Soft Pre-Open Sets In Soft Bitopological Spaces

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Abstract

In this work, I introduce the concept of soft bitopological space on a soft set and some definitions on soft pre-open set on soft bitopological space. Also introduce soft pre separation axioms, Spre- $T'_0$, Spre-$T'_1$ and Spre-$T'_2$, with study some properties in soft bitopological space.

Keywords: Soft set; Soft pre-open; Soft topology; Soft bitopological spaces.

1. Introduction

Many classical methods have been used to solve some complicated problems in engineering economics and environment. For instance, the interval mathematics, theory of fuzzy, theory of probability, and sets which can consider as mathematical tools for dealing with uncertainties since all these theories have their own problems and difficulties. In [2], the author in 1999 introduced the notion of soft set, which is free of difficulties in solving aforementioned problems, and it has been applied over many different fields. In 2011, Naim Cagman and his colleagues introduced a new concept of soft set called soft topology define by using the soft power set of soft set [6]. In 2011[7], the authors defined the concept of “soft topology on the collection of soft sets over ” with some basic notations of soft topological spaces. In [6], the notion of soft topology was more general than that in [7]. Therefore, algebraists continue investigating the work of Cagman [6] and follow their notations and mathematical formalism. In 2013 J. Subhashini and C. Sekar defined soft pre-open sets [4] by following Cagman’s theory of soft topology. Therefore, this paper has introduced soft bitopological space relying on [6] and defined the soft pre-open set of soft bitopological space. Also, I have discussed soft pre separation axioms, Spre-$T'_0$, Spre-$T'_1$ and Spre-$T'_2$.

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2. Preliminaries

Through this section, I give an introduction some important definitions and facts about soft topology and recall primary definition soft set that need in this work.

2.1. Definition [2]

The set of ordered pairs \( F_A = \{ (x, f_A(x)) : x \in E, f_A(x) \in P(U) \} \) represents a soft set \( F_A \) on the universe \( U \), where \( f_A : A \rightarrow P(U) \) is mapping and \( f_A \) is called an approximate function of the soft set \( F_A \). However, the set of all soft sets over \( U \) is denoted by \( S_S(U) \).

2.2. Definition [5]

Let \( F_A \in S_S(U) \). If \( f_A(x) = \emptyset \) for all \( x \in A \), then \( F_A \) is called an empty set and denoted by \( F_\emptyset \).
Moreover, if \( f_A = U \) for all \( x \in A \), then \( F_A \) is called an \( A \)-universal soft set and denoted by \( F_A^\hat{\cdot} \). But, if \( A = E \), then \( A \)-universal soft set is called universal soft set and denoted by \( F_E^\sim \).

2.3. Definition [5]

Let \( F_A, F_B \in S_S(U) \). If \( f_A(x) \subseteq f_B(x) \) for all \( x \), then \( F_A \) is a soft subset of \( F_B \) and denoted by \( F_A \subseteq F_B \).

2.4. Definition [5]

Let \( F_A, F_B \in S_S(U) \). Then, the soft union is denoted by \( F_A \cup F_B \), however, the soft intersection is denoted by \( F_A \cap F_B \). Also, the soft difference of \( F_A \) and \( F_B \) is denoted by \( F_A \Delta F_B \), are defined by the approximate functions
\[ f_{A \cup B}(x) = f_A(x) \cup f_B(x), \quad f_{A \cap B}(x) = f_A(x) \cap f_B(x), \quad f_{A \Delta B}(x) = f_A(x) \Delta f_B(x), \]
respectively, on the other hand, the soft complement \( F_A^c \) of \( F_A \) is defined by the approximate function
\[ f_A^c(x) = f_A^c(x), \quad \text{where} \quad f_A^c(x) \text{ is the complement of the set } f_A(x); \text{ that is, } \quad f_A^c(x) = U - f_A(x) \text{ for all } x \in A. \]
It is easy to see that \( (F_A^c)^c = F_A \) and \( F_\emptyset^c = F_E \).

2.5. Proposition [5]

Let \( F_A, F_B, F_C \in S_S(U) \). Then,
\[ F_A \cup F_A = F_A, \quad F_A \cap F_A = F_A. \]
2. $F_A \cap F_\emptyset = F_A, F_A \cap F_\emptyset = F_\emptyset$.

