

# Evaluation of Lean Distribution Systems of Electricity Production Networks with Mathematical Programming and Simulation Method in Photovoltaic Power Generations

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## Abstract

Nowadays waste of energy is one of the most important problems power plants are faced with all over the world. Because of costly energy sources, notably fossil energy, renewable energy technologies are becoming more indispensable. Since no specific published record has yet been found of studying mathematical programming and simulation method applied to solar power plants and photovoltaic power generations in buildings as a lean manufacturing method, in this paper, by using mathematical programming and simulation method, we propose an approach consisting of the combination of mathematical and economic models used for distribution systems of electricity transmission networks in solar power plants installed out of the city, and photovoltaic power generations in buildings of the city.

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The main approach is to use the simulation method for both models and compare the results with each other. Finally, by analyzing the test results, the optimum process of solar energy is presented. This aims- above all- to lead to cost reduction by appraising the percentage of country energy demand in distribution systems of electricity transmission networks. Moreover, the paper goes on to discuss constructing new solar power plants, or using new methods and technologies to satisfy the country demand which has the minimum rate of energy waste and the maximum earned value. Besides, system costs and the rate of energy waste during the transmission are considered as two criteria for comparison, and then by the test results of the simulation, the optimum procedure of the suggested models is presented.

**Keywords:** Mathematical programming; electricity production; solar power plants; simulation method; lean manufacturing; photovoltaic power generations.

## **1. Introduction**

There is a pressing need to accelerate the development of advanced clean energy technologies all over the world in order to decrease the problem of global warming as well as the threatening major change to the climate caused by dirty old fossil fuels. Solar energy is believed to be a kind of renewable, sustainable and endless energy [1]. Solar energy, when used in applications such as lighting, space heating, and water heating, is a proven viable alternative to non-renewable energy sources [2, 3]. Therefore, using solar power plants can be a reasonable replacement for old power plants which cause atmospheric pollution on a massive scale. The most important application of solar power plants is electricity production which provides energy for performing different activities as a vital demand for a modern society. To add value to the distributed nature of solar generated electricity, it is important to know the PV capacity of the different regions of a city when installing a PV power plant in order to select the feeder with the greatest capacity credit [4-11]. The photovoltaic electricity generation in Taiwan had the potential of 36.1 TWh, accounting for 16.3% of the total domestic electricity consumption [12]. The daily average solar radiations of Gaize and Florianopolis were ranged from 14 to 27 MJ/m<sup>2</sup> and from 2.46 to 5.72 kWh/m<sup>2</sup> respectively [13, 14]. The annual average solar radiation was obtained between 750 kWh/m<sup>2</sup> and 2485 kWh/ m<sup>2</sup> for Lake Van Region in Turkey [15]. The rooftop solar photovoltaic potentials were quantified as 885.1 kWh and 10 TWh in [16, 17], respectively. The hybrid photovoltaic/thermal solar system increased the production of electrical and thermal energy by 38% [3]. The cumulative installed photovoltaic power increased to 1.45 GW in USA, 2 GW in Japan and 5.3 GW in Germany by the end of 2006–2008, respectively [18]. Most of the studies given above generally use Weibull model, Rayleigh distribution model, maximum entropy theory, artificial neural networks, feature extraction algorithms, and geographical information systems in the stage of analyzing wind and solar power potentials [19]. Besides, the above researches show the importance of PV solar power plants utilized in different countries.

There have been different researches to study the performance and financial improvement of various solar power plants and PV generations. In 1931, a description of a solar chimney power plant was presented by Gunther [20]. Pasumarthi and Sherif built a complete mathematical model and analyzed the experimental and theoretical performances of a demonstration solar chimney power system [21]. Reference [22] presented a thermodynamic cycle analysis of the solar chimney power plant for the calculation of limiting performance,

efficiency, and the relationship between the main variables including chimney friction, system, turbine and exit kinetic energy losses. Reference [23] studied the correlation between electricity spot market prices and PV generation, which they found to give a good indication of the additional value of PV electricity. Considering the high solar potential of most Greek territories, an integrated study was conducted by [24] based on long term solar potential experimental measurements in order to determine the optimum configuration of a photovoltaic system at representative locations all-over Greece. Zhao and his colleagues [25] presented the design optimization of a photovoltaic/thermal (PV/T) system using both non-concentrated and concentrated solar radiations. The system consisted of a photovoltaic (PV) module using a silicon solar cell and a thermal unit based on the direct absorption collector (DAC) concept. Shah and his colleagues [26] studied the impact of large scale PV generation on power system oscillation, through which the inter-area oscillation was specially considered. The effect of PV on inter-area mode was investigated in New England–New York test system for different levels of penetrations and operating conditions. The analysis revealed that increased PV penetration could affect the critical inter-area mode detrimentally. Besides, several works have been done to address the challenges introduced by the proliferation of PV generations [26-35]. Forero and his colleagues [34] introduce a system developed for monitoring PV solar plants using a novel procedure based on virtual instrumentation. The measurements and processing of the data were taken care of using high precision I/O modular field point (FP) devices as hardware, a data acquisition card as software and the package of graphic programming, Lab VIEW. The system was able to store and display both the collected data of the environmental variables and the PV plant electrical output parameters, including the plant I–V curve. Mediavilla and his colleagues [35] studied real PV production from two 100 kWp grid-connected installations located in the same area, both of which experienced the same fluctuations in temperature and radiation. Data sets on production were collected over an entire year and both installations were compared under various levels of radiation. Reference [36] proposed a simple constrained optimization method. The objective function was the electrical output power, and the constraints were the current–voltage characteristic and the energy balance at the level of a series–parallel PV modules. The method depicted the influence of various climates on the optimal PV cells interconnections. Also, it should not be left unmentioned that the output of PV modules changes seasonally in proportion to changes in solar radiation [36-48].

