

# Optimal Placement of Phasor Measurement Unit on Electrical Grid Using a Hybrid Technique

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## Abstract

This paper presents a methodology to determine the optimal location of phasor measurement units (PMUs) in any network to make it observable. This proposed methodology is based on network connectivity information for the optimal placement of Phasor Measurement Unit (PMU) that minimizes the cost of installation and provide the entire power system observability. The method is based on hybridizing the Particle Swarm Optimization and tabu-search (PSO-TS) algorithm. The tabu-lists are used within the PSO algorithm: the first one aims to differentiate the best solutions obtained by particles while the second prevent local optimal solutions non-respecting the constraints. The proposed algorithm is tested on IEEE 14-bus and IEEE 30-bus systems.

**Keywords:** Phasor Measurement Units (PMUs); Particle Swarm Optimization (PSO); Tabu-search (TS); hybrid method; Network connectivity.

## 1. Introduction

In modern world, countless countries around the world are affected by power failures, which are caused by factors such as lack of investment in power system infrastructure, bad maintenance culture and increase in load at the consumer end which stresses the power transmission and distribution system.

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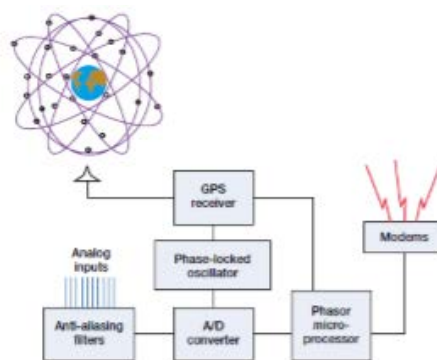
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Consequently, power companies suffer from losses of billions of dollars, and inconvenience to private and business customers. Presently, many devices are developed to provide near instantaneous measurements (phase, current, and voltage). One of those devices is called Phasor Measurement Units (PMU), [1]. It provides synchronized measurements of real-time phasors of bus voltages and branch currents. Synchronicity among PMUs is achieved by same-time sampling of the voltage and the current waveforms using a common synchronizing signal from the Global Positioning System (GPS). Obtaining phasors from different buses of a power system with the same time-space can increase performances of monitored control system in various fields of application, such as state estimation and stability analysis [2]. Appropriate methodology is needed to determine the optimal placements of PMUs in a power system network. In addition to its ability to measure voltage and current phasors, a PMU may include other features such as protective actions in power system.

The scope of the present paper is limited to the optimal location of PMUs in power system and use of PMUs for state estimation. A power system is called fully observable only when all of its states can be uniquely determined [3]. The goal of the present work is to find the minimum number of PMUs to make the system topologically observable, as well as the optimal locations of these PMUs. In modern years, there has been significant research work on finding the minimum number for the optimal placement problem of PMUs. In [4], a bisecting search method was implemented to find the minimum number of PMUs to make the system observable. In [5], the authors used a simulated annealing technique to find the optimal PMU locations. In [6], a genetic algorithm (GA) was used to find the optimal PMU locations. The minimum number of PMUs needed to make the system observable was found using a bus-ranking methodology. The authors in [7] and [8] used integer programming to determine the minimum number of PMUs. In [9], the author used the condition number of the normalized measurement matrix as a criterion for selecting candidate solutions, along with binary integer programming to select the PMU locations. The author in [10] used the tabu search (TS) algorithm to solve the optimal PMU placement problem. However, the number of PMUs was determined by trial and error and only their locations were determined using the TS algorithm. The authors in [11] proposed a particle swarm optimization (PSO)-based algorithm to find both the optimal number and locations for the PMUs to be placed, but their study did not cover cases such as the loss of the PMU in a network. The author in [12] used an exhaustive search approach to determine the minimum number and optimal placement of the PMUs for state estimation, considering single branch outages only. In [13], the authors proposed a method to find the optimal conventional measurement set that is necessary for complete system numerical observability, considering a single measurement loss and single-branch outage contingency. Next, the positions of these measurements were rearranged by a heuristic algorithm in order to minimize the number of PMU placement sites. However, both the essential and the redundant conventional measurement sets were found by trial and error. The authors in [14] proposed a PMU placement algorithm that was based on a metaheuristic method. First, PMUs were placed at the most important network nodes, and then the iterated local search algorithm was used to minimize the number of PMUs needed to observe the network. The work limited the location and number of PMUs to be placed and did not consider the system during contingencies. In [15] and [16], the authors used binary PSO and modified binary PSO algorithms to find the optimal number and location of PMUs to maintain system observability. The proposed algorithms studied the system in the case of normal operation and in the case of the loss of a single branch or single measurement.