3. $F_A \cap F_B = F_B, F_A \cap F_B = F_A$.

4. $F_A \cap F_A^c = F_A, F_A \cap F_A^c = F_\emptyset$.

5. $F_A \cap F_B = F_B \cap F_A, \widetilde{F_B} \cap F_A = F_B \cap F_A$.

6. $(F_A \cap F_B) \cap F_C = F_A \cap (F_B \cap F_C), (F_A \cap F_B) \cap F_C = F_A \cap (F_B \cap F_C)$.

7. $F_A \cap (F_B \cap F_C) = (F_A \cap F_B) \cap (F_A \cap F_C) \cap (F_B \cap F_C)$.

8. $F_A \cap (F_B \cap F_C) = (F_A \cap F_B) \cap (F_A \cap F_C) \cap (F_B \cap F_C)$.

2.6. Definition [6]

Let $F_A \in S_S(U)$. The soft power set of $F_A$ is defined by $\bar{P}(F_A) = \{ F_{A_i} : F_{A_i} \subseteq F_A, i \subseteq I \subseteq N \}$ and its cardinality is defined by $| \bar{P}(F_A) | = 2^{\sum_{x \in f_A(x)} | f_A(x) |}$, where $| f_A(x) |$ is the cardinality of $f_A(x)$.

2.7. Definition [6]

Let $F_A \in S_S(U)$. A soft topology on $F_A$, denoted by $\tau$, is a collection of soft subsets of $F_A$ having the following properties:

- $F_\emptyset, F_A \in \tau$.

- $\{ F_{A_i} \subseteq F_A, i \subseteq I \subseteq N \} \subseteq \tau \Rightarrow \bigcup_{i \in I} F_{A_i} \in \tau$.

- $\{ F_{A_i} \subseteq F_A, 1 \leq i \leq n, n \in N \} \subseteq \tau \Rightarrow \bigcap_{i=1}^{n} F_{A_i} \in \tau$.

2.8. Definition [6]

Let $(F_A, \tau)$ be a soft space on $F_A$. Every element of $\tau$ is called a soft open sets.

2.9. Definition [6]

Let $(F_A, \tau)$ be a soft space on $F_A$ and $F_B \subseteq F_A$. Then, the collection
\( \widehat{\tau}_B = \{ F_A : F_A \subseteq \widehat{\tau}, i \subseteq I \} \) is called a soft subspace topology on \( F_B \). Hence, \( (F_B, \widehat{\tau}_B) \) is called a soft topological subspace of \( (F_A, \widehat{\tau}) \).

2.10. Definition [6]

Let \((F_A, \widehat{\tau})\) be a soft space on \( F_A \) and \( F_B \subseteq F_A \). The soft interior of \( F_B \), denoted \( F_B^\circ \), is defined as the union of all soft open subsets of \( F_B \). Note that \( F_B^\circ \) is the biggest soft open set that is contained by \( F_B \).

2.11. Theorem [6]

Let \((F_A, \widehat{\tau})\) be a soft space on \( F_A \) and \( F_B, F_C \subseteq F_A \). Then,

1. \((F_B^\circ)^* = F_B^\circ \)

2. \( F_B \subseteq F_C \). Then, \( F_B^\circ \subseteq F_C^\circ \)

3. \( F_B^\circ \cap F_C^\circ = (F_B \cap F_C)^\circ \)

4. \( F_B^\circ \cup F_C^\circ \subseteq (F_B \cup F_C)^\circ \)


Let \((F_A, \widehat{\tau})\) be a soft space on \( F_A \) and \( F_B \subseteq F_A \). Then, the soft closure of \( F_B \), denoted \( \overline{F}_B \) is defined as the soft intersection of all soft closed superset of \( F_B \). Note that \( \overline{F}_B \) is the smallest soft closed set that containing \( F_B \).

2.13. Theorem [6]

Let \((F_A, \widehat{\tau})\) be a soft space on \( F_A \) and \( F_B, F_C \subseteq F_A \). Then,

1. \( (\overline{F}_B) = \overline{F}_B \)

2. \( (\overline{F}_B)^\circ = (F_B^\circ)^* \)

3. \( F_B \subseteq F_C \). Then, \( \overline{F}_B \subseteq \overline{F}_C \).
2.14. **Theorem [6]**

Let \((F_A, \tau)\) be a soft space on \(F_A\) and \(F_B \subseteq F_A\) Then \(F_B \subseteq F_B \subseteq \overline{F_B}\).

