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Stochastic Model for Rainfall Occurrence Using Markov Chain Model in Kurdufan State, Sudan

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Abstract

This paper will attempt to demonstrate the potential benefits of using Stochastic Processes for modeling and interpreting historical rainfall records by the examination of weekly rainfall occurrence using Markov Chains as the driving mechanism. The weekly occurrence of rainfall was modeled by two-state first and second order Markov chain. While the amount of rainfall of a rainy week was approximated by taking the maximum likelihood estimation method to predict transition probability matrices of rainfall sequences during the rainy season. Daily rainfall data for 21 years was collected from two meteorological stations located in Kurdufan State (Sudan). The data indicated that the season starts effectively, on 8_{th} week of June at El-Obied station and sixth week of June at Kadugli station. The transition probability matrix of Markov chain model found to be homogeneous and remained constant over the period study. Accordingly, the Index of Drought-proneness degree (ID) was found to be higher in Elobied than Kadugli Station and the hypothesis is accepted at 5% level of significant with P-value (0.151).

Keywords: Kordofan; Markov Chains; rainfall; stochastic process; Transition matrix; (;).

1. Introduction

(Sudan enjoys an extremely diversified ecological system that provides immense fertile land of about (80) million hectares. A large number of livestock (about 121 million heads of sheep, goats cattle and camels), natural pastures of about 24 million hectares, forest area of about (64) million hectares in addition to considerable water resources from rivers, seasonal streams and rain with annual amount of (109) billions cubic of water [1].

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The paper problem that, existing rainfall data is generally available for most areas on monthly basis or means. There is a need to know the probability of having a dry or wet period having a consecutive period of 2 or 3 weeks during the rainy season. Such knowledge will enable us to propose calendar for farmers and irrigation engineers suggesting the start and end of rainy season.

The main objectives of this paper is to increases the understanding of the agricultural planners and irrigation engineers to identifying the areas where agricultural development should be focused as a long term drought mitigation strategy. In addition, this study will contribute toward a better understanding of the climatology of drought in a major drought-prone region of the Sudan. In addition, this study may help the agronomists and agricultural scientists to decide the timing of cultivation and introduce new crops.

Markov chains (MC) have been widely used with daily rainfall models. The first stochastic model of the temporal precipitation with Markov chain (two –state first order) introduced by Gabrial and Neuman in 1962 [2]. Richardson in 1981 used first order Markov chain along with an exponential distribution to describe the daily rainfall distribution in the (USA) [3]. Akaike in 1974 used similar Markov chain to simulate the daily rainfall occurrence [4]. James. Reference [5] also used "statistical Modeling of Daily Rainfall Occurrence. All these studies has revealed that the generated data using Markov chain along with suitable probability distribution preserve the seasonal and statistical characteristics of historical rainfall data.

The rest of the paper is structured as follows: In Section two, we outline the Markov modeling estimation of transition probabilities. In section three, we present the source of data. Section four, presents and discusses the results. In section, five summary was provided including conclusions and recommendations.

2. Markov chains modeling:

The theory of stochastic process deals with system, developing in time or space in accordance with probabilistic laws. Its concept is based on expanding the random variable concept to include time. The function $X_{(t,s)}$ is called a stochastic process, when X random variable, a function of S possible outcomes of an experiment (state space), t is the parameter set of process (time), so that the set of possible values of an individual random variables, $X_{n(xt)}$,of a stochastic process $X_{n,} n \geq 1, X_{(t),} t \in T$ is known as it's state space. Markov Chains are the simplest mathematical models for the random phenomena evolving time [6].

The stochastic process with discrete parameter space

$X_{n,}$ (n = 0,1,2,….) or the stochastic process with continuous parameter $X_{n,} n \geq 0$ is called the Markov chain . $P[X_{n+1,} = j / X_n = i, X_{n-1} = i, n-1........X_1 = i, X_0] = P_{ij}$ ……..(1)

For all states $i_1,\ldots\ldots, i_{n-1,} i, j$ and $n \geq 1$.

We refer to this fundamental equation as the Markov property, the future depend on the past through the present. The random variables $X_0; X_1;......., X_{n-1}, \cdots$ are dependent. Since the probability, Pij is non-negative process then:

$$\sum_{j=0}^{k} Pij = 1 \qquad 0 \le Pij \le 1 \forall i, j \ge 0 \quad \text{………………..} \ (2)$$

2.1 Probability Transition Matrix

If there is a limit, state n elements then transition probability for i and j values can be organized in a matrix called probability transition matrix [7].

$$P_{ij} = \Pr(X_{n+1} = j / X_n = i \ \text{ Satisfy } P_{ij} > 0 \ , \ \sum P_{ij} = 1 \text{ for all j}$$

These probabilities can be written in the following matrix form:

$$P = \begin{pmatrix} p_{11} & p_{12} & .... & p_{1n} \\ p_{21} & p_{22} & .... & p_{2n} \\ : & : & : & : \\ p_{n1} & p_{n2} & ..... & p_{nn} \end{pmatrix}$$

This matrix is called the probability transition matrix of Markov chain.