Although there have been many researches related to solar power plants and PV generations, notably PV ones, there has hardly ever been a significant attempt to present mathematical programming and simulation method applied to solar power plants and photovoltaic power generations in buildings as a lean manufacturing method. Thus, in this paper, it has been aimed to study solar power plants and photovoltaic generations in buildings. The former is installed out of a city where different parameters are considered including the cost of production, the costs of power transition, the additional costs of power, set up costs for every solar power plant, etc. However, the latter is a lean manufacturing model consisting of solar panels for every building in the city separately. Similarly, different aspects of the model are also presented including solar panels capacity, the number of needed solar panels, the price of every solar panel, etc. In order to analyze the two models and compare the results with each other, the optimized model is considered for both the above- mentioned, applying the simulation method to the models until the test results are observed. Finally, the test results of the simulation and mathematical programming are presented to indicate the advantages and disadvantages of the two models.

Besides, the study can guide users whether to utilize the electricity production obtained from the construction of solar power plants out of the city, or from the photovoltaic generations installed in every building.

To make the paper clear, the following sections are presented with the aim of focusing on the discussed subjects.

## **2. Solar energy for electricity production**

Using solar energy as a source of producing electricity has certain advantages and disadvantages. Although there are many advantages of using solar energy- including the decrease of environmental pollution [49], its being cost effective compared to the projected high cost of oil [50, 51], the fact that it is renewable, sustainable, endless [1], and easy to use [52], etc., some important drawbacks of using it to generate electricity cannot be denied. One of the main disadvantages is the initial cost of the equipment used to harness the energy from the sun. It is obvious as the price of solar panels decreases, the use of solar cells to generate electricity will rise [53, 54]. To be efficient in providing a source of electricity, a solar energy installation requires a large area to host the system. This may be a turn-off in areas where space is short, or expensive. Pollution can be a disadvantage to solar panels, as pollution can degrade the efficiency of photovoltaic cells. Clouds also provide the same influence since they can reduce the energy of the sun's rays. This certain disadvantage is more of an issue with older solar components, as newer designs integrate technologies to overcome the worst of these effects. Also, solar energy is only useful when the sun is shining, so it requires a backup power. During the night, the expensive solar equipment will remain unused despite the fact that the use of solar battery chargers can help to reduce the influence of this drawback; therefore, the location of solar panels can have an effect on the performance. Besides, solar plants typically produce just a fraction of their rated capacity. For instance, TEP operates one of the largest solar PV arrays in the United States, a 5-MW system. But over two years of operation, the capacity factor for that generator has averaged 19%, meaning it produced only 19% of its rated capacity most of the time [55]. It should be noted that solar energy as a renewable source of electricity generation has some limitations which need special consideration in order to improve efficiency and reduce costs. In this paper, authors have tried to apply mathematical programming and simulation method to two models of PV solar power plants and PV generations using the lean manufacturing method for the latter in order to represent the test results for both models.

## **3. Lean manufacturing**

Lean manufacturing emphasizes various parameters including those of reducing waste, boosting efficiency, and obtaining continuous improvement. It focuses on minimizing costs, maximizing customer options, speeding up delivery, and increasing the quality of products and services. Proponents of lean manufacturing have identified roughly eight major types of wastes, namely overproduction, waiting for the next process step, unnecessary transport of materials, over processing of parts, inventories more than minimum amounts, unnecessary movement by employees, production of defective parts, and underutilization of human resources. More broadly, waste can be defined as any activity that does not add value to the product or service. The guiding principle in lean manufacturing is the elimination of non-value-adding activities through continuous improvement efforts. In general, waste consumes resources but creates no value for customers. Waste reduction is thus essential and can

include elimination of excess usage of utilities and materials. The focus of the multidimensional approach is on cost reduction, by eliminating the non-value-adding activities and using tools such as cellular manufacturing. It also focuses on total productive maintenance, production smoothing, setup reduction and the like to omit the waste [56-60].

Modeling is considered for two cases in this paper. In the first case, it is made for solar power plants installed out of the city whereas the second is the consideration of lean manufacturing for photovoltaic generations in buildings in urban areas. Therefore, by applying lean manufacturing to the second case electricity transition costs are omitted. Also, waste transfer and additional processes, costs of electricity production and of land needed for solar power plants are subtracted. It is obvious that analyzing the two cases indicate the optimal model.

#### **4. Simulation method**

Simulation is an imitation of the process or a system over a period of time. In fact, a simulation is used when a problem cannot be solved by modeling in the real world. By using the simulation technique, the model's behavior is discussed and analyzed over a period of time. Therefore, simulation is one of the most common and accepted means of operations research and systems analysis. The followings explain why simulation is considered:

- The simulation of complex systems and sub-systems is feasible.
- Information, organizational, and environmental changes can be simulated and the effects can also be observed.
- By changing the simulation inputs and examining the outputs, comprehensive knowledge of effective variables can be obtained.
- Simulation can be used to test designs before implementing them.
- Simulation can be used for research and analytical purposes [61].