2.15. **Definition**

Let \((F_A, \tau_1)\) and \((F_A, \tau_2)\) be the two different soft topologies on \(F_A\). Then \((F_A, \tau_1, \tau_2)\) is called a soft bitopological space.

2.16. **Example**

Let \(U = \{u_1, u_2, u_3, u_4\}\), \(E = \{w_1, w_2, w_3\}\), \(A = \{w_1, w_2\}\) such that \(A \subseteq E\) and \(F_A = \{(w_1, \{u_1, u_2\}), (w_2, \{u_3, u_4\})\}\) then

\(F_{A_1} = \{(w_1, \{u_1\})\}\)

\(F_{A_2} = \{(w_1, \{u_2\})\}\)

\(F_{A_3} = \{(w_1, \{u_1, u_2\})\}\)

\(F_{A_4} = \{(w_2, \{u_3\})\}\)

\(F_{A_5} = \{(w_2, \{u_4\})\}\)

\(F_{A_6} = \{(w_2, \{u_3, u_4\})\}\)

\(F_{A_7} = \{(w_1, \{u_1\}), (w_2, \{u_3\})\}\)

\(F_{A_8} = \{(w_1, \{u_1\}), (w_2, \{u_4\})\}\)

\(F_{A_9} = \{(w_1, \{u_1\}), (w_2, \{u_3, u_4\})\}\)

\(F_{A_{10}} = \{(w_1, \{u_2\}), ((w_2, \{u_4\})\}\)
\[ F_{A_1} = \{(w_1, \{u_2\}), (w_2, \{u_4\})\} \]
\[ F_{A_2} = \{(w_1, \{u_2\}), (w_2, \{u_3, u_4\})\} \]
\[ F_{A_3} = \{(w_1, \{u_1, u_2\}), (w_2, \{u_1\})\} \]
\[ F_{A_4} = \{(w_1, \{u_1, u_2\}), (w_2, \{u_4\})\} \]
\[ F_{A_5} = F_A \]
\[ F_{A_6} = F_\phi \]

Then \( \tilde{\tau}_1 = \{F_\phi, F_A\} \) and \( \tilde{\tau}_2 = \{F_\phi, F_A, F_{A_2}, F_{A_4}\} \) are a soft topology of \( F_A \) then \( (F_A, \tilde{\tau}_1, \tilde{\tau}_2) \) is a soft bitopological space.

2.17. Definition

Let \((F_A, \tilde{\tau}_1, \tilde{\tau}_2)\) be a soft bitopological space over \( F_A \) and \( F_B \subseteq F_A \). Then \( \tilde{\tau}_1 F_B = \{F_\phi \cap F_B : F_\phi \in \tilde{\tau}_1, i \in I \subseteq N\} \) and \( \tilde{\tau}_2 F_B = \{F_\phi \cap F_B : F_\phi \in \tilde{\tau}_2, i \in I \subseteq N\} \) are said to be the relative topologies on \( F_B \). Then \((F_B, \tilde{\tau}_1 F_B, \tilde{\tau}_2 F_B)\) is called a relative soft bitopological space of \((F_A, \tilde{\tau}_1, \tilde{\tau}_2)\).

2.18. Theorem

If \((F_A, \tilde{\tau}_1, \tilde{\tau}_2)\) is a soft bitopological space then \( \tilde{\tau}_1 \cap \tilde{\tau}_2 \) is a soft topological space over \( F_A \).

Proof :-

• \( F_\phi, F_A \in \tilde{\tau}_1 \cap \tilde{\tau}_2 \).

• Let \( \{F_{A_i}, i \in I\} \) be a family of soft sets in \( \tilde{\tau}_1 \cap \tilde{\tau}_2 \) \( \Rightarrow F_{A_i} \in \tilde{\tau}_1 \) and \( F_{A_i} \in \tilde{\tau}_2 \) for all \( i \in I \). Therefore \( \bigcup_{i \in I} F_{A_i} \in \tilde{\tau}_1 \) and \( \bigcup_{i \in I} F_{A_i} \in \tilde{\tau}_2 \). Thus \( \bigcup_{i \in I} F_{A_i} \in \tilde{\tau}_1 \cap \tilde{\tau}_2 \).

