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Space-time as Dark Energy and Dark Matter

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Abstract

In this paper we modify Einstein's field equations, and write down the cosmological term and the metric tensor as an energy momentum tensor of space-time,and interpret space-time as a form of energy, not vacuum energy as such.The energy momentum tensor of space-time has equivalents of both mass and pressure components.The acceleration of the universe, Dark Energy and Dark Matter are explained in terms of the energy momentum tensor of space-time.

Keywords: Space-time energy momentum tensor; Space-time as energy; Einstein's field equations; Dark Energy; Dark Matter; Universe with acceleration.

1. Introduction

We write,following Hemantha and de Silva [1, 2],Einstein's Field Equations in General Relativity in the form,

$$R^{\mu\nu} - \frac{1}{2}\bar{R}g^{\mu\nu} = \kappa T^{\mu\nu} - \Lambda g^{\mu\nu}$$

(1.1)

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where $R^{\mu\upsilon}$,$g^{\mu\upsilon}$, $T^{\mu\upsilon}$ are the Ricci tensor, metric tensor and the energy momentum tensor of ordinary matter and radiation respectively,$\bar{R}$ is the Ricci scalar,Λ is the cosmological term and $\kappa = \frac{-8\pi G}{c^2}$. The Λ term with the metric tensor, is written on the right hand side of the equations as we take it to represent the energy momentum tensor due to space- time. This is a new interpretation and the first term on the right hand side of the equation represents the usual energy momentum tensor due to matter and radiation, and the second term represents the energy momentum tensor due to space-time. The space-time itself is considered as a source of energy-momentum.

The space time is determined by not only ordinary matter and radiation but by space time itself as well. The space time (space and time) in Newtonian formulation was given and matter and radiation existed in a given space time. In the Einsteinian formulation space time was determined by matter and radiation. In the present formulation space time is determined by space time itself in addition to matter and radiation.

In the original version of equations of Einstein the Λ term is written on the left hand side of the equations as a geometrical tensor, and the right hand side of the equations consists only of the energy- momentum tensor due to matter and radiation.

The Λ term, as a positive constant, and written on the left hand side of the field equations in the given sign convention, gives rise to a field that repels particles and objects, rather than to one that attracts them. Einstein, as well known, introduced the term in order to obtain a static solution instead of the expanding universe that was obtained by solving the original equations without the Λ term. In our formulation Λ term could be a variable either positive or negative.

2. $\Lambda g^{\mu\upsilon}$ as a tensor representing energy momentum

When the Λ term with the metric tensor is written on the right hand side of the equations as a term that represents the energy momentum tensor of the space-time itself,it presents a different picture altogether. The Λ term with the metric tensor now represents a space-time that is a source for the space- time itself, while having an energy momentum of its own. This energy of the space time is not the same as the vacuum energy in general, however, we do not rule out the possibility of space time energy giving the vacuum energy at some epoch in the evolution of the universe.

Now the left hand side of the equation (1.1) is divergenceless, and hence the right hand side also should be divergenceless. If $\Lambda$is considered as a constant, then since the metric tensor is divergenceless it implies that the energy momentum tensor of ordinary matter and radiation is also divergenceless as in the original equations of Einstein. In this case the energy of the space time is “conserved” as the energy momentum tensor of the space-time is also divergenceless.

However, if Λ is not a constant, then $\Lambda g^{\mu\upsilon}$ is not divergenceless implying that the energy momentum tensor is also not divergenceless, for the sum of the two terms on the right hand side of the equation (1.1) has to be divergenceless.

This implies that it is the total of the energy momentum of matter and radiation, and that of the space-time that remains a constant. This is a new phenomenon arising out of consideration of space-time as a form of energy. The space-time being a form of energy can be converted to other forms of energy and vice versa. This is different from the *C*-field introduced by Hoyle, (see for example [3]), in his formulation of the steady state theory. In the formulation of Hoyle the *C* –field contributed to continuous creation of matter, increasing the energy of the universe. There was no "exchange" of energy between matter and energy, and space - time.