In this paper, optimum models are considered for solar power plants installed out of the city and photovoltaic generations in buildings. After ensuring the simulation method, optimization and simulation are used. Simulation-optimization is a series of research activities which is increasingly being used in practical simulation applications. The recent advancements in simulation-optimization techniques for complex systems involve how they are applied in practice. A deeper exploratory process may be provided by simulation-optimization which is the process of finding the combination of decision variables corresponding to the best performance of a system evaluated through the output of a simulation model of this system [62].

##### ***4.1. Structure of the model***

Linear and nonlinear Programming is considered as a classic method of optimization [63]. In mathematics, nonlinear programming (NLP) is the process of solving a system of equalities and inequalities, collectively termed constraints, over a set of unknown real variables, along with an objective function for the purpose of

maximizing or minimizing, where some of the constraints or the objective functions are nonlinear [64]. Here, in this paper, we use these models to model solar power plants and photovoltaic power generations. In this section, the structure of optimized model with the combination of lean manufacturing approach and nonlinear programming are presented, in the first and second model, we try to minimize and optimize costs of produced power for solar power plants installed out of the city and photovoltaic power generations in buildings respectively. Finally, in the third model, the number of solar panels used for buildings in the city and solar power plants out of the city accorded with the power production demand are presented and compared.

#### **4.2. Assumptions**

Some assumptions are considered in this section in order to solve the model and place constraints on it. The assumptions explain the first and second models which are power plants installed out of the city and photovoltaic power generations as a lean manufacturing in buildings respectively.

- a) Power Plant in this model is a solar photovoltaic (PV) system
- b) All solar panels have the same performance.

#### **4.3. The first model (Optimization model of solar power plants)**

This model is based on nonlinear programming and solar power plants analysis. Some parameters, variables and decision making are considered by this model.

##### **4.3.1. Index**

There are two indexes in this model. The former is the number of power plants installed out of the city. However, the latter is the time period.

j: index of Power Plants (j=1,2,3, ..., m)

t: index of Time period (t=1,2,3, ..., T)

##### **4.3.2. Parameters**

Some parameters are entered into the model to obtain variables. The parameters are as fixed data. Therefore, the decision making is presented by the data.

$C_{jt}$ : Cost per kilowatt hour power produced by (j) power plant in period t (Constant number).

$V_{jt}$ : Cost per kilowatt hour power transferred from (j) power plant in period t (Constant number).

$H_{jt}$ : Cost of excess power produced by (j) power plant in period t (Constant number).

$D_{jt}$ : Power Consumption received from (j) power plant (Continuous uniform probability distribution).

$R_j$ : Setup cost for (j) power plant

$\Delta_{jt}^{\max}$ : Maximum capacity (j) power plant in period t (Constant number).

$\Delta_{jt}^{\min}$ : Minimum capacity (j) power plant in period t (Constant number).

F: Certain number of power plants (Constant Number).

#### 4.3.3. Variables

The purpose of this model is to obtain the best and optimized variables used for solar power plants. The decision making is based on the variables shown below.

$Y_j$ : If (j) power plant use. (1-0)

$Z_{jt}$ : Power produced by (j) power plants per kilowatt hour in period t.

$K_{jt}$ : Energy surplus produced by (j) power plants in period t.

$NPW_{jt}$ : Net present value for (j) power plant in period t.

$Z_{optimum}$ : Objective Function of primary

#### 4.3.4. Proposed model

Operation research method is utilized in this model to obtain optimal variables. This model has some limitations and parameters. The parameters are important data in solar power plants. Thus, the decision making is presented by the data [65, 66]. Also, the method of engineering economics is considered in order to calculate costs over a period of time [67].

$$Min Z_{optimum} = \sum_{j=1}^m R_j Y_j + \sum_{j=1}^m \sum_{t=1}^T (NPW_{jt} + V_{jt}) . Y_j . Z_{jt} + \sum_{j=1}^m \sum_{t=1}^T H_{jt} . K_{jt} . (1+i)^{-t}, (1)$$

$$S.t. \Delta_{jt}^{\min} \leq Y_j . Z_{jt} \leq \Delta_{jt}^{\max}, \forall j = 1, 2, \dots, m, \forall t = 1, 2, \dots, T, (2)$$

$$K_{jt} = Y_j . Z_{jt} - D_{jt}, \forall j = 1, 2, \dots, m, \forall t = 1, 2, \dots, T, (3)$$

$$\sum_{j=1}^m Y_j = F, \forall j = 1, 2, \dots, m, (4)$$

$$Y_j = \{0,1\}, Z_{jt} \geq 0, K_{jt} \in Free, \forall j = 1,2,\dots,m, \forall t = 1,2,\dots,T, (5)$$

**4.4. The second model (Optimization model of photovoltaic power generations)**

This model analyzes photovoltaic power generations installed in the buildings of the city based on a mathematical model. The decision making is presented accorded with the parameters which are vital to the photovoltaic power generations analysis.