• Let \( F_{A_i} \in \tilde{\tau}_1 \cap \tilde{\tau}_2, 1 \leq i \leq n', n' \in N \). Then \( F_{A_i} \in \tilde{\tau}_1 \) and \( F_{A_i} \in \tilde{\tau}_2 \) \( \Rightarrow 1 \leq i \leq n', n' \in N \). Since \( \bigcap_{i=1}^{n'} F_{A_i} \subseteq \tilde{\tau}_1 \) and \( \bigcap_{i=1}^{n'} F_{A_i} \subseteq \tilde{\tau}_2 \). Therefore \( \bigcap_{i=1}^{n'} F_{A_i} \in \tilde{\tau}_1 \cap \tilde{\tau}_2, 1 \leq i \leq n', n' \in N \).
2.19. Remark

If \((F_A, \tau_1, \tau_2)\) is a soft bitopological space then \(\tau_1 \cup \tau_2\) is not a soft topological space over \(F_A\).

2.20. Example

Let us consider 2.16 and let \(\tau_1 = \{ F_\emptyset, F_A, F_A \} \) and \(\tau_2 = \{ F_\emptyset, F_A, F_A \} \) are soft topology of \(F_A\) then \((F_A, \tau_1, \tau_2)\) is soft bitopological space . Now \(\tau_1 \cup \tau_2 = \{ F_\emptyset, F_A, F_A, F_A \} \). If take \(F_A, F_A\), \(F_A \cup F_A = \{(x_1, \{u_1\}), (x_1, \{u_2\})\} \notin \tau_1 \cup \tau_2\). Thus \(\tau_1 \cup \tau_2\) is not soft topology on \(F_A\).

3. Some Definition of Soft Pre-open set in soft bitopological space

In this section introduce some definitions of soft pre-open set , soft pre-closed , soft pre-neighborhood , soft pre-closure and soft pre-interior on soft bitopological space .

3.1. Definition

Let \((F_A, \tau_1, \tau_2)\) be soft bitopological space and let \(F_B \subseteq F_A\) , \(F_B\) is called soft pre-open set with respect to the two soft topological spaces \(\tau_1\) and \(\tau_2\) if \(F_B \subseteq (F_B)^\circ\).

3.2. Notes

1- The set of all soft pre-open set with respect to the two soft topologies is denoted by \(\text{Pre}(F_A)\).

2- The relative soft bitopological space for \(F_B\) with respect to Soft pre-open sets is the collection \(\text{Pre}(F_A)_{F_B}\) given by \(\text{Pre}(F_A)_{F_B} = \{F_C \subseteq F_B : F_C \in \text{Pre}(F_A)\}\)

3- Any \(\tau_2\)-open soft set is not necessarily to be soft pre-open set.

4- Any soft pre-open set is not necessarily to be of \(\tau_1\)-open (\(\tau_2\)-open ) soft set.

3.3. Example

Let us consider example 2.20 , \((F_A, \tau_1, \tau_2)\) is soft bitopological space . Take \(F_A \subseteq F_A\) then \(F_A \subseteq (F_A)^\circ \Rightarrow F_A\) is soft per-open set with respect to the two soft topological spaces \(\tau_1\) and \(\tau_2\).

3.4. Remarks

1- The intersection of any soft pre-open sets is not necessary a soft pre-open set.
2- The union of any soft pre-open sets is soft pre-open set with respect to soft bitopological space.

The example to part (1) is simply. The following proof explain the part (2) of the remark 3.4. Let \( F_B \) and \( F_C \) be a two soft pre-open sets with respect to soft bitopological space \( (F_A, \tau_1, \tau_2) \). i.e. \( F_B \subseteq (F_B)^{\tau_1} \) and \( F_C \subseteq (F_C)^{\tau_2} \) with respect to the two soft topological spaces \( \tau_1 \) and \( \tau_2 \). Since \( F_B \cup F_C = (F_B \cup F_C)^{\tau_1 \cup \tau_2} \) then

\[
F_B \cup F_C \subseteq (F_B \cup F_C)^{\tau_1 \cup \tau_2} \quad \text{with respect to the two soft topological spaces } \tau_1 \text{ and } \tau_2.
\]

3.5. Definition

Let \( (F_A, \tau_1, \tau_2) \) is soft bitopological space and let \( F_B \subseteq F_A \). \( F_B \) is called soft pre-closed set of \( F_A \) if and only if \( F_B \) is soft pre-open set of \( F_A \).