The above formulation does not imply that the space - time would interact with electromagnetic radiation, it only means that the energy of space - time could be converted to other forms of energy and vice versa.

3. Robertson Walker Space Times

For different values of μ and ν in the equation (1.1), we obtain the following two independent equations with four unknown variables R, ρ, Λ and p, in the case of the Robertson-Walker metrics.

$$-\frac{3\dot{R}^2}{R^2c^2}-\frac{3k}{R^2}= \kappa\rho-\Lambda$$
(3.1)

$$\frac{k}{R^2}+\frac{\dot{R}^2+2R\ddot{R}}{R^2c^2} = \frac{\kappa p}{c^2}+\Lambda$$
(3.2)

where $k=-1,0,1$ and a dot denotes differentiation with respect to cosmic time.

From the above equations we find that the space time energy tensor gives rise to a component equivalent to that of ρ the density of ordinary matter and radiation as in the equation (3.1), and to a component equivalent that of p the pressure due to ordinary matter and radiation as in the equation (3.2).

Since $\kappa = \frac{-8\pi G}{c^2}$ is negative, it is clear that when Λ is positive, it represents a positive density of space time energy as in equation (3.1), and a negative pressure due to space time energy as in equation (3.2). When Λ is negative, it represents a negative density and a positive pressure.

We define Λ' equal to $\frac{\Lambda c^2}{8\pi G}$ so that Λ' has same dimensions as of density $\rho[ML^{-3}]$ and could be compared with ρ. Thus $\Lambda'[gcm^{-3}]$ gives the density of energy due to the space-time, and $\rho[gcm^{-3}]$ gives the energy density of ordinary matter and radiation. Λ' resembles the vacuum energy but has a different identification as the energy density of space - time. We may define the vacuum energy as the energy of space- time in the absence of ordinary matter and radiation.

From equations (3.1) and (3.2) we obtain

$$\frac{d}{dt}[(\rho+\Lambda')R^3]+3\left(\frac{p}{c^2}-\Lambda'\right)R^2\dot{R}=0$$

(3.3)

When $\Lambda' = 0$,the above equation reduces to the usual equation of "conservation of energy" (see for example Mc.Vittie [4]),

$$\frac{d}{dt}(\rho R^3) + 3\left(\frac{p}{c^2}\right)R^2\dot{R} = 0$$
(3.4)

The equation (3.3) can also be written as $\frac{d}{dt}[\rho R^3] + 3\left(\frac{p}{c^2}\right)R^2\dot{R} + \dot{\Lambda}'R^3 = 0$
(3.5)

The equations (3.3) and (3.5) demonstrate that the energy of space - time can be converted to energy of ordinary matter and radiation, and vice versa. When Λ' is a constant, the equation (3.5) reduces to the usual equation (3.4), since the space - time energy is "conserved "separately in that case.

It should be noted that Λ'appears with both the density and pressure of ordinary matter and energy in equation (3.3) as it should be. Λ'acts as a density and as well as pressure. It implies that when off diagonal terms are zero, the trace of $\Lambda g^{\mu\nu}$gives rise to-2Λ, due to negative pressure arising out of Λ and this makes the space –time expands, when $\Lambda$is positive.

4. de - Sitter space

We illustrate some of these ideas in the case when $\Lambda(\Lambda')$ is a constant with the well known de-Sitter Universe, where the energy momentum tensor of matter and radiation is null.

Eliminating k in equations (3.1) and (3.2), we have, in the case of a matter-radiation free universe

$$3\ddot{R} - \Lambda c^2 R = 0$$
(4.1)

where Λ is a constant.

If Λ is a positive constant,we obtain the de-Sitter universe that expands forever.

