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# Modelling Volatility of Daily Stock Returns: Is GARCH(1,1) Enough?

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**Abstract**

Volatility in financial markets has attracted growing attention by academics, policy makers and practitioners during the past two decades. First, volatility receives a great deal of concern from policy makers and financial market participants because it can be used as a measurement of risk. Second, High volatility of return in financial market may discourage investors to invest in stock market and hence greater uncertainty. So we need to estimate the appropriate volatility model to capture the volatility. In this paper, we study the performance of simple GARCH model. We apply the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model of different lag order to model volatility of stock returns of four Bangladeshi Companies on Dhaka Stock Exchange (DSE). Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) are used to select the best GARCH(p,q) model. From the empirical results, it is found that the distribution of daily returns are non-normal with negative skewness and pronounced excess kurtosis. Result shows that, GARCH(1,1) is the best than other GARCH(p,q) models in modeling volatility for the daily return series of DSE.

***Keywords:*** Volatility; stock return; DSE; GARCH(1,1).

## 1. Introduction

Volatility is the most important variable in the pricing of derivative securities, whose trading volume has quadrupled in recent years.

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It is not the same as risk. According to [27], volatility is defined as a statistical measure of the dispersion of returns for a given security or market index and it can either be measured using the standard deviation or variance between returns from that same security or market index.

The daily volatility is the measure of fluctuations within a day and is calculated as the square of the daily return series. The variable return is denoted by R and is calculated as: $R_t = \log(x_t / x_{t-1})$; where, $x_t$ is the daily closing price index. And thus the daily volatility is just the square of the daily return as: $\sigma_t = R_t^2$ Volatility forecasts are fundamental in several financial applications. For instance, volatility inputs are widely used for portfolio optimization, hedging, risk management, and pricing of options and other type of derivatives [25]. Given that financial volatility is a measure of risk, policy makers often rely on market volatility to have an idea of the vulnerability of financial markets and the economy [20]. Furthermore, many decisions are taken anticipating what could occur in the future, thus, a forecast of the volatility of financial variables is a relevant piece of information. So we need to estimate the appropriate volatility models that capture volatility well. In this paper, we focus upon one aspect of GARCH models, namely, choosing the best GARCH model among all other lag order of GARCH models. There are many very good surveys covering the mathematical and statistical properties of GARCH models. See, for example, [3], [4], [18], [19], [8] and [26]. There are also several comprehensive surveys that focus on the forecasting performance of GARCH models including [20], [21], and [1]. However, there are relatively few surveys that focus on the practical econometric issues associated with estimating GARCH models and forecasting volatility. This paper, which draws heavily from [28], gives a tour through the empirical analysis of univariate GARCH models for financial time series.

There is observed considerable uncertainty and volatility both in the emerging and mature stock markets. Great concern is about the fluctuating returns of their investments due to the market risk and variation in the market price speculation as well as the unstable business performance, [6]. In the real world of financial markets, investors and financial analysts are generally more interested in the profit or loss of the stock over a period of time that is; the increase or decrease in the price, than in the price self.

Modelling volatility in financial markets is important because it sheds further light on the data generating process of the returns, [17] and the riskiness associated with the asset since volatility is related to risk, [24]. Due to the usefulness of volatility, various models have been developed since Engel's paper of 1982.

Engle (1982), [22] studied on ARCH and Bollerslev (1986), [23] on GARCH models, and revealed that, these models were designed to deal with the assumption of non-stationarity found in real life financial data. He further pointed out that these models have become widespread tools for dealing with time series heteroscedasticity. The ARCH and GARCH models treat heteroscedasticity as a variance to be modelled. The goal of such models is to provide a volatility measure like a standard deviation that can be used in financial decisions concerning risk analysis, portfolio selection and derivative pricing.