However, the authors did not only depend on the PMU to maintain system observability, they also used the existing conventional measurements. In addition, the algorithm allowed the existence of solutions violating the observability constraint and did not neglect them from the beginning or during the search as in this paper.

The optimal solution was then determined as the solution with minimum number of PMUs and the minimum number of unobservable buses. Integer programming was used in [17] to find the optimal location of the PMUs to make the system observable. However, the number of PMUs was pre-set to 10% of the bus number, chosen according to their priority index. To keep the system observable, the PMUs were fixed at these locations and the conventional supervisory control and data acquisition measurements were optimally located. Effective utilization of this technology is very useful in mitigating blackouts and learning the real time behaviour of the power system. Figure 1 shows the basic block diagram Phasor Measurement Unit device [18]



**Figure 1:** Block diagram of the Phasor measurement unit

## 2. Material and Methods

This project is concerned about the optimal placement of phasor measurement units (PMUs) so as to make a system completely observable for normal operating condition as stated earlier, but the required process through which the scope of this research was accomplished have not been discussed in details yet. This section is therefore included to unravel the misery behind the scope of this research in due course.

### 2.1 Material

A hybrid system of Tabu-search algorithm and particle swarm optimization algorithm was introduced, as the name implies, the proposed method combines the strength of the individual technique to overcome the weaknesses of the other technique. The combination of PSO and TS joins the strength of PSO's global search with TS's local search to help vary the search and overcome PSO's premature convergence to poor quality local optima.

- **Tabu-search:** it is a 'higher level' heuristic procedure for solving optimization problem, designed to guide other methods to escape the trap of local optimality (Glover, 1990). The overall approach is to avoid entrainment in cycles by forbidding or penalizing moves which take the solution, in the next

iteration, to points in the solution space previously visited, It uses a flexible structures memory (to permit search information to be exploited more thoroughly than by rigid memory systems or memory less systems), conditions for strategically constraining and freeing the search process(embodied in tabu restrictions and aspiration criteria), and memory functions of varying time spans for intensifying and diversifying the search (reinforcing attributes historically found good and driving the search into new regions).

- **Particle swarm optimization (PSO):** Particle swarm optimization (PSO) is an artificial intelligence (AI) technique that can be used to find approximate solutions to extremely difficult or impossible numeric maximization and minimization problems, it optimizes a problem by iteratively trying to improve a candidate solution with regard to a given measure of quality. PSO optimizes a problem by having a population of candidate solutions, here dubbed particles, and moving these particles around in the search-space according to simple mathematical formulae over the particle's position and velocity.

Combining the unique characteristics of Tabu-search algorithm and particle swarm optimization algorithm is able to achieve optimum placement of PMU using less computation time.

## 2.2 Methods

### 2.2.1 Optimal Placement of PMU

PMUs are very expensive and it is not cost-effective to place a PMU at every bus in the system. PMUs should be minimally placed at the buses that provide maximum observability for the system. An optimization problem is formulated that minimizes the PMU installation cost and maximizes system observability. A complete optimal PMU placement algorithm should minimize total cost of PMUs and maximize observability.