**4.4.1. Parameters**

I: Interest Rate

T: Production Life (operation period)

Q: Operational Costs

C: Cost for Purchase One Solar Panel

A: Total Consumption criterion quantity

B: Production Capacity of Each Solar Panel

N: Number of Solar Panels Needed

**4.4.2. Variables**

Z: Electricity Produced by each Solar Panel

F: Objective Function in Second model

**4.4.3. Proposed model**

This model is for photovoltaic power generations installed in buildings of the city. It is based on mathematical models [68, 69].

$$F = \frac{A.C.Q.Z}{B} + \int \sum_{t=1}^T \frac{A.Q.Z}{B} (1+I)^{-t} .dZ, (5) \Rightarrow F = \frac{A.C.Q.Z}{B} + \int \frac{A.Q.Z}{B} \left( \frac{1 - \left( \frac{1}{(1+I)} \right)^T}{I(1+1)} \right) .dZ, (6)$$



$$\Rightarrow F = \frac{A.C.Q.Z}{B} + \frac{A.Q.Z^2}{2.B} \left[ \frac{1 - \left( \frac{1}{1+I} \right)^T}{I(I+1)} \right], (7) \Rightarrow \frac{\partial F}{\partial Z} = 0, (8) \Rightarrow \frac{A.C.Q}{B} + \frac{A.Q.Z}{B} \left[ \frac{1 - \left( \frac{1}{1+I} \right)^T}{I(I+1)} \right] = 0, (9) \Rightarrow \frac{A.Q.Z}{B} \left[ \frac{1 - \left( \frac{1}{1+I} \right)^T}{I(I+1)} \right] = -\frac{A.C.Q}{B}, (10) \Rightarrow Z^* = \left[ \frac{-C.I(I+1)}{1 - \left( \frac{1}{1+I} \right)^T} \right], (11)$$

The derivative of the model is calculated to obtain the optimal amount of Z\*, then by entering the amount, the optimized model is presented:

$$F = \frac{A.C.Q}{B} \cdot \left[ \frac{-C.I(I+1)}{1 - \left( \frac{1}{1+I} \right)^T} \right] + \frac{A.Q}{2.B} \cdot \left[ \frac{-C.I(I+1)}{1 - \left( \frac{1}{1+I} \right)^T} \right]^2 \cdot \left[ \frac{1 - \left( \frac{1}{1+I} \right)^T}{I(I+1)} \right], (12) \Rightarrow$$

$$F = \frac{A.C.Q}{B} \cdot \left[ \frac{-C.I(I+1)}{1 - \left( \frac{1}{1+I} \right)^T} \right] + \frac{A.Q.C^2}{2.B} \cdot \left[ \frac{I(I+1)}{1 - \left( \frac{1}{1+I} \right)^T} \right], (13) \Rightarrow F = \frac{-AC^2.Q.I(I+1)}{2B \cdot \left[ 1 - \left( \frac{1}{1+I} \right)^T \right]}, (14)$$

#### 4.4.4. Dependence on the number of solar panels with power plants

In the first model, Z<sub>i</sub> is calculated by the following formula:

$$z_i = \frac{-\sum_{j=1}^m \int_a^x H_j \cdot K_j \cdot dK_j (1+i)^{-t} - \sum_{j=1}^m R_j \cdot Y_j}{\sum (NPW_j) Y_j}, (13)$$

In the second model, Z<sub>i</sub> can be calculated as the first model:

$$Z = \frac{-N.C.Q + \sqrt{(N.C.Q)^2 + 2F.N.Q \cdot \left[ \frac{1 - \left( \frac{1}{1+I} \right)^T}{I(I+1)} \right]}}{N.Q \cdot \left[ \frac{1 - \left( \frac{1}{1+I} \right)^T}{I(I+1)} \right]}, (14)$$

In order to calculate  $N^*$ , the two above formulas are set equal to each other as the following:

$$\frac{-N.C.Q + \sqrt{(N.C.Q)^2 + 2F.N.Q \left( \frac{1 - \left(\frac{1}{1+I}\right)^T}{I(I+1)} \right)}}{N.Q \left( \frac{1 - \left(\frac{1}{1+I}\right)^T}{I(I+1)} \right)} = \frac{-\sum_{j=1}^m \int_a^x H_j \cdot K_j \cdot dK_j (1+i)^{-t} - \sum_{j=1}^m R_j \cdot Y_j}{\sum (NPW_j) Y_j}, \quad (15)$$

Then,  $N^*$  is obtained from the above equality. It is conspicuous that by entering the parameters of the third model, the variables are obtained continuously, so when  $Z^*$  is entered,  $N^*$  is obtained.

## 5. Simulation with arena software

Simulation is the combination of methods and applicable tools which are appropriate for real systems. In this paper, it is used as a suitable method for the operation analysis of solar power plants and solar power generations since the study of physical systems is very difficult and expensive or even impossible in certain cases. This process was performed through Arena simulation software. Arena helps, demonstrates, predicts, and measures system strategies in order to bring out an effective, efficient and optimized performance. Thus, it is a very popular simulation modeling software. Also, it protects business by analyzing the impact of data, without causing disruptions in services. When the life of a business is at stake, Arena improves business performance. This method was used for the analysis of solar power plants and solar power generations in the first and second model [70].