2.2 One step transition probability

The Markov chain $\{X_n, n = 0,1,2,\cdots\}$ with state space $S = \{0,1,2,\cdots\}$ is probability transition for random process from *i* to j.

$$P(X_{n+1} = j \mid X_n = i) = P(X_1 = j \mid X_0 = i) \ \ \forall n$$

The probability of making a transition from state i to state j in one step is denoted p_{ij}

For a Markov chain with 2 states, the matrix is called the one-step transition matrix [8].

$$\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

For a Markov chain with 3 states, the one-step transition matrix is

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$$

2.3 The n- step transition probability

The probability transition of random processes $\{X_n : n = 0,1,2,\cdots\}$ from state i to state j after n step is called the n-step transition probability defined as:

$$p_{ij}^{(n)} = P(X_{n+m} = j \mid X_m = i), \quad n \geq 0,\ i, j = 0,1,2,...$$

This indicates the probability transition of random processes from state i to state j after n step [9]. We can write it in term of matrix $P^{(n)}$ as follow:

$$P^{(n)} = \begin{pmatrix} p_{00}^{(n)} & p_{01}^{(n)} & p_{02}^{(n)} & \cdots \\ p_{10}^{(n)} & p_{11}^{(n)} & p_{12}^{(n)} & \cdots \\ p_{20}^{(n)} & p_{21}^{(n)} & p_{22}^{(n)} & \cdots \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

The interpretation of matrix is:

1- If n =1 then $p_{ij}^{(n)}$ becomes the probability transition of random processes, from state i to state j, with one step denoted by p_{ij}.
2- If n = 0 then $p_{ij}^{(0)} = P(X_0 = j \mid X_0 = i) = \begin{cases} 1, & i = j, \\ 0, & i \neq j. \end{cases}$
3- For all, n = 0,1,.. the matrix $P^{(n)}$ became a random process satisfying the two above properties.

2.4 Chapman – Kolmogorov equation

Chapman – Kolmogorov equation is my help us to predict and forecast for several steps or several years in the future. The probability transition of random processes from state to state after n + m step [6].

If $\{X_n : n = 0,1,2,\cdots\}$ Markov chain with limit m states and transition probability matrix (TPM) $P = (p_{ij})$ then:

$$p_{ij}^{(n)} = \sum_{k=1}^{m} p_{ik}^{(r)} \; p_{kj}^{(n-r)}, \quad \forall\, r = 1, 2, ..., n-1. \qquad \text{.................}(3)$$

2.5 Maximum Likelihood Function

When have a Markov Chain with state 0, 1 , 2 , 3 , ….. with unknown transition matrix P, the likelihood function is:

$$L = \prod_{i,\, j \in s} .P_{ij}^{nij} \qquad \text{…………….} \qquad (4)$$

n_{ij} = the number of times has state j following state i, to maximize the function:

$$\sum_{i \in s} p_{ij} = 1$$

Indicates that each row of transition matrix is equal to 1 and then:

$$\sum_{i \in s} p_{ij} = \sum \frac{n_{ij}}{\sum n_{i.}} \qquad \text{……………} \qquad (5)$$

Where n_{ij} the transition count for $(i, j)^{th}$ cell and $n_{i.}$ is the i^{th}, row total transition count. Therefore, the random variable n_{ij} depends on the parameter P_{ij}

$$\ln L = \sum n_{ij} L P_{ij} \qquad \text{…………………….} \qquad (6)$$

2.6 *Notations*

For the purpose of this paper, some notions are explained as follows [10].

1- $P_m(W_i / W_{i-1})$ = conditional probability of a wet week on week (i) given a wet week on week $(i-1)$ in a certain period m.

2- $P_m(D_i / W_{i-1})$ = conditional probability of a dry week on week (i) given a wet week on week $(i-1)$ in a certain period m.

3- (W_i / D_{i-1}) = conditional probability of a wet week on week (i) given a dry week on week $(i-1)$ in a certain period m.

4- (D_i / D_{i-1}) = conditional probability of a dry week on week (i) given a dry week on week $(i-1)$ in a certain period m.

