}$ is soft per-open set with respect to the two soft topological spaces $\tilde{\tau}_1$ and $\tilde{\tau}_2$.

3.4. Remarks

1- The intersection of any soft pre-open sets is not necessary a soft pre-open set

2- The union of any soft pre-open sets is soft pre-open set with respect to soft bitopological space .

The example to part (1) is simply . The following proof explain the part (2) of the remark 3.4 . Let F_B and F_C be a two soft pre-open sets with respect to soft bitopological space $(F_A,\tilde{\tau}_1,\tilde{\tau}_2)$. i.e. $F_B \tilde{\subseteq} (\overline{F}_B)^{\circ}$ and $F_C \tilde{\subseteq} (\overline{F}_C)^{\circ}$ with respect to the two soft topological spaces $\tilde{\tau}_1$ and $\tilde{\tau}_2 \Rightarrow F_B \tilde{\cup} F_C \tilde{\subseteq} (\overline{F}_B)^{\circ} \tilde{\cup} (\overline{F}_C)^{\circ} \tilde{\subseteq} (\overline{F}_B \tilde{\cup} \overline{F}_C)^{\circ}$. Since $\overline{F}_B \tilde{\cup} \overline{F}_C = (\overline{F_B \tilde{\cup} F_C})$ then $F_B \tilde{\cup} F_C \tilde{\subseteq} (\overline{F_B \tilde{\cup} F_C})^{\circ}$ with respect to the two soft topological spaces $\tilde{\tau}_1$ and $\tilde{\tau}_2$.

3.5. Definition

Let $(F_A,\tilde{\tau}_1,\tilde{\tau}_2)$ is soft bitopological space and let $F_B \tilde{\subseteq} F_A$, F_B is called soft pre-closed set of F_A if and only if $F_B^{\tilde{c}}$ is soft pre-open set of F_A .

3.6. Definition

Let $(F_A,\tilde{\tau}_1,\tilde{\tau}_2)$ is soft bitopological space and $\alpha \in F_A$, $F_B \tilde{\subseteq} F_A$ is said to be soft pre-neighborhood of a point α if there is a soft pre-open set F_C such that $\alpha \in F_C \tilde{\subseteq} F_B$. The set of all soft pre-neighborhoods of a point α is denoted by Spre-$\tilde{\upsilon}(\alpha)$.

3.7. Definition

Let $(F_A,\tilde{\tau}_1,\tilde{\tau}_2)$ is soft bitopological space , and $F_B \tilde{\subseteq} F_A$. A point $\alpha \in F_A$ is said to be soft pre-interior point of F_B with respect to the two soft topological spaces $\tilde{\tau}_1$ and $\tilde{\tau}_2$ if there is a soft pre-open F_C such that $\alpha \in F_C \tilde{\subseteq} F_B$. The set of all soft pre-interior points of F_B with respect to the two soft topological spaces $\tilde{\tau}_1$ and $\tilde{\tau}_2$ denoted by Spre-int(F_B).

3.8. Definition

Let $(F_A,\tilde{\tau}_1,\tilde{\tau}_2)$ is soft bitopological space . A point α is called soft pre-limit point of soft subset F_B of F_A with respect to the two soft topological spaces $\tilde{\tau}_1$ and $\tilde{\tau}_2$ if and only if for each a soft pre-open set F_C containing another point different from α in F_B , that is $(F_C/\{\alpha\}) \tilde{\cap} F_B \neq \phi$. The set of all soft pre-limit points of F_B be denoted by Spre-lm(F_B).

3.9. Definition

Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is soft bitopological space , and $F_B \tilde{\subseteq} F_A$, the intersection of all soft pre-closed sets containing F_B is called soft pre-closure of F_B , and is denoted by Spre-cl(F_B).

In the year 2014 J. Subhashinin and Dr. C.Sekar [3] by depending on the [6] and [4] introduces soft pre separation axioms , soft P$T_\circ$-space and some of its properties in the soft topological spaces . Now begin to important section to discuss soft pre separation axioms and some result .

4. The Separation Axioms in Soft Bitopological Space

In section four , I introduce some soft pre separation axioms , Spre-$T_\circ$, Spre-T_1 and Spre-T_2 and illustrate transmission this Properties to The relative soft bitopological space with some result of soft pre separation axioms .