The assumption that variance is constant through time is statistically inefficient and inconsistent, [13]. In real life, financial data for instance stock market returns data, variance changes with time (a phenomenon termed as heteroscedasticity), hence there is need for studying models which accommodate this possible variation in variance. Many studies have suggested that volatility of returns in stock markets world over can be modelled and forecasted using the GARCH type models.

Financial time series usually exhibit stylized characteristics. Firstly, it was observed by [23], that financial returns displayed volatility clustering meaning that large changes in the price of an asset are often followed by other large changes, and small changes are often followed by other small changes. Secondly, [9] demonstrated that financial data exhibit leptokurtosis meaning that the distribution of the returns is fat-tailed. Finally, [11] introduced the leverage effect meaning that volatility is higher after negative shocks than after positive shocks of the same magnitude. A good volatility model, then, must be able to capture and reflect these stylized facts, [22]. The daily returns exhibit the “stylized facts” of volatility clustering as well as non-normal empirical distribution. Researchers gave documented these and many other stylized facts about the volatility of economic and financial time series. Reference [4] gave a complete account of these facts. The GRCH model is capable of explaining many of those stylized facts. The four most important ones are: volatility clustering, fat tails, volatility mean reversion, and asymmetry.

## 2. Data and Methodology

### *2.1 Data*

We employ daily observations of stock market indices of four Bangladeshi companies namely Bangladesh Export Import Company Limited (BEICL), Beximco Pharmaceuticals Limited (BPL), Prime Bank Limited (PBL) and Arab Bangladesh Bank Limited (ABBL) for the Period January 2000 to November 2014. The data were provided by the Dhaka Stock Exchange (DSE) library.

### *2.2 GARCH(p,q) Model*

The Autoregressive Conditional Heterscedasticity (ARCH) model by Engle (1982) [22] and its generalization, GARCH by Bollerslev (1986) [23] are the major and widely used methodologies in modeling and forecasting volatility of financial time series. The standard GARCH (p, q) model expresses the variance at time, t as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

$\sigma_t^2$ is the conditional variance, $\varepsilon_t$ the residual returns, defined as; $\varepsilon_t = \sigma_t z_t$ and $z_t \sim N(0,1)$ i.e are standardized residual returns. $\omega$, $\alpha_i$ and $\beta_j$ are the parameters to be estimated. In order for the variance to be positive the necessary condition is that $\omega > 0$, $\alpha_i \geq 0$ (for $i = 1,\ldots,q$) and $\beta_i \geq 0$ (for $j = 1,\ldots,p$). For $p = 0$, equation (1) reduces to an ARCH(q) model and for $p = 0 = q$, equation(1) reduces to simply white noise. In this model, the conditional variance only depends on the magnitude, and not the sign, of the underlying asset. Large ARCH coefficients, $\alpha_i$ imply that volatility reacts significantly to market movements while large

GARCH coefficients, $\beta_j$ indicate that shocks are persistent, [12].

It can be shown that any GARCH(p,q) process can be written in an ARMA(p,q) representation [15]. As an alternative to conditional normal distribution. Bollerslev (1987)[6], and Kaiser (1996)[14] use student-t distribution while Nelson (1991)[16], Kaiser (1996)[14] suggests Generalized Student-t distribution. In this study, the assumption of conditional normality is used estimation.

### *2.3 Model Selection*

In financial modelling, one of the main challenges is to select a suitable model from a candidate family to characterize the underlying data. The choice of a good model in the application of time series analysis is crucial; the total process cannot be automated since the context is all important and there is never a perfect or unique model. Model selection criteria provide useful tools in this regard and assesses whether a fitted model offers an optimal balance between goodness-of-fit and parsimony. Ideally, a criteria will identify candidate models that are either too simplistic to accommodate the data or unnecessarily complex. The most common model selection criteria are the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC).

AIC= T ln(residual sum of squares) + 2n,

BIC= T ln(residual sum of squares) + nln(T)

Where, T is the number of usable observation, and n is the number of parameter to be estimated. In practice, the model with smallest AIC or BIC is selected as the best model

According to [10], A desirable model is one that minimizes the AIC and the BIC.