### 5.1. The first model (Simulation modeling of solar power plants)

The main aim of the simulation model analysis is to evaluate the effects of uncertainties on the demand rates. Since the demand rates are uncertain, they can be characterized by a certain probability distribution. In this section, to investigate the solar power plants by a simulation method, the Arena software is utilized to ensure the accuracy of the model. Figure 1 demonstrates the simulation modeling of the first model.

### 5.2. The Second model (Simulation of solar power generations)

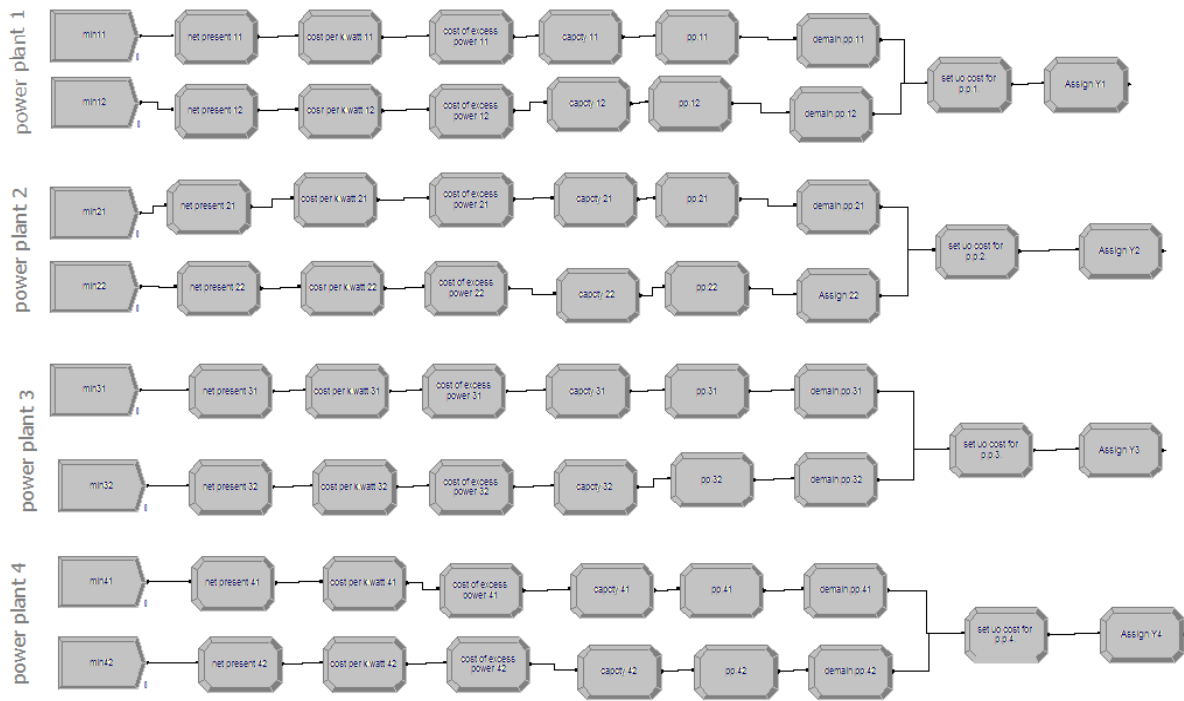
Figure 2 illustrates the simulation of the second model based on the solar power generations data. These data can be different for every solar power generation installed in buildings of the city.

## 6. Results of the optimization and simulation modeling

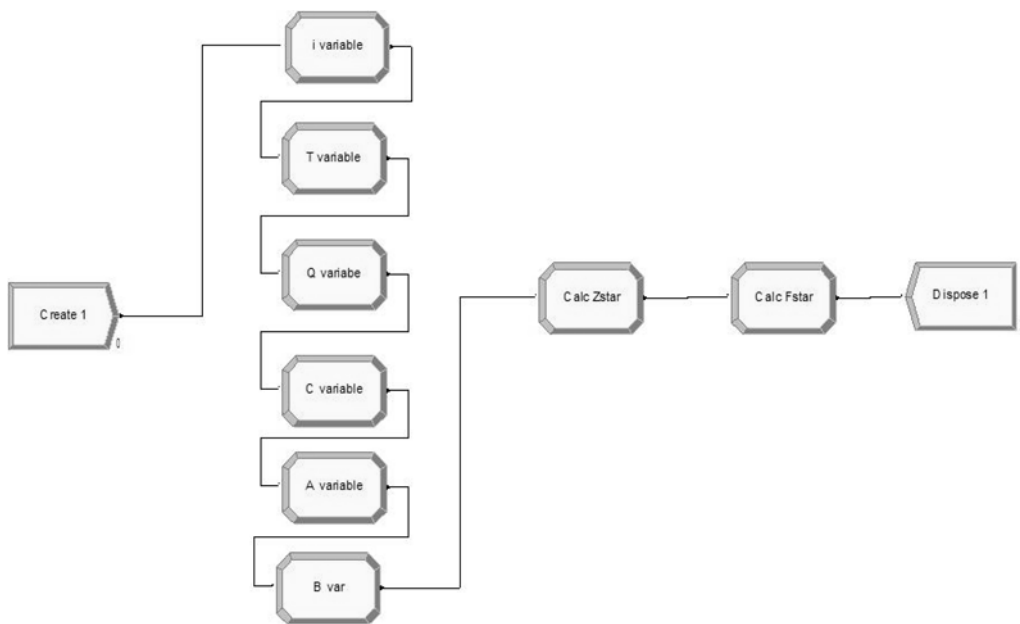
In this section, an example of the analysis of solar power plants installed out of the city and solar power generations in buildings of the city is presented. Therefore, the output of the first and second optimum models and the simulation model analysis are examined.