If Λ is a negative constant, the equation is similar to that of simple harmonic motion. We have solutions where R takes the form$R = A\cos(\omega t + \alpha)$and we take the modulus of$R$as it has to be positive. Thus Rwould change from zero to zero for suitable values of cosmic time$t$,and then would start a different cycle. This corresponds to a cyclic universe.

If Λ is zero, Rincreases linearly with time.

In all the three cases the energy density of space-time remains constant and is "conserved" on its own as there is no ordinary matter and radiation. In this case the energy of the space-time could be identified as the vacuum energy.

Now consider the de Sitter space-time written in the form

$$ds^2 = (1 - \frac{1}{3}\Lambda r^2)c^2dt^2 - \frac{dr^2}{(1 - \frac{1}{3}\Lambda r^2)} - r^2(d\Theta^2 + sin^2\Theta d\varphi^2)$$

Jayakody[5] has shown that the angular momentum $h$per unit mass of a particle that describes a circle of coordinate radius a can be written as

$$h^2 = \frac{-\Lambda a^4}{3}c,$$

implying that no circular motion is possible if $\Lambda$is non negative, as could be expected since positive Λgives rise to a repulsion.

5. Λ considered as a variable

In this case the energy of space - time does not remain a constant and can be converted to other forms of energy represented by the energy momentum tensor for ordinary matter and radiation. In order to illustrate this and also to illustrate that the space - time energy could be interpreted as dark energy that accelerates the universe as observed by Perlmutter and his colleagues [6,7] and Reiss and his colleagues [8] we proceed as follows. This is only an illustration.

We now begin a discussion on solving the two equations (3.1) and (3.2), for R under the boundary conditions stated below. However in the process, ρ and Λ also have to be found as they are not prescribed as functions of time. Only p is prescribed as we consider zero pressure models. As there are three variables in effect (namely$R$, ρ and Λ), in addition to k, we could assume a solution of the form $R = R(t)$ and substitute it in the two equations (3.1) and (3.2), to obtain ρ and Λ as functions of t, the cosmic time, and see whether the solution would agree with the values of ρ andΛ at the present epoch..

Further since we are interested in solutions that give an expanding universe with an acceleration in the present epoch, and since the Universe had been expanding with a deceleration in the previous epoch commencing with the big bang, it is seen that $\ddot{R}(t)$ has changed from negative to positive at the onset of acceleration. This implies that $\ddot{R}(t)$ is equal to zero at the onset of acceleration.

In solving the above equations we assume the following boundary conditions.

$R(t) = 0$ at $t = 0$, this would correspond to the usual big bang model at $t = 0$.

The present observations tell us that the ratio of dark energy to that of ordinary matter is $\frac{7}{3}$, (see for example[9]).

Since we assume that Λ' correspond to dark energy or space-time energy,we should take $\frac{\Lambda'}{\rho} = \frac{7}{3}$ at the present epoch.

Since according to observations the onset of acceleration has taken place when the redshift was in the range 1.2 - 1.6 (see for example[8]), we could take the range of $z$to be $z = 0.2 - 0.6$ at the onset of acceleration. This implies that $\ddot{R}(t)$should be equal to zero in the range $z = 0.2 - 0.6$.

We demand that the density ρ should be positive for all values ofthe cosmic time$t$, and should lie between $4.5 \times 10^{-30}\, gcm^{-3}$and $1.8 \times 10^{-29}\, gcm^{-3}$ at the present epoch. We take the density to lie in the above range as different authors have quoted these values.

We make no demand on Λ'in general, as Λ' could be either positive or negative, in different epochs. However we would prefer to have Λ' in the range $1.9 \times 10^{-30}\, gcm^{-3} < \Lambda' < 7.7 \times 10^{-29}\, gcm^{-3}$, at the present epoch, as a value in this interval is quoted by different authors as the density of dark energy.