4.1. Definition

Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is soft bitopological space , then $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is called Spre-$T_\circ$ space if and only if for all pair of soft point $\alpha_1, \alpha_2 \in F_A$ such that $\alpha_1 \neq \alpha_2$, there exists soft pre-open set F_B containing α_1 but not α_2 or soft pre-open set F_C containing α_2 but not α_1 .

4.2. Theorem

A soft bitopological space $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is Spre-$T_\circ$ space if and only if for each distinct soft points α_1, α_2 in F_A , Spre-cl($\{\alpha_1\}$) $\neq$ Spre-cl($\{\alpha_2\}$) .

Proof :-

Let $\alpha_1, \alpha_2 \in F_A$ such that $\alpha_1 \neq \alpha_2$ and Spre-cl($\{\alpha_1\}$) $\neq$ Spre-cl($\{\alpha_2\}$) .

Then there exists at least one soft point α_3 in F_A such that , $\alpha_3 \in$ Spre-cl($\{\alpha_1\}$) but $\alpha_3 \notin$ Spre-cl($\{\alpha_2\}$) . Suppose $\alpha_3 \in$ Spre-cl($\{\alpha_1\}$) , to show that $\alpha_1 \notin$ Spre-cl($\{\alpha_2\}$) . If $\alpha_1 \in$ Spre-cl($\{\alpha_2\}$) , then $\{\alpha_1\} \tilde{\subset}$ Spre-cl($\{\alpha_2\}$) . So Spre-cl($\{\alpha_1\}$) $\tilde{\subset}$ Spre-cl(Spre-cl($\{\alpha_2\}$))=Spre-cl($\{\alpha_2\}$) , hence $\alpha_3 \in$ Spre-cl($\{\alpha_1\}$) , then $\alpha_3 \in$ Spre-cl($\{\alpha_2\}$) which is contradiction . Hence $\alpha_1 \notin$ Spre-cl($\{\alpha_2\}$) , consequently $\alpha_1 \in F_A$ -Spre-cl($\{\alpha_2\}$) but Spre-cl($\{\alpha_2\}$) is soft pre-closed , so F_A-Spre-cl($\{\alpha_2\}$) is soft pre-open which contains α_1 but not α_2 . It follows that $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is Spre-$T_\circ$ space .

Conversely , since $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is Spre-$T_\circ$ space, then for each tow distinct soft points $\alpha_1, \alpha_2 \in F_A$ there exists soft pre-open set F_B such that $\alpha_1 \in F_B$, $\alpha_2 \notin F_B$. $F_A - F_B$ is soft closed set which does not contain α_1 but contains α_2 , by definition (3.9) Spre-cl($\{\alpha_2\}$) is the soft intersection of all soft pre-closed which contain $\{\alpha_2\}$. Thus , Spre-cl($\{\alpha_2\}$) $\tilde{\subseteq} F_A - F_B$ then $\alpha_1 \notin F_A - F_B$ .This implies that $\alpha_1 \notin$ Spre-cl($\{\alpha_2\}$) . So we have $\alpha_1 \in$ Spre-cl($\{\alpha_1\}$) , $\alpha_1 \notin$ Spre-cl($\{\alpha_2\}$) . Therefore Spre-cl($\{\alpha_1\}$) $\neq$ Spre-cl($\{\alpha_2\}$)

4.3. Theorem

Every soft subspace of Spre- $T_\circ$ space is Spre- $T_\circ$ space .

Proof :-

Let $(F_B, \tilde{\tau}_1 F_B, \tilde{\tau}_2 F_B)$ be a soft sub space of Spre- $T_\circ$ space $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$. To prove that the soft sub space is Spre- $T_\circ$ space , let $\beta_1, \beta_2 \in F_B$ such that $\beta_1 \neq \beta_2$. Since $F_B \tilde{\subseteq} F_A$ then $\beta_1 \neq \beta_2 \in F_A$ and $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is Spre- $T_\circ$ space , then there is a soft pre-open set F_C in F_A , such that $\beta_1 \in F_C, \beta_2 \notin F_C$. So $F_C \tilde{\cap} F_B$ is soft pre-open set in F_B and $\beta_1 \in F_C \tilde{\cap} F_B$ and $\beta_2 \notin F_C \tilde{\cap} F_B$. Hence $(F_B, \tilde{\tau}_1 F_B, \tilde{\tau}_2 F_B)$ is Spre- $T_\circ$ space .