**6.1. Results of the first model (Optimization modeling of solar power plants)**

In this section the results of the first model according to the data are presented in table 1.



**Figure 1:** simulation of the first optimum model



**Figure 2:** simulation of the second optimum model

**Table 1:** The data of the first model for low demand

The input of the first model (optimization model) – low demand									The output of the first model (optimization model)		
Row	$R_j$	$NPW_{jt}$	$V_{jt}$	$H_{jt}$	$\Delta_{jt}^{\min}$	$\Delta_{jt}^{\max}$	$D_{jt}^{\min}$	$Y_j$	$Z_{jt}$	$K_{jt}$	
j=1	t=1	5,000,000,000	35,000	34,000	12,000	3,000	4,000	4,000	1	4000	0
j=1	t=2	5,000,000,000	45,000	40,000	13,000	3,500	4,500	4,200		4200	0
j=2	t=1	3,500,000,000	45,000	35,000	18,000	3,200	4,000	4,300	1	4300	0
j=2	t=2	3,500,000,000	25,000	30,000	20,000	3,300	4,000	4,000		4000	0
j=3	t=1	4,000,000,000	32,000	25,000	14,000	3,100	4,000	3,900		0	3900
j=3	t=2	4,000,000,000	30,000	29,000	15,000	3,200	4,000	3,800		0	3800
										0	
j=4	t=1	3,000,000,000	28,000	40,000	9,000	3,400	4,000	4,000	1	0	0
j=4	t=2	3,000,000,000	50,000	39,000	15,000	3,500	4,500	4,100		4100	0

**Table 2:** The data of the first model for medium demand

The input of the first model (optimization model) – medium demand									The output of the first model (optimization model)		
Row	$R_j$	$NPW_{jt}$	$V_{jt}$	$H_{jt}$	$\Delta_{jt}^{\min}$	$\Delta_{jt}^{\max}$	$D_{jt}^{\text{average}}$	$Y_j$	$Z_{jt}$	$K_{jt}$	
j=1	t=1	5,000,000,000	35,000	34,000	12,000	3,000	4,000	4,500	1	0	5000
j=1	t=2	5,000,000,000	45,000	40,000	13,000	3,500	4,500	4,700		5200	0
j=2	t=1	3,500,000,000	45,000	35,000	18,000	3,200	4,000	4,800	1	5300	0
j=2	t=2	3,500,000,000	25,000	30,000	20,000	3,300	4,000	4,500		0	5000
j=3	t=1	4,000,000,000	32,000	25,000	14,000	3,100	4,000	4,400	1	5900	0
j=3	t=2	4,000,000,000	30,000	29,000	15,000	3,200	4,000	4,300		5800	0
j=4	t=1	3,000,000,000	28,000	40,000	9,000	3,400	4,000	4,500	0	4000	5000
j=4	t=2	3,000,000,000	50,000	39,000	15,000	3,500	4,500	4,600		4100	5100

The results of the optimization model are presented according to different input data shown on three tables including low, medium, and high demand respectively.

In table 1, the input data of the optimization model are entered into the Lingo 8 software and the output is given on the right.

The optimum amount of the objective function is obtained from the software ( $Z=43600$ ) [64].

Table 2 demonstrates the input and output data of the second sample (medium demand). The output data and the optimum amount of the objective function ( $Z=51300$ ) are calculated by the Lingo 8 software.

In table 3, the input and output data of the third sample (high demand) are illustrated. As mentioned earlier, the output data and the optimum amount of the objective function ( $Z=36000$ ) are obtained from the Lingo 8 software.

**Table 3:** The data of the first model for medium demand

Input of primary model (optimization) - high demand									Output of primary model (optimization)		
Row		$R_j$	$NPW_{jt}$	$V_{jt}$	$H_{jt}$	$\Delta_{jt}^{\min}$	$\Delta_{jt}^{\max}$	$D_{jt}^{\max}$	$Y_j$	$Z_{jt}$	$K_{jt}$
j=1	t=1	5,000,000,000	35,000	34,000	12,000	3,000	4,000	5000	1	0	4500
j=1	t=2	5,000,000,000	45,000	40,000	13,000	3,500	4,500	5200		4700	4500
j=2	t=1	3,500,000,000	45,000	35,000	18,000	3,200	4,000	5300	1	4800	0
j=2	t=2	3,500,000,000	25,000	30,000	20,000	3,300	4,000	5000		0	0
j=3	t=1	4,000,000,000	32,000	25,000	14,000	3,100	4,000	4900	1	5400	0
j=3	t=2	4,000,000,000	30,000	29,000	15,000	3,200	4,000	4800		5300	0
j=4	t=1	3,000,000,000	28,000	40,000	9,000	3,400	4,000	5000	0	0	4500
j=4	t=2	3,000,000,000	50,000	39,000	15,000	3,500	4,500	5100		0	4600

**6.2. Results of the second model (Optimization modeling of photovoltaic power generations)**

In this section, the presented results of the second model according to the data are shown in table 4.