## 3. Result and Discussion

### *3.1 Basic Statistics of daily return series*

**Table 1:** Descriptive statistics of daily return series

| Statistic | BEICL | BPL | PBL | ABBL |
|---|---|---|---|---|
| Mean | 5.60785E-05 | 0.000171 | -0.00064 | -0.00028 |
| Median | -0.00284 | -0.00191 | -0.00034 | -0.001 |
| Maximum | 2.77259 | 0.72022 | 0.35017 | 0.27173 |
| Minimum | -2.515 | -0.73868 | -2.37547 | -2.31656 |
| Standard Deviation | 0.145447254 | 0.037098 | 0.052 | 0.051855 |
| Kurtosis | 185.0965682 | 99.76489 | 1184.42 | 1101.642 |
| Skewness | 3.681612972 | -0.59467 | -26.0862 | -25.6926 |

From the above table we observe that for every company skewness are negative and kurtosis value is greater than 3. Hence we can say that the return series has leptokurtic distribution.

**Table 2:** Shapiro-Wilk Normality test of daily reurn series of selected companies

| Company Name | Statistics | Significant |
|---|---|---|
| BEICL | W=0.2498 | p-value $< 2.2\times10^{-16}$ |
| BPL | W=0.67 | p-value $< 2.2\times10^{-16}$ |
| PBL | W=0.3446 | p-value $< 2.2\times10^{-16}$ |
| ABBL | W=0.3825 | p-value $< 2.2\times10^{-16}$ |

From the above Shapiro-Wilk normality test we observe that for each company p value is approximately zero i.e. it's value is less than 0.05 that leads null hypothesis is rejected. So we can say that the return series is non-normal which support descriptive statistic returns.