### 2.2.2 Problem Formulation

#### Cost Function Formulation

The installation cost  $F(x)$  is directly linked to a minimal number of PMUs to be placed. The minimization function may take the following form:

$$F(X) = \text{Min} \sum_{i=1}^n (w_i * x_i) \quad (1)$$

Subjected to the constraint  $G(x) \geq b$

Where  $w_i$  is the cost of PMU installed at  $i^{\text{th}}$  bus.  $w_i$  is defined related to connectivity of buses.

$$\frac{\text{total no of buses} - \text{total no of connection of } i^{\text{th}} \text{ bus}}{\text{total no of buses}} \quad (2)$$

#### Constraints Formulation

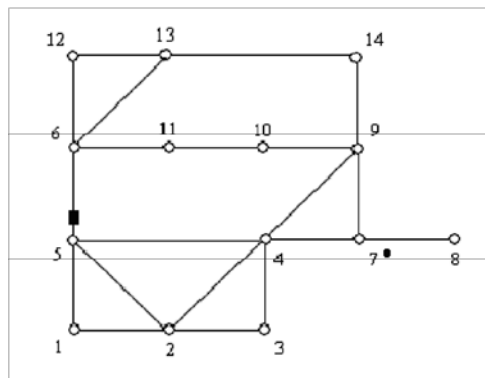
Taking IEEE 14 bus system as an example

Where 'x' in equation (1) is the binary decision variable vector for PMU, whose entries are defined as

$$\begin{cases} 1 & \text{if it is connecting to } j \\ 1 & \text{if } i = j \\ 0 & \text{other} \end{cases} \quad (3)$$

Matrix A can be obtained by transforming its entries into binary form.

Consider a 14-bus system, as shown in Figure.2,



**Figure 2:** IEEE- 14 bus system

Binary connectivity matrix (A) for IEEE- 14 bus system can be obtained from equation 3

|   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |

The constraints can be formed as

$$g_1 = x_1 + x_2 + x_5 \geq 1$$

$$g_2 = x_1 + x_2 + x_3 + x_4 + x_5 \geq 1$$

$$g_3 = x_2 + x_3 + x_4 \geq 1$$

$$g_4 = x_2 + x_3 + x_4 + x_5 + x_7 + x_9 \geq 1$$

$$g_5 = x_1 + x_2 + x_4 + x_5 + x_6 \geq 1$$

$$g_6 = x_5 + x_6 + x_{11} + x_{12} + x_{13} \geq 1$$

$$g_7 = x_4 + x_7 + x_8 + x_9 \geq 1$$

$$g_8 = x_7 + x_8 \geq 1$$

$$g_9 = x_4 + x_7 + x_9 + x_{10} + x_{14} \geq 1$$

$$g_{10} = x_9 + x_{10} + x_{14} \geq 1$$

$$g_{11} = x_6 + x_{10} + x_{11} \geq 1$$

$$g_{12} = x_6 + x_{12} + x_{13} \geq 1$$

$$g_{13} = x_6 + x_{12} + x_{13} + x_{14} \geq 1$$

$$g_{14} = x_9 + x_{13} + x_{14} \geq 1$$

The first constraint  $g_1 \geq 1$  implies that at least one PMU must be placed at either one of buses 1, 2 or 5 in order to make 1 observable. Similarly, the second constraint  $g_2 \geq 1$  indicates that at least one PMU should be installed at any one of the buses 1, 2, 3, 4 or 5 in order to make our bus 2 observable. The operator '+' serves as the logical 'OR' and the use of 1 in the right hand side of the inequality ensure that at least one of the variables appearing in the sum will be non-zero.

### 2.2.3 Proposed Algorithm

#### Hybridization of PSO-TS

Though PSO can successfully search through a large search space and find an optimal solution, it converges

prematurely toward local optima. This implies that the algorithm often is subjected to partial optimization. Instead of finding the global optimal solution, it finds the local optimal solution within the swarm and this is because it becomes trapped in a local minimum. Tabu-search compensate for the constraint existing in the PSO approach to the problem.

The hybridization of PSO with TS algorithm combines a slightly modified form of PSO with a standard Tabu Search. As shown in figure 6, the flowchart provides a top-level overview of the architecture of the algorithm. As depicted in the flowchart, PSO serves as the driving force of the PSO-TS algorithm.