**Table 4:** Input and output of the second model

	Alternative 1	Alternative 2	Alternative 3	Alternative 4	Alternative 5
Made in	Korea	china	Taiwan	U.S.A	Japan
I	0.25	0.12	0.18	0.10	0.13
T	60 month	12 month	48 month	36 month	24 month

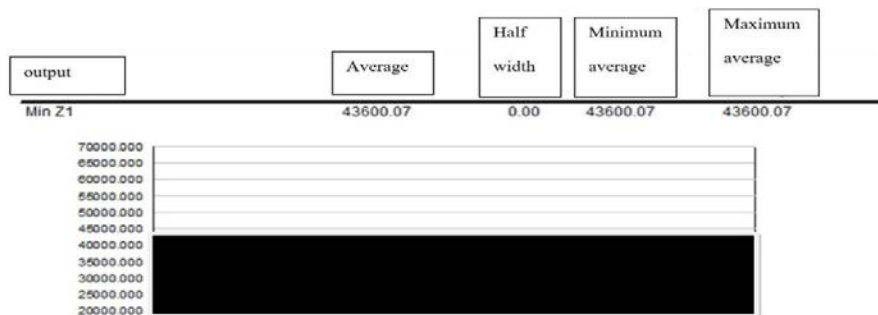
Q	80 YEAR/\$	10 YEAR/\$	50 YEAR/\$	90 YEAR/\$	50 YEAR/\$
C	410\$	170\$	250\$	390\$	433\$
A	456250 W/YEAR	32850 W/YEAR	423400 W/YEAR	279225 W/YEAR	175200 W/YEAR
B	250 W	90 W	290 W	255 W	240 W
Z*	191.23	213.248	109.66	172.50	138.71
F*	5723554104 \$	66160192 \$	1000674035 \$	3315114965 \$	1096156222 \$

**6.3. Results of the equality of the first and second model**

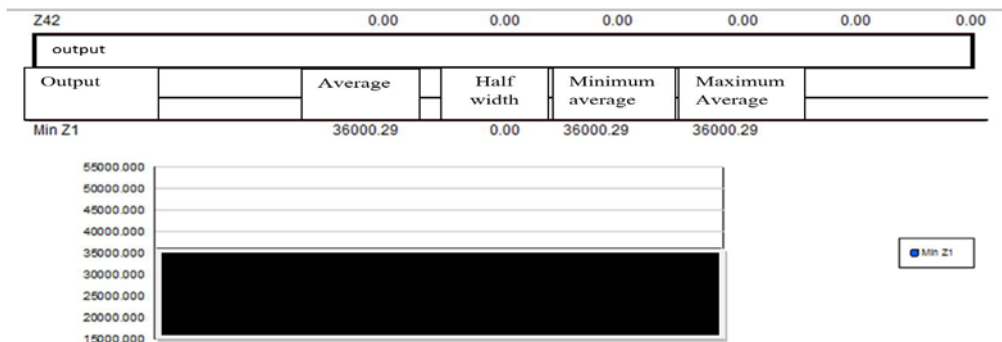
The optimum number of solar panels used for the photovoltaic power generations installed in buildings of the city is  $10 \times 10^{28}$  ( $N^* = 10 \times 10^{28}$ ) based on the optimum amount of Z obtained from the above model.

**6.4. Results of the first model (Simulation modeling of solar power plants)**

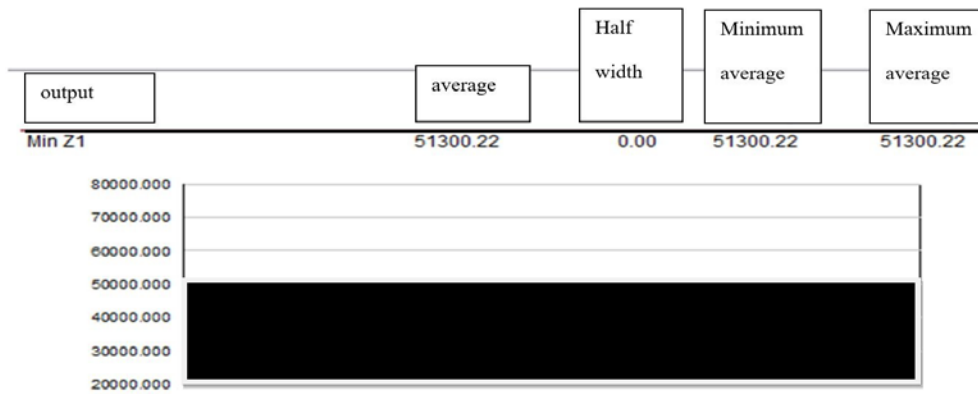
Figure 3-5 shows the optimum amount of Z using simulation method for low, medium, and high demand in the case of deterministic parameters.



**Figure 3:** image of the optimum amount of Z for three samples in the case of deterministic parameters (Part 1)



**Figure 4:** image of the optimum amount of Z for three samples in the case of deterministic parameters (Part 2)



**Figure 5:** image of the optimum amount of Z for three samples in the case of deterministic parameters (Part 3)

In order to compare the results of the simulation and optimum models for the three mentioned samples, table 5 indicates the optimum amount of the objective function related to each sample. As it can be seen in table 5, the differences between the simulation and optimum models are negligible for the three samples.