4.4. Definition

Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is soft bitopological space , then $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is called Spre- T_1 space if and only if for all pair of soft point $\alpha_1, \alpha_2 \in F_A$, there are two soft pre-open sets F_B, F_C such that F_B contains α_1 but not α_2 and F_C contains α_2 but not α_1 .

4.5. Theorem

Every soft subspace of Spre- T_1 space is Spre- T_1 space .

Proof :-

Let $(F_B, \tilde{\tau}_1 F_B, \tilde{\tau}_2 F_B)$ be a soft sub space of Spre- T_1 space $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$. To prove that the soft sub space is Spre- T_1 space , let $\beta_1, \beta_2 \in F_B$ such that $\beta_1 \neq \beta_2$. Since $F_B \tilde{\subseteq} F_A$ then $\beta_1 \neq \beta_2 \in F_A$ and $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is Spre- $T_\circ$ space , then there exists two soft pre-open set F_C , F_D in F_A , such that $\beta_1 \in F_C$ but $\beta_2 \notin F_C$ and $\beta_2 \in F_D$ but $\beta_1 \notin F_D$.

Then we obtain two soft set $F_{C_1} = F_C \tilde{\cap} F_B, F_{D_1} = F_D \tilde{\cap} F_B$ are soft pre-open sets in F_B , we have $\beta_1 \in F_{C_1}$ but $\beta_2 \notin F_{C_1}; \beta_2 \in F_{D_1}$, but $\beta_1 \in F_{D_1}$. Hence $(F_B, \tilde{\tau}_1 F_B, \tilde{\tau}_2 F_B)$ is Spre- T_1 space .

4.6. Theorem

If Every singleton soft subset of soft bitopological space $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is soft pre-closed , then $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is Spre-T_1 space .

Proof :-This is clearly seen .

4.7. Theorem

A soft bitopological space $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is a Spre-T_1 space if and only if Spre-cl$(\{\alpha\}) = \phi$, for each $\alpha \in F_A$.

Proof :-This is clearly by using prove contradiction .

4.8. Definition

Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is soft bitopological space , then $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is called Spre-T_2 space (Spre-Hausdorf) if and only if for each pair of distinct soft point $\alpha_1, \alpha_2 \in F_A$, there exists two soft pre-open sets F_B, F_C in F_A such that $\alpha_1 \in F_B, \alpha_2 \in F_C$ and $F_B \tilde{\cap} F_C = \phi$.

4.9. Theorem

Each soft subspace of Spre-T_2 space is Spre-T_2 space .

Proof :-

Let $(F_B, \tilde{\tau}_1 F_B, \tilde{\tau}_2 F_B)$ be a soft sub space of Spre-T_2 space $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ and let $F_B \neq \phi$ be a soft subset of F_A , and $\alpha_1 \neq \alpha_2 \in F_B$ then $\alpha_1, \alpha_2 \in F_A$, since $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is Spre-T_2 space ,there exists two soft pre-open sets F_D, F_C in F_A such that $\alpha_1 \in F_D, \alpha_2 \in F_C$ and $F_D \tilde{\cap} F_C = \phi$. So $F_D \tilde{\cap} F_B, F_C \tilde{\cap} F_B$ are soft pre-open sets in F_B and $\alpha_1 \in F_D \tilde{\cap} F_B, \alpha_2 \in F_C \tilde{\cap} F_B$; and $(F_D \tilde{\cap} F_B) \tilde{\cap} (F_C \tilde{\cap} F_B) = (F_D \tilde{\cap} F_C) \tilde{\cap} F_B = \phi$. Hence $(F_B, \tilde{\tau}_1 F_B, \tilde{\tau}_2 F_B)$ is a Spre-T_2 space .

4.10. Theorem

Each singleton soft subset of Spre-T_2 space is a soft pre-closed .

Proof :-This is clearly seen .

5. Conclusion

In the conclusion of a work paper , many of the basic concepts on soft bitopological space , introduced soft bitopology . Furthermore , introduced relative soft bitopological space , soft pre-open set and some definitions on bitopology by soft pre-open set as (soft pre-closed , soft pre-neighborhood , soft pre-interior ,soft pre-limit point and soft pre-closure) these definitions using in other sections from the work and introduce some soft pre separation axioms and studied Properties on soft bitopological space with some important results, one could study the soft ideal bitopology and get some important results too .

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