Using equations (3.1) and (3.2) with $p = 0$ we obtain the following expressions for density ρ of the homogeneous universe and the density of energy due to space-time which is represented by Λ'.

$$\rho = \frac{1}{4\pi G}\left(\frac{kc^2+\dot{R}^2-R\ddot{R}}{R^2c^2}\right) \tag{5.1}$$

$$\Lambda' = \frac{1}{8\pi G}\left(\frac{kc^2+\dot{R}^2+2R\ddot{R}}{R^2c^2}\right) \tag{5.2}$$

Using the boundary condition$\frac{\Lambda'}{\rho} = n,$ where n is a constant at the present epoch, and substituting the above expressions for ρ and Λ' , we have

$$\frac{1}{(8\pi G)}\left(\frac{kc^2+\dot{R}^2+2R\ddot{R}}{R^2c^2}\right)\left(\frac{(4\pi G)\,R^2c^2}{kc^2+\dot{R}^2-R\ddot{R}}\right) = n, \tag{5.3}$$

which reduces to

$$(2n-1)(kc^2+\dot{R}^2\,) - 2(n+1)R\ddot{R} = 0$$

i.e. $\ddot{R} = \frac{(2n-1)(kc^2+\dot{R}^2)}{2(n+1)\,R}$, at the present epoch. (5.4)

At the present epoch this implies that $\ddot{R} > 0$, if $k = 0$ or 1 and$n > \frac{1}{2}$. If $n = \frac{1}{2}$, $\ddot{R} = 0$ for all values of k. It implies that for values of $k = 0$ or 1 the universe changes from deceleration to acceleration when $n = \frac{1}{2}$. Further if $k = -1$, $\ddot{R} < 0$, when $n > \frac{1}{2}$ at the present epoch assuming that $\dot{R}$ is less than c, indicating a deceleration. Under these conditions the universe changes from acceleration to deceleration when $n = \frac{1}{2}$.

It follows that, assuming $k = 0$ or 1, the Universe expands with acceleration at the present epoch since $n = \frac{7}{3}$

(boundary condition (ii)).

Further even if Λ' is a constant the universe changes from deceleration to acceleration when ρ decreases to the value $2\Lambda'$, if $k = 0$ or 1, and the other way around when $k = -1$. Reference [10] has found many solutions but unfortunately none of them satisfies all the boundary conditions, though he has solutions that give inflation as well. Katugampala and de Silva [11] also have given solutions involving both acceleration and deceleration of the universe though the solutions do not satisfy some of the boundary conditions.

6. A solution with deceleration and acceleration

Though it is difficult to find a model for the universe satisfying all the boundary conditions stated, Katugamapala [10] has shown that the model

$$R = b\sqrt{(1 - \cos^3 \omega t)} \tag{6.1}$$

where b and ω are constants, satisfies inflation, an acceleration at the present epoch, taken as

$t_0 = 4.26 \times 10^{17} s$ $(13.69 \times 10^9\, years)$, time from the big bang.

From (5.4) we have

$$\dot{R} = \frac{3b\omega \cos^2 \omega t \sin \omega t}{2\sqrt{1 - \cos^3 \omega t}}$$

(6.2)

$$\ddot{R} = \left(\frac{3b\omega^2}{2}\right)\left[\frac{-3\cos^6 \omega t + \cos^4 \omega t + 6\cos^3 \omega t - 4\cos \omega t}{2(1 - \cos^3 \omega t)^{3/2}}\right] \tag{6.3}$$

$\cos \omega t = 0$, is a solution of $\ddot{R} = 0$, and it is clear that when $\cos \omega t = 0$, $\dot{R} = 0$, making values of t corresponding to $\cos \omega t = 0$ points of inflection for R as a function of t.

$\cos \omega t = 0$ Implies $\omega t = \frac{\pi}{2} (0 < \omega t < \pi)$.