**Table 5:** Comparison of the simulation and optimum models for low, medium, and high demand

The output of the simulation model	The output of the optimum model
Z=43600.07	Z=43600
Z=51300.22	Z=51300
Z=36000.29	Z=36000

**6.5. Results of the second model (Simulation modeling of the photovoltaic power generations)**

Table 6 shows the simulation data of the second model based on the kind of the solar panels and other effective parameters which are considered in the second model.

In this table, the alternatives are the various types of solar panels which are different for every country. As it is depicted in table 6, five alternatives are presented. By considering the data,  $F^*$  and  $Z^*$  are simulated.

**Table 6:** Simulation output of the second model (Adapted from [71])

A 95% confidence interval for the objective function ( $F^*$ ) (the output of the simulation model)	The probability distribution of the variable T Years	
[5662487708, 5733023128]	Triangular(4,5,6)	Alt 1
[65545901.87, 69445381.87]	Triangular(.5,1,1.5)	Alt 2
[993496430.9,1003419410.9]	Triangular(3.5,4,5)	Alt 3
[3279248413.7,3333170673.7]	Triangular(2.5,3,3.5)	Alt 4
[2286367411,2344968310.7]	Triangular(1.5,2,2.5)	Alt 5

**Table 7:** The output of first and second methods

Value of the objective function (The output of the optimum model)	Value of the objective function (The output of the simulation model)	
5703803620	5703803620	Alt 1
66160192	66160192	Alt 2
1000674035	1000674035	Alt 3
3315114965	3315114965	Alt 4
2317898362	2317898362	Alt 5

To balance the first and second models, the output data of a single solar power generation are indicated in table 8 since the data shown in table 7 are for a number of solar power generations. Thus, the following calculations are performed to show the data for a single solar power generation.

compare the results of the simulation and optimum models,  $F^*$  of the two models related to photovoltaic power generations are presented in table 8.

The results of the simulation and mathematical models indicate that the differences between the two models are negligible ( $|\text{The output of the simulation model} - \text{The output of the mathematical model}| < 0.01$ ).

$$\text{Alt 1 : } n = \frac{A}{B} = \frac{456250}{250} = 1825 \times 191.23 = 349159 \tag{16}$$

$$\frac{5703803620}{349159} = \mathbf{16346.08}$$

$$\text{Alt 2 : } \frac{32850}{90} = 365 \times 213.24 = 77832.6 \tag{17}$$

$$\frac{66160192}{77832.6} = \mathbf{850.03}$$

$$\text{Alt 3 : } \frac{423400}{290} = 1460 \times 109.66 = 160103.6 \tag{18}$$

$$\frac{1000674035}{160103.6} = \mathbf{6250.16}$$

$$\text{Alt 4 : } \frac{279225}{255} = 1095 \times 172.50 = 188887.5 \tag{19}$$

$$\frac{3315114965}{188887.5} = \mathbf{17550.73}$$

$$\text{Alt 5 : } \frac{172500}{240} = 730 \times 138.71 = 101258.3 \tag{20}$$

$$\frac{2317898362}{101258.3} = \mathbf{22890.94}$$



**Table 8:** Comparison of the simulation and optimum models in the case of deterministic parameters for a single solar power generation.

Value of the objective function (The output of the simulation model)	Value of the objective function (The output of the optimum model)	
16346.08	16346.08	Alt 1
850.03	850.03	Alt 2
6250.16	6250.16	Alt 3
17550.73	17550.73	Alt 4
22890.94	22890.94	Alt 5

## 7. Conclusion

In this paper, by using mathematical programming and simulation methods, an approach is presented including the combination of mathematical and economical models used for distribution systems of electricity transmission networks in solar power plants installed out of the city and photovoltaic power generations in buildings of the city as a lean manufacturing process. It was because there was no specific published record of comparing the simulation and mathematical methods applied to both solar power plants and photovoltaic power generations. The two models were compared and the results showed that the mathematical and simulation models have practically the same results. It can be concluded that the first and second models have the same operation, which is really useful for using both methods as reliable techniques of analyzing the operation of solar power plants and photovoltaic power generation utilized in different buildings. It is obvious that the operation analysis obtained from the two models is a procedure used to determine the efficiency. Although different results can be obtained from different situations based on various data of the first and second models, in this paper, photovoltaic power generations installed in the buildings produce better results in comparison with solar power plants installed out of the city due to the comparison of the value of  $Z$  and  $F^*$ . For instance, according to table 5 and table 8, the value of  $Z$  for solar power plants (Low demand) was 43600 ( $Z=43600$ ) and the value of  $F^*$  for photovoltaic power generations in Alt1 (Korea) was 16346.08 ( $F^*=16346.08$ ). Therefore, it can be concluded that solar power generations have better economic performance in comparison with solar power plants. Consequently, the main objective of this paper is to present the two mentioned methods as reliable techniques for analyzing and evaluating different solar power plants and photovoltaic power generations for the sake of introducing the optimum model which has the minimum rate of energy waste and the maximum efficiency.

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