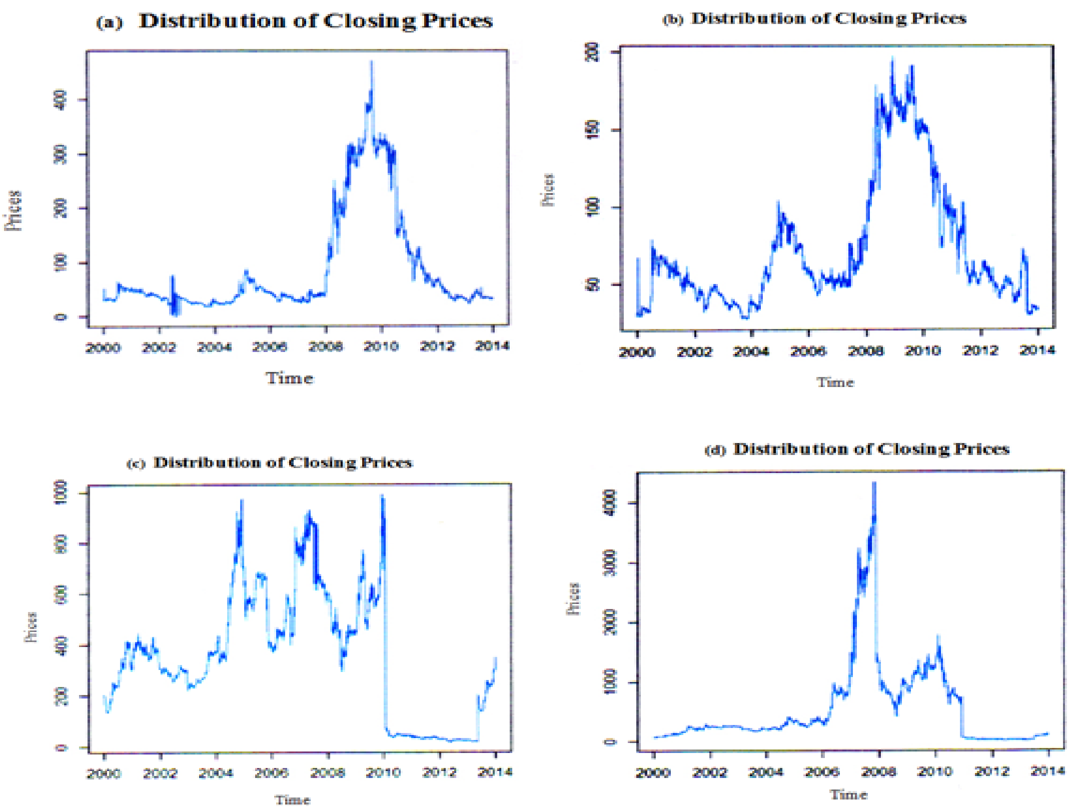


**Figure 1:** Distribution of Closing Prices of the selected companies from January, 200 to November, 2014.

From the above plot, we observe that over the period of study the prices seem to be trending, suggesting perhaps that the mean and variance gas been changing. So we say that the daily closing price of Bangladesh Export Import Company Limited(a), Beximco Pharmaceuticals Limited (b), Prime Bank Limited (c) and Arab Bangladesh Bank Limited (d) is not stationary.