3.6. Definition

Let \( (F_A, \tau_1, \tau_2) \) is soft bitopological space and \( \alpha \in F_A \). \( F_B \subseteq F_A \) is said to be soft pre-neighborhood of a point \( \alpha \) if there is a soft pre-open set \( F_C \) such that \( \alpha \in F_C \subseteq F_B \). The set of all soft pre-neighborhoods of a point \( \alpha \) is denoted by \( \text{Spre-}(\alpha) \).

3.7. Definition

Let \( (F_A, \tau_1, \tau_2) \) is soft bitopological space, and \( F_B \subseteq F_A \). A point \( \alpha \in F_A \) is said to be soft pre-interior point of \( F_B \) with respect to the two soft topological spaces \( \tau_1 \) and \( \tau_2 \) if there is a soft pre-open \( F_C \) such that \( \alpha \in F_C \subseteq F_B \). The set of all soft pre-interior points of \( F_B \) with respect to the two soft topological spaces \( \tau_1 \) and \( \tau_2 \) denoted by \( \text{Spre-int}(F_B) \).

3.8. Definition

Let \( (F_A, \tau_1, \tau_2) \) is soft bitopological space. A point \( \alpha \) is called soft pre-limit point of soft subset \( F_B \) of \( F_A \) with respect to the two soft topological spaces \( \tau_1 \) and \( \tau_2 \) if and only if for each a soft pre-open set \( F_C \) containing another point different from \( \alpha \) in \( F_B \), that is \( (F_C \setminus \{\alpha\}) \cap F_B \neq \emptyset \). The set of all soft pre-limit points of \( F_B \) be denoted by \( \text{Spre-lm}(F_B) \).

3.9. Definition
Let \((F_A, \tau_1, \tau_2)\) is soft bitopological space, and \(F_B \subseteq F_A\), the intersection of all soft pre-closed sets containing \(F_B\) is called soft pre-closure of \(F_B\), and is denoted by \(\text{Spre-cl}(F_B)\).

In the year 2014 J. Subhashinin and Dr. C.Sekar [3] by depending on the [6] and [4] introduces soft pre separation axioms, soft \(PT_s\)-space and some of its properties in the soft topological spaces. Now begin to important section to discuss soft pre separation axioms and some result.

4. The Separation Axioms in Soft Bitopological Space

In section four, I introduce some soft pre separation axioms, \(\text{Spre-}T_s\), \(\text{Spre-}T_1\) and \(\text{Spre-}T_2\) and illustrate transmission this Properties to The relative soft bitopological space with some result of soft pre separation axioms.

4.1. Definition

Let \((F_A, \tau_1, \tau_2)\) is soft bitopological space, then \((F_A, \tau_1, \tau_2)\) is called \(\text{Spre-}T_s\) space if and only if for all pair of soft point \(\alpha_1, \alpha_2 \in F_A\) such that \(\alpha_1 \neq \alpha_2\), there exists soft pre-open set \(F_B\) containing \(\alpha_1\) but not \(\alpha_2\) or soft pre-open set \(F_C\) containing \(\alpha_2\) but not \(\alpha_1\).

4.2. Theorem

A soft bitopological space \((F_A, \tau_1, \tau_2)\) is \(\text{Spre-}T_s\) space if and only if for each distinct soft points \(\alpha_1, \alpha_2\) in \(F_A\), \(\text{Spre-cl}\{\alpha_1\} \neq \text{Spre-cl}\{\alpha_2\}\).

Proof :-

Let \(\alpha_1, \alpha_2 \in F_A\) such that \(\alpha_1 \neq \alpha_2\) and \(\text{Spre-cl}\{\alpha_1\} \neq \text{Spre-cl}\{\alpha_2\}\).

Then there exists at least one soft point \(\alpha_3\) in \(F_A\) such that \(\alpha_3 \in \text{Spre-cl}\{\alpha_1\}\) but \(\alpha_3 \notin \text{Spre-cl}\{\alpha_2\}\).