Thus, for each week, four elements in the transition matrix were to be determined in first order Markov chain. For a second order chain, eight elements of the transitional probability matrix were to be determined.

1- $P_m(W_i / W_{i-1}W_{i-2})$ = conditional probabilities of a wet week following two a wet week in certain period m.

2- $P_m(W_i / W_{i-1}D_{i-2})$ 3- $P_m(W_i / D_{i-1}W_{i-2})$ 4- $P_m(W_i / D_{i-1}D_{i-2})$

5- $P_m(D_i / D_{i-1}D_{i-2})$ 6- $P_m(D_i / W_{i-1}D_{i-2})$ 7- $P_m(D_i / D_{i-1}W_{i-2})$

8- $P_m(W_i / W_{i-1}W_{i-2})$

2.6.1 Initial Probability

$$P_D = F_D / n \quad \text{..........................} \quad (7)$$

$$P_W = F_W / n \quad \text{..........................} \quad (8)$$

2.6.2 Conditional Probabilities

$$P_{DD} = F_{DD} / F_D \quad \text{..........................} \quad (9)$$

$$P_{WW} = F_{WW} / F_W \quad \text{..........................} \quad (10)$$

$$P_{WD} = 1 - P_{DD} \quad \text{..........................} \quad (11)$$

$$P_{DW} = 1 - P_{WW} \quad \text{..........................} \quad (12)$$

2.5.3. Consecutive dry and wet week probabilities:

$$2D = P_{Dw1}.P_{DDw1} \text{.................................} (13)$$

$$2W = P_{Ww1}.P_{WWWw2} \text{...........................} (14)$$

$$3D = P_{Dw1}.P_{DDw2}.P_{DDw3}......................(15)$$

$$3W = P_{Ww1}.P_{WWw2}.P_{WWw3}.....................(16)$$

Where,

Wet week: A week with rainfall of 7mm or more.

Dry week: A week with rainfall of less than 7mm.

$P_D =$ Probability of the dry week

$P_W =$ Probability of the wet week

$F_D =$ Number of dry weeks

$F_W =$ Number of wet weeks

n - Number of years of data

$P_{DD} =$ Probability (conditional) of a dry week preceded by a dry week

P_{WW} = Probability (conditional) of a wet week preceded by a wet week

P_{WD} = Probability (conditional) of a wet week preceded by a dry week

$P_{DW} =$ Probability (conditional) of a dry week preceded by a wet week

$F_{DD} =$ Number of dry weeks preceded by another dry week

$F_{WW} =$ Number of wet weeks preceded by another wet week

2D = Probability of two consecutive dry weeks.

2W = Probability of two consecutive wet weeks.

3D = Probability of three consecutive dry weeks.

3W = Probability of three consecutive wet weeks.

$P_{Dw1} =$ Probability of the dry week (first week)

$P_{DDw2} =$ Probability of the second dry week, given the preceding week dry

$P_{DDw3} =$ Probability of the third dry week, given the preceding week dry

$P_{Ww1} =$ Probability of the wet week (first week)

$P_{WWw2} =$ Probability of the second wet week, given the preceding week wet

$P_{WWw3} =$ Probability of the third wet week, given the preceding week wet

3. Source of data

The present work is based on data related to the autumn season (May-November) daily rainfall reported by two stations in Sudan. Elobied in North Kurdofan state (longitude 12:30° and 14:30 North, and 29° and 32° East) and Kadougli in south Kurdofan (latitudes 90° 45′ and 12° 45′ N, and longitudes 29° 15′ and 32° 30′ E), over priod of 20 years (1990-2009) from Kadougli station, and 21 years (1990-2010) from Elobied station [1]. We transfer the original daily data to weekly data by dividing the month into four classes. The autumn season (1-Mayto 30-November), have (28 Standard Metrological Weeks (SMW).

4. Results and Discussion

4.1 Conditional Probabilities

We can compute the conditional probability states of the (SMW) according to the following formulas

$$p_{dw} = \frac{N_{dw}}{N_{dw} + N_{dd}} \ldots(17),\ p_{dd} = \frac{N_{dd}}{N_{dd} + N_{dw}} \ldots(18),\ p_{wt} = \frac{N_{wd}}{N_{wd} + n_{ww}} \ldots.(19),\ p_{ww} = \frac{N_{ww}}{N_{ww} + N_{wd}} .(20)$$

4.2 Initial Probability

Markov chains could give probability of spell lengths within a given period as well as probability of a specified amount of rain within a given period. To compute the initial probability from data we can use the equations (7) and (8) as follow:

Table 1: Conditional probabilities of (SMW)

		Elobied Station				***Kadugli Station***			
Class	*SMW*	P_{dd}	P_{dw}	P_{wd}	P_{ww}	P_{dd}	P_{dw}	P_{wd}	P_{ww}
1-7may	1	0.95	0.05	0.50	0.50	0.88	0.12	0.67	0.33
8-15may	2	0.81	0.19	0.60	0.40	0.71	0.29	0.67	0.33
16-22may	3	0.82	0.18	0.75	0.25	0.82	0.18	1.00	0.00
23-31may	4	0.67	0.33	0.83	0.17	0.64	0.36	0.56	0.44
1-7jun	5	0.75	0.25	0.60	0.40	0.50	0.50	0.60	0.40
8-15jun	6	0.50	0.50	0.56	0.44	0.30	0.70	0.60	0.40
16-22jun	7	0.30	0.70	0.55	0.45	0.40	0.60	0.60	0.40
23-30jun	8	0.58	0.42	0.44	0.56	0.67	0.33	0.27	0.73
1-7jul	9	0.00	1.00	0.33	0.67	0.43	0.57	0.31	0.69
8-15jul	10	0.40	0.60	0.19	0.81	0.43	0.57	0.23	0.77
16-22jul	11	0.00	1.00	0.25	0.75	0.33	0.67	0.21	0.79
23-31jul	12	0.00	1.00	0.05	0.95	0.00	1.00	0.18	0.82
1-7aug	13	0.00	1.00	0.11	0.89	0.00	1.00	0.19	0.81
8-15aug	14	0.00	1.00	0.24	0.76	0.00	1.00	0.12	0.88
16-22aug	15	0.33	0.67	0.06	0.94	0.33	0.67	0.06	0.94
23-31aug	16	0.40	0.60	0.13	0.88	0.33	0.67	0.21	0.79
1-7sep	17	0.00	1.00	0.18	0.82	0.40	0.60	0.20	0.80
8-15sep	18	0.00	1.00	0.17	0.83	0.33	0.67	0.21	0.79
16-22sep	19	0.36	0.64	0.70	0.30	0.46	0.54	0.86	0.14
23-30sep	20	0.46	0.54	0.75	0.25	0.50	0.50	0.63	0.38
1-7oct	21	0.22	0.78	0.50	0.50	0.00	1.00	0.25	0.75
8-15oct	22	0.62	0.38	0.50	0.50	0.18	0.82	0.78	0.22
16-22oct	23	0.50	0.50	0.86	0.14	0.33	0.67	0.73	0.27
23-31oct	24	0.95	0.05	1.00	0.00	0.75	0.25	0.75	0.25
1-7nov	25	0.89	0.11	0.67	0.33	0.64	0.36	0.67	0.33
8-15nov	26	0.89	0.11	0.67	0.33	0.88	0.12	0.50	0.50
e16-22nov	27	0.89	0.11	1.00	0.00	1.00	0.00	-	-
23-30nov	28	1.00	0.00			1.00	0.00	-	-

4.3 Consecutive dry and wet week probabilities

For a second order chain, there are eight elements of the transitional probability matrix to be determined. We can also drive a third order equation to calculate three consecutive dry or a wet days.

1- P_{DDw2} means that the probability of the second week being dry, given the preceding week is dry, denoted by 2D.

2- P_{WWw2} means that the probability of the second week being wet, given that the preceding week is wet, denoted by 2W.

3- P_{DDw3} means that the Probability of the third week being dry, given that the preceding week is dry, denoted by 3D.

P_{WWw3} means that the Probability of the third week being wet, given that the preceding week is wet, denoted by 3W.

By equations (13), (14), (15), and (16) we can calculate the 2D, 2W, 3D, and 3W respectively as in the table (3).

The analysis of consecutive dry and wet spells (Table 3) during rainy season reveals that there is an interval limits (90-10) % chances that 2 consecutive dry weeks may occur during (1st -8th) SMW and (19th -27th) SMW . Similarly, the probabilities of occurrence of three consecutive dry weeks are also very high with interval limits (71-14) % during the (1st -6th) SMW and (20th -27th) SMW.

The probability of occurrence of two consecutive wet weeks are (10-80) % during (5th – 18th) SMW and the probability of occurrence of three consecutive wet weeks are (14-57) % during (7th – 18th) for ElObied Station data.