Unlike other models where the search population is independently operated, the hybrid PSO-TS embeds TS within PSO. The diversification of the search provided by TS helps PSO avoid its limitation of premature convergence toward local optima. The PSO algorithm drives the overall search and begins the process by a random search through the solution space. Once initial pbest and a global gbest are obtained, the pbests are passed to Tabu Search to explore the nearby area of the swarm. TS take in these pbests and establish a local search boundary centered on each pbest. The search examines the nearby area for a potential better solution. If a better solution is found, TS returns this updated solution to PSO. PSO then updates the swarm based on the best solution found so far. The pbest and gbest along with the particles' velocities and positions are updated. The iteration continues until the stopping criterion is met.

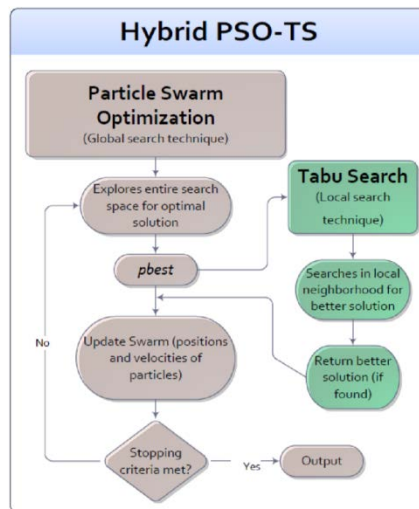


Figure 3: Top-level flowchart of PSO-TS

**2.2.4 Optimal PMU Placement (OPP) problem without considering ZIBs**

Create an initial swarm of randomly generated  $n$  no. of binary string and each elements of swarm is a potential solution ( $x$ ), also define maximum iteration count ( $itmax$ ). Length of each element of binary string equals to the total bus of the system. The value 1 at  $d^{th}$  position in  $x$  represents  $d^{th}$  bus with PMUs and 0 represents  $d^{th}$  bus without PMUs.

Form bus to bus connectivity matrix ( $A$ ) using system data.

Form the weight matrix ( $W$ ) for the installation cost of PMU at each bus using matrix  $A$  using equation 3.3.

- take potential solution  $i = 1$ .
- obtain the constraint observability matrix ( $G$ ).
- form the weighted element matrix,  $W_i = x * W$ .
- check if each element of  $G$  is greater than '0'. If yes, the system will be completely observable by the potential solution  $x$ , otherwise system will not be completely observable by
- define the cost function as the sum of the elements of weighted element matrix ( $W_i$ ) i.e.  $\sum_{i=1}^n w_i x_i$  in case of full observability and otherwise a very high value (taken as 1000 for this problem) as we are dealing with a minimization problem maintaining full observability of the network.
- check if  $i \geq n$ . If not, go to step 5, otherwise step 10
- assign  $f_{best} = \text{Min} \sum_{i=1}^n (w_i * x_i)$  and  $g_{best} = i^{\text{th}}$  potential solution
- set *iteration count* = 1
- assign *best\_list* =  $p_{best}$ , *tabu list* is been initialized
- run modified PSO taking information generated in previous steps (1-10) as initial solution and update the position and velocity vectors of the swarm. potential solution set  $X$  gets updated.
- repeat *step 4* to *step 10*.
- check if the solution converged or maximum iteration number reached. If the answer is no in both the cases increase the *iteration count* by 1 and go to *step 12*, otherwise go to *step 15*.
- take global best solution ( $g_{best}$ ) as the location of PMUs in this network and  $f_{best}$  as the minimum value of cost function. Print  $g_{best}$  and  $f_{best}$ .

### 3. Result and Discussion

**Table 1:** The method of hybridization of PSO-TS is tested on IEEE-14 and IEEE-30 bus system, the table below shown the result

| Minimum number of PMU needed | Bus location of the PMUs           |
|------------------------------|------------------------------------|
| 4                            | 2,6,7, 9                           |
| 10                           | 2, 4, 6, 9, 10, 12, 15, 19, 25, 27 |

### 4. Conclusion

Development of phasor measurement unit has improve monitoring of the system accurately with help of GPS technology. As phasor measurement units are expensive optimal placement algorithms have been developed to install PMUs in power system. Conventional state estimation algorithm estimates the state of power system which is no longer needed if the power system is equipped with PMU.



In this project a new algorithm is developed based on the conventional power system state estimation model, the model of state estimation containing PMU is developed. The algorithm was tested on IEEE 14 and 30 bus system.

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