Suppose \(\alpha_3 \in \text{Spre-cl}\{\alpha_1\}\), to show that \(\alpha_1 \notin \text{Spre-cl}\{\alpha_2\}\). If \(\alpha_1 \in \text{Spre-cl}\{\alpha_2\}\), then \(\alpha_1 \in \text{Spre-cl}\{\alpha_3\}\). So \(\text{Spre-cl}\{\alpha_1\} \subseteq \text{Spre-cl}\{\alpha_3\}\), hence \(\alpha_3 \in \text{Spre-cl}\{\alpha_1\}\), then \(\alpha_3 \in \text{Spre-cl}\{\alpha_2\}\) which is contradiction. Hence \(\alpha_1 \notin \text{Spre-cl}\{\alpha_2\}\), consequently \(\alpha_1 \in F_A\)-\text{Spre-cl}(\{\alpha_2\}) but \text{Spre-cl}(\{\alpha_1\}) is soft pre-closed, so \(F_A\)-\text{Spre-cl}(\{\alpha_2\}) is soft pre-open which contains \(\alpha_1\) but not \(\alpha_2\). It follows that \((F_A, \tau_1, \tau_2)\) is \(\text{Spre-}T_s\) space.

Conversely, since \((F_A, \tau_1, \tau_2)\) is \(\text{Spre-}T_s\) space, then for each tow distinct soft points \(\alpha_1, \alpha_2 \in F_A\) there
exists soft pre-open set $F_B$ such that $\alpha_1 \in F_B$, $\alpha_2 \notin F_B$. $F_A - F_B$ is soft closed set which does not contain $\alpha_1$ but contains $\alpha_2$, by definition (3.9) Spre-cl($\{\alpha_2\}$) is the soft intersection of all soft pre-closed which contain $\{\alpha_2\}$. Thus, Spre-cl($\{\alpha_2\}$) $\subseteq$ $F_A - F_B$ then $\alpha_1 \notin F_A - F_B$. This implies that $\alpha_1 \notin$ Spre-cl($\{\alpha_2\}$). So we have $\alpha_1 \in$ Spre-cl($\{\alpha_1\}$), $\alpha_1 \notin$ Spre-cl($\{\alpha_2\}$). Therefore Spre-cl($\{\alpha_1\}$) $\neq$ Spre-cl($\{\alpha_2\}$)

4.3. Theorem

Every soft subspace of Spre-$T_\alpha$ space is Spre-$T_\alpha$ space.

Proof :-

Let $(F_B, \tilde{r}_1 F_B, \tilde{r}_2 F_B)$ be a soft sub space of Spre-$T_\alpha$ space $(F_A, \tilde{r}_1, \tilde{r}_2)$. To prove that the soft sub space is Spre-$T_\alpha$ space, let $\beta_1, \beta_2 \in F_B$ such that $\beta_1 \neq \beta_2$. Since $F_B \subseteq F_A$ then $\beta_1 \neq \beta_2 \in F_A$ and $(F_A, \tilde{r}_1, \tilde{r}_2)$ is Spre-$T_\alpha$ space, then there is a soft pre-open set $F_C$ in $F_A$, such that $\beta_1 \in F_C, \beta_2 \notin F_C$. So $F_C \cap F_B$ is soft pre-open set in $F_B$ and $\beta_1 \in F_C \cap F_B$ and $\beta_2 \notin F_C \cap F_B$. Hence $(F_B, \tilde{r}_1 F_B, \tilde{r}_2 F_B)$ is Spre-$T_\alpha$ space.

4.4. Definition

Let $(F_A, \tilde{r}_1, \tilde{r}_2)$ is soft bitopological space, then $(F_A, \tilde{r}_1, \tilde{r}_2)$ is called Spre-$T_\alpha$ space if and only if for all pair of soft point $\alpha_1, \alpha_2 \in F_A$, there are two soft pre-open sets $F_B, F_C$ such that $F_B$ contains $\alpha_1$ but not $\alpha_2$ and $F_C$ contains $\alpha_2$ but not $\alpha_1$.

4.5. Theorem

Every soft subspace of Spre-$T_\alpha$ space is Spre-$T_\alpha$ space.