Let R_a be the value of R at the point of inflection, which is the point at which acceleration of the Universe begins after a phase of deceleration. Then $R_a = b\sqrt{(1 - \cos^3 \frac{\pi}{2})} = b$

Writing $T = \cos \omega t_0$, and substituting (6.1), (6.2) and (6.3) in the equation (5.3) with $n = \frac{7}{3}$ for the present epoch t_0 and rearranging the terms we have the following expression for the unknown b, in terms of ω and t_0.

$$b^2 = \left(\frac{c^2}{\omega^2}\right) \frac{44(1 - T^3)}{(-81T^6 - 39T^4 + 360T^3 - 240T)} \tag{6.4}$$

Taking the redshift at the onset of acceleration as 1.4 we have $\frac{R_O}{R_a} = 1.4$, which givescos $\omega t_0 = -0.9864$, and hence $\omega t_0 = 2.96\, rad$.

As $t_0 = 4.26 \times 10^{17} s$, we have $\omega = 6.94 \times 10^{-18} rads^{-1}$. However, this leads to negative values of $\rho$for certain values of t, and we have to discard this value. In order to obtain positive values for ρ for all time t we take the value $\omega = 5.32 \times 10^{-18} rads^{-1}$ which gives b = 6.14 × $10^{27} cm$, corresponding to a redshift of 1.26, at the onset of acceleration. The density of the universe at the present epoch is $1.22 \times 10^{-29}\, g.cm^{-3}$ agreeing with the observations as stated under boundary condition (iv).

From $R = b\sqrt{(1 - \cos^3 \omega t)}$, we find that $\frac{dR}{dt}$as$t$ tends to zero is $\sqrt{\frac{3}{2}} b\omega$, which is of the order of the velocity of light for the stated values.

An interesting feature is that Λ'decreases from very high values at $t = 0$ to zero at $t = 1.5 \times 10^{17}\ s$, and is negative till $t = 1.02 \times 10^{18}\ s$ becoming negative again at

$t = 1.35 \times 10^{18}\ s$. The significance of negative values of Λ' will be discussed under dark matter in the next section.

6. Schwarzschild – de Sitter Metric with Λ

It is not only dark energy that could be represented by Λ. We may identify Λas representing dark matter (see for example Trimble [12]) as well. We write down the field equations in the absence of ordinary matter and radiation, in the form

$$R^{\mu\nu} - \frac{1}{2}\bar{R} g^{\mu\nu} = -\Lambda g^{\mu\nu} \tag{6.1}$$

and write the Schwarzschild metric in the form

$$ds^2 = (1 - \frac{2m}{r} - \frac{1}{3}\Lambda r^2)c^2 dt^2 - \frac{dr^2}{(1 - \frac{2m}{r} - \frac{1}{3}\Lambda r^2)} - r^2 (d\Theta^2 + sin^2\Theta d\varphi^2)$$

(6.2)

for a spherically symmetric object.

The geodesic equations for constant $r = a$, and $\theta = \frac{\pi}{2}$ can be written as (Jayakody [5])

$$a^2 \frac{d\varphi}{ds} = h$$

(6.3)

and

$$h^2 = [\frac{3m-\Lambda a^3}{3(a-3m)}]\, a^2c^2 \qquad (6.4)$$

eliminating $\frac{dt}{ds}$, where$h$ is a constant, $m = \frac{GM}{c^2}$, M being the mass of the spherically symmetric object. Since *a* can be assumed to be greater than 3*m*,(6.4) implies that $\Lambda < \frac{3m}{a^3}$.

The equation (6.3)gives the angular momentum h per unit mass of a particle describing a circular path of coordinate radius *a* around a spherically symmetric distribution of mass *M*, and the equation (6.4) expresses *h* at a given a in terms of m and Λ.

If Λ =0 and *m* is negligible the equation (6.4) reduces to

$$h^2 = mac^2 \qquad (6.5)$$

corresponding to the Newtonian equation. When *m* =0, the equation (6.4) reduces to

$h^2 = \frac{-\Lambda a^4}{3}c$ discussed under section 4.