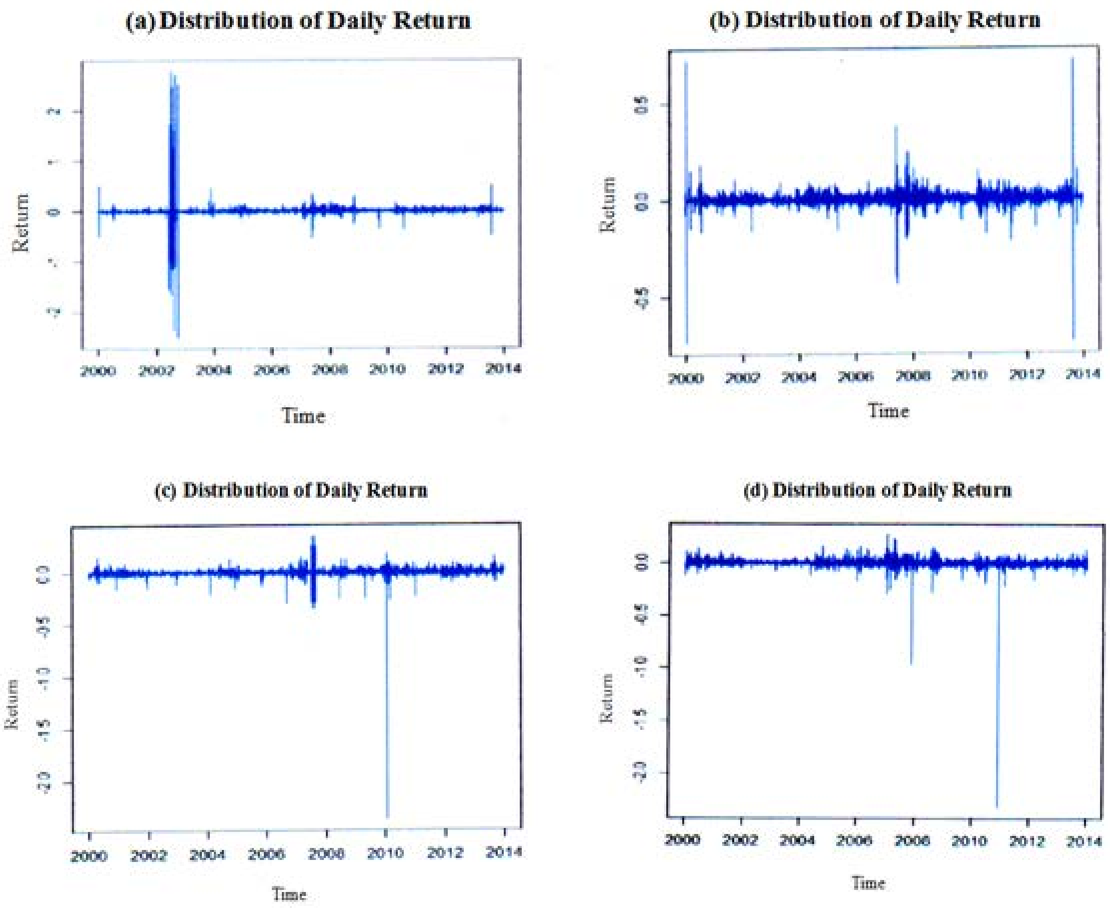


**Figure 2:** Distribution of Daily Return Series Data of selected companies from January, 200 to November, 2014.

From the above figure we observe that the return series appears to be stable with an average return of approximately zero: however the volatility or variability of the data changes over time. In fact, the data shows volatility clustering, that is highly volatile periods tend to be clustered together means there is no visual evidence of serial correlation in the return but there is evidence of serial correlation in the amplitude of the returns.

### *3.2 Empirical Results*

We consider six GARCH(p,q) models; GARCH(1,1), GARCH(1,2), GARCH(1,3), GARCH(2,1), GARCH(2,2) and GARCH(2,3) estimate the parameters and compare their performance. The results are showed in the table below:

From the above table we can see that among all the GARCH models GARCH(1,1) model gives the smallest values of AIC and BIC for all of the four companies. Hence we may conclude that GARCH(1,1) is the best model to capture stock returns volatility.

**Table 3:** Estimation result of GARCH(p,q) models

| Company Name | (p,q) | AIC | BIC | Company Name | (p,q) | AIC | BIC |
|---|---|---|---|---|---|---|---|
| BEICL | (1,1) | 2.880520 | 2.779989 | PBL | (1,1) | 2.776297 | 2.899568 |
| | (1,2) | 2.884531 | 2.964062 | | (1,2) | 2.781991 | 2.916489 |
| | (1,3) | 2.880525 | 2.969998 | | (1,3) | 2.776465 | 2.923190 |
| | (2,1) | 2.926913 | 3.006445 | | (2,1) | 2.783876 | 3.006445 |
| | (2,2) | 2.883689 | 2.793162 | | (2,2) | 2.788570 | 2.935295 |
| | (2,3) | 2.892359 | 2.991774 | | (2,3) | 2.781538 | 2.940490 |
| BPL | (1,1) | 3.325846 | 3.375004 | ABBL | (1,1) | 2.971700 | 3.040600 |
| | (1,2) | 3.331304 | 3.390294 | | (1,2) | 2.976600 | 3.055400 |
| | (1,3) | 3.331304 | 3.390294 | | (1,3) | 2.981400 | 3.070100 |
| | (2,1) | 3.336856 | 3.405677 | | (2,1) | 2.976600 | 3.055400 |
| | (2,2) | 3.327572 | 3.386561 | | (2,2) | 2.981500 | 3.070100 |
| | (2,3) | 3.332474 | 3.401295 | | (2,3) | 2.986400 | 3.084900 |

### *3.3 Estimation results of GARCH(1,1) Model for selected companies*

**Table 4:** Estimation result of GARCH(1,1) model for BEICL

| Parameter | Estimate | Standard Error | P-value |
|---|---|---|---|
| $\mu$ | -0.008770 | 0.053557 | 0.869923 |
| $AR(1)$ | -0.737088 | 0.172442 | 0.000019 |
| $AR(2)$ | 0.127805 | 0.055662 | 0.021670 |
| $MA(1)$ | 0.840257 | 0.164716 | 0.000000 |
| $\omega$ | 0.037266 | 0.022867 | 0.103175 |
| $\alpha_1$ | 0.108179 | 0.050049 | 0.030658 |
| $\beta_1$ | 0.872005 | 0.053553 | 0.000000 |