Proof :-

Let $(F_B, \tilde{r}_1 F_B, \tilde{r}_2 F_B)$ be a soft sub space of Spre-$T_\alpha$ space $(F_A, \tilde{r}_1, \tilde{r}_2)$. To prove that the soft sub space is Spre-$T_\alpha$ space, let $\beta_1, \beta_2 \in F_B$ such that $\beta_1 \neq \beta_2$. Since $F_B \subseteq F_A$ then $\beta_1 \neq \beta_2 \in F_A$ and $(F_A, \tilde{r}_1, \tilde{r}_2)$ is Spre-$T_\alpha$ space, then there exists two soft pre-open set $F_C, F_D$ in $F_A$, such that $\beta_1 \in F_C$ but $\beta_2 \notin F_C$ and $\beta_2 \in F_D$ but $\beta_1 \notin F_D$.

Then we obtain two soft set $F_{C_1} = F_C \cap F_B, F_{D_1} = F_D \cap F_B$ are soft pre-open sets in $F_B$, we have $\beta_1 \in F_{C_1}$ but $\beta_2 \notin F_{C_1}; \beta_2 \in F_{D_1}$, but $\beta_1 \notin F_{D_1}$. Hence $(F_B, \tilde{r}_1 F_B, \tilde{r}_2 F_B)$ is Spre-$T_\alpha$ space.
4.6. **Theorem**

If every singleton soft subset of soft bitopological space \((F_A, \tilde{\tau}_1, \tilde{\tau}_2)\) is soft pre-closed, then \((F_A, \tilde{\tau}_1, \tilde{\tau}_2)\) is Spre-\(T_1\) space.

**Proof** :- This is clearly seen.

4.7. **Theorem**

A soft bitopological space \((F_A, \tilde{\tau}_1, \tilde{\tau}_2)\) is a Spre-\(T_1\) space if and only if Spre-cl\(\{\{\alpha\}\}\) = \(\phi\), for each \(\alpha \in F_A\).

**Proof** :- This is clearly by using prove contradiction.

4.8. **Definition**

Let \((F_A, \tilde{\tau}_1, \tilde{\tau}_2)\) be a soft bitopological space, then \((F_A, \tilde{\tau}_1, \tilde{\tau}_2)\) is called Spre-\(T_2\) space (Spre-Hausdorff) if and only if for each pair of distinct soft point \(\alpha_1, \alpha_2 \in F_A\), there exists two soft pre-open sets \(F_B, F_C\) in \(F_A\) such that \(\alpha_1 \in F_B, \alpha_2 \in F_C\) and \(F_B \cap F_C = \phi\).

4.9. **Theorem**

Each soft subspace of Spre-\(T_2\) space is Spre-\(T_2\) space.

**Proof** :-

Let \((F_B, \tilde{\tau}_1 F_B, \tilde{\tau}_2 F_B)\) be a soft sub space of Spre-\(T_2\) space \((F_A, \tilde{\tau}_1, \tilde{\tau}_2)\) and let \(F_B \neq \phi\) be a soft subset of \(F_A\), and \(\alpha_1 \neq \alpha_2 \in F_B\) then \(\alpha_1, \alpha_2 \in F_A\), since \((F_A, \tilde{\tau}_1, \tilde{\tau}_2)\) is Spre-\(T_2\) space, there exists two soft pre-open sets \(F_D, F_C\) in \(F_A\) such that \(\alpha_1 \in F_D, \alpha_2 \in F_C\) and \(F_D \cap F_C = \phi\). So \(F_D \cap F_B, F_C \cap F_B\) are soft pre-open sets in \(F_B\) and \(\alpha_1 \in F_D \cap F_B, \alpha_2 \in F_C \cap F_B\); and \((F_D \cap F_B) \cap (F_C \cap F_B) = (F_D \cap F_B) \cap F_B = \phi\). Hence \((F_B, \tilde{\tau}_1 F_B, \tilde{\tau}_2 F_B)\) is a Spre-\(T_2\) space.

4.10. **Theorem**

Each singleton soft subset of Spre-\(T_2\) space is a soft pre-closed.

**Proof** :- This is clearly seen.
5. Conclusion

In the conclusion of a work paper, many of the basic concepts on soft bitopological space, introduced soft bitopology. Furthermore, introduced relative soft bitopological space, soft pre-open set and some definitions on bitopology by soft pre-open set as (soft pre-closed, soft pre-neighborhood, soft pre-interior, soft pre-limit point and soft pre-closure) these definitions using in other sections from the work and introduce some soft pre separation axioms and studied Properties on soft bitopological space with some important results, one could study the soft ideal bitopology and get some important results too.

References