Dark matter had to be formulated [12] as it was noted that the right hand side of the equation (6.5) with the mass of the object could not account for the large *h* that was observed. However, from the equation (6.4) it is clear that a larger *h* could be accounted for at given *a,* for suitable values of Λ. Assuming that the mass required to generate the velocity observed is five times the mass of the object, we find that Λ that satisfies the equation

$[\frac{3m-\Lambda a^3}{3(a-3m)}]\, a^2c^2 = 5mac^2$ could account for the velocities around the central body. This

$$\Lambda = \frac{-3m\,(4a-15m)}{a^4} \qquad (6.6)$$

which could be satisfied if 4*a*> 15 *m*, and $\Lambda < 0$. In our formulation $\Lambda$is a variable, and for some models of the universe it should be possible to find suitable values satisfying (6.6). The model discussed under the previous section gives negative Λ for certain values of cosmic time *t.* If the mass required to account for high values of *h* is *n* times the mass of the object, then we have

$[\frac{3m-\Lambda a^3}{3(a-3m)}]\, a^2c^2 = nmac^2$, giving $\Lambda = \frac{-3m[(n-1)a-3nm]}{a^4}$.

Λ can be positive if$m > \frac{(n-1)a}{3n}$. Thus even if the pressure due to space –time is negative, it is possible to have circular motion of high values of *h*, for suitable values of *m*, *n* and *a*, as *m* would overcome the effect of Λ, provided that $\Lambda < \frac{3m}{a^3}$ as seen from (6.4). Jayakody [5] has also shown that for a ray of light passing at a distance r_0 from the centre of the spherically symmetric body the angle of deflection is given by 2δ where

$$\delta = \left(\frac{2\varepsilon}{3r_0} + \frac{5\pi\varepsilon^2}{24r_0^2}\right)\left(1 + \frac{r_0^2\Lambda}{6} - \frac{2\varepsilon^2}{9r_0^2}\right)^{-1}$$

where $\varepsilon = 3m$.

Neglecting terms of order higher than 1 of Λ and terms of order higher than 2 of ε, Jayakody [5] has shown that

$$2\delta \approx \frac{4m}{r_0} + \frac{15\pi m^2}{4r_0^2} - \left(\frac{2r_0 m}{3} + \frac{5\pi m^2}{8}\right)\Lambda$$

This agrees with the Schwarzschild value $\frac{4m}{r_0}$as a first approximation, and also could be seen to be greater than that value when $\Lambda<0$. Thus negative Λ increases the angle of deflection of a light ray passing near a spherically symmetric object. It is possible to obtain values of $2\delta>\frac{4m}{r_0}$ even when $\Lambda>0$ for suitable values of r_0 and m.

7. Conclusion

Field equations in General Relativity are written with the cosmological term considered as a variable, on the right hand side, interpreting the corresponding tensor not as a geometric tensor but as the energy momentum tensor of the space – time. With this formulation the space – time itself contributes to the energy momentum that determines the metric tensor. The energy of the space- time and that of the ordinary matter and radiation is conserved as a whole, thus the energy of the space – time is "converted" to other forms of energies, and vice versa. This implies that matter could be created not as in the C – field theory but at the expense of the energy of space – time. This energy is different from the vacuum energy, eliminating the so called coincidence problem and the cosmological constant problem, as $\Lambda$is a variable.

Λalso gives rise to acceleration of the universe at the present epoch with possible deceleration in the future as $\ddot{R}$ could be negative for certain values of Λ. The energy of the space - time is identified as dark energy. It should be emphasized that it is the energy momentum tensor of the space - time that is important as the acceleration of the universe is caused by the negative pressure due to Λ in the relevant energy momentum tensor.

Λ is also identified with dark matter, especially when it has negative values. As Cho [13] has mentioned there is little hope for finding WIMPS to explain dark matter and we are of the view that our hypothesis is an answer to the problems discussed in literature.

8. Limitations and recommendations

We are limited and constrained by not coming out with a detailed model to explain the expansion of the universe and we recommend the working out of a detailed model and also calculations in regard to weak lensing in connection with dark matter.

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