The estimated GARCH(1,1) model is

$$y_t = \sigma_t \varepsilon_t$$
$$\sigma_t^2 = 0.037266 + 0.108179 y_{t-1}^2 + 0.872005 \sigma_{t-1}^2$$

**Table 5:** Estimation result of GARCH(1,1) model for BPL

| Parameter | Estimate | Standard Error | P-value |
|---|---|---|---|
| $\mu$ | -0.098950 | 0.064661 | 0.125943 |
| $AR(1)$ | 0.081125 | 0.058056 | 0.162306 |
| $\omega$ | 0.229814 | 0.178479 | 0.197877 |
| $\alpha_1$ | 0.128896 | 0.079344 | 0.104263 |
| $\beta_1$ | 0.743204 | 0.167655 | 0.000009 |

The estimated GARCH(1,1) model is

$$y_t = \sigma_t \varepsilon_t$$
$$\sigma_t^2 = 0.229814 + 0.128896 y_{t-1}^2 + 0.743204 \sigma_{t-1}^2$$

**Table 6:** Estimation result of GARCH(1,1) model for PBL

| Parameter | Estimate | Standard Error | P-value |
|---|---|---|---|
| $\mu$ | -0.006297 | 0.054151 | 0.907429 |
| $AR(1)$ | -0.298520 | 0.036563 | 0.000000 |
| $AR(2)$ | -0.228377 | 0.037511 | 0.000000 |
| $AR(3)$ | -0.867137 | 0.046045 | 0.000000 |
| $MA(1)$ | 0.350125 | 0.029572 | 0.000000 |
| $MA(2)$ | 0.355290 | 0.019806 | 0.000000 |
| $MA(3)$ | 0.956256 | 0.024805 | 0.000000 |
| $\omega$ | 0.064968 | 0.034740 | 0.061466 |
| $\alpha_1$ | 0.206160 | 0.071804 | 0.004090 |
| $\beta_1$ | 0.742640 | 0.065376 | 0.000000 |

The estimated GARCH(1,1) model is

$$y_t = \sigma_t \varepsilon_t$$
$$\sigma_t^2 = 0.077882 + 0.332405 y_{t-1}^2 + 0.357867 \sigma_{t-1}^2 + 0.262910 \sigma_{t-3}^2$$

**Table 7:** Estimation result of GARCH(1,1) model for ABBL

| Parameter | Estimate | Standard Error | P-value |
|---|---|---|---|
| $\mu$ | 0.005663 | 0.053638 | 0.915925 |
| $AR(1)$ | -0.424958 | 0.317697 | 0.181020 |
| $AR(2)$ | -0.056589 | 0.070146 | 0.419820 |
| $MA(1)$ | 0.526438 | 0.316239 | 0.095976 |
| $\omega$ | 0.000795 | 0.002470 | 0.747601 |
| $\alpha_1$ | 0.000000 | 0.002461 | 1.000000 |
| $\beta_1$ | 0.999000 | 0.000055 | 0.000000 |

The estimated GARCH(1,1) model is

$$y_t = \sigma_t \varepsilon_t$$
$$\sigma_t^2 = 0.000795 + 0.999000\sigma_{t-1}^2$$

**4. Conclusion**

The volatility of Dhaka Stock Exchange (DSE) returns of four selected companies BEICL, BPL, PBL and ABBL has been modelled for a period of 01/01/2000 to 30/11/2014 using different GARCH(p,q) models; GARCH(1,1), GARCH(1,2), GARCH(1,3), GARCH(2,1), GARCH(2,2) and GARCH(2,3). From the empirical results obtained, we can conclude the following: Firstly, it was found that the return series of DSE are not normally distributed. Secondly, the DSE return series also exhibit volatility clustering and leptokurtosis as seen from the high excess kurtosis values. Over all, GARCH(1,1) performed best in modeling volatility of DSE stock returns.

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