For Kadugli Station analysis of consecutive dry and wet spells explained that, during rainy season there is being interval limits (13-63) % chances that two consecutive dry weeks may occur during (1st -9th) SMW and (16th -27th) SMW . Similarly, the probabilities of occurrence of three consecutive dry weeks are low- compared with that of ElObied Station data - with interval limits (20-60) % during (1st -4th) SMW and (20th -27th) SMW.

The probability of occurrence of two consecutive wet weeks are (13-63) % during (4th – 23rd) SMW and the probability of occurrence of three consecutive wet weeks are (20-80)% during (5th – 21st)SMW.

Table (4) shows that the probability of a week being a wet after two weeks (ElObied Station). This insures that the starting point occurs with probability more than 51% and the probability of end point correspond to 5% and reaches a higher point of 94% at 12nd SMW. In addition, table (4) shows that the probability of a week being a wet after two weeks (Kadugli Station). In a station the starting point occurs with probability more than 55% and the probability of end point correspond to 13% and reaches a higher point equal to 91% at 12nd SMW.

Table 2: No of dry and wet, initial probability for SMW and percentage (Elobied Station)

		Elobied Station						*Kadugli Station*					
		No of dry and wet		*Initial probability for SMW*		*Initial probability for SMW%*		No of dry and wet		*Initial probability for SMW*		*Initial probability for SMW%*	
Class	*SMW*	Dry	Wet	PD	PW	PD%	PW%	Dry	wet	PD	PW	PD%	PW%
1-7may	1	19	2	0.90	0.10	90.48	9.52	17	3	0.85	0.15	85	15
8-15may	2	16	5	0.76	0.24	76.19	23.81	14	6	0.7	0.3	70	30
16-22may	3	18	3	0.86	0.14	85.71	14.29	17	3	0.85	0.15	85	15
23-31may	4	13	6	0.62	0.29	61.90	28.57	12	8	0.6	0.4	60	40
1-7jun	5	13	6	0.62	0.29	61.90	28.57	10	10	0.5	0.5	50	50
8-15jun	6	11	10	0.52	0.48	52.38	47.62	9	11	0.45	0.55	45	55
16-22jun	7	9	12	0.43	0.57	42.86	57.14	10	10	0.5	0.5	50	50
23-30jun	8	11	10	0.52	0.48	52.38	47.62	10	10	0.5	0.5	50	50
1-7jul	9	5	16	0.24	0.76	23.81	76.19	7	13	0.35	0.65	35	65
8-15jul	10	5	16	0.24	0.76	23.81	76.19	6	14	0.3	0.7	30	70
16-22jul	11	4	17	0.19	0.81	19.05	80.95	5	15	0.25	0.75	25	75
23-31jul	12	1	20	0.05	0.95	4.76	95.24	3	17	0.15	0.85	15	85
1-7aug	13	2	19	0.10	0.90	9.52	90.48	3	17	0.15	0.85	15	85

8-15aug	14	4	17	0.19	0.81	19.05	80.95	2	18	0.1	0.9	10	90
16-22aug	15	2	19	0.10	0.90	9.52	90.48	2	18	0.1	0.9	10	90
23-31aug	16	4	17	0.19	0.81	19.05	80.95	5	15	0.25	0.75	25	75
1-7sep	17	3	18	0.14	0.86	14.29	85.71	5	15	0.25	0.75	25	75
8-15sep	18	2	19	0.10	0.90	9.52	90.48	5	15	0.25	0.75	25	75
16-22sep	19	11	10	0.52	0.48	52.38	47.62	12	8	0.6	0.4	60	40
23-30sep	20	14	7	0.67	0.33	66.67	33.33	12	8	0.6	0.4	60	40
1-7oct	21	8	13	0.38	0.62	38.10	61.90	5	15	0.25	0.75	25	75
8-15oct	22	12	9	0.57	0.43	57.14	42.86	10	10	0.5	0.5	50	50
16-22oct	23	13	8	0.62	0.38	61.90	38.10	10	10	0.5	0.5	50	50
23-31oct	24	20	1	0.95	0.05	95.24	4.76	16	4	0.8	0.2	80	20
1-7nov	25	18	3	0.86	0.14	85.71	14.29	16	6	0.8	0.3	80	30
8-15nov	26	19	2	0.90	0.10	90.48	9.52	16	4	0.8	0.2	80	20
e16-22nov	27	19	2	0.90	0.10	90.48	9.52	20	0	1	0	100	0
23-30nov	28	21	0	1.00	0.00	100.00	0.00	20	0	1	0	100	0

Table 3: the probability of occurrence of two and three consecutive (wet and dry) weeks and Percentages limits

			Stations	
			ElObied	Kadugli
2 consecutive	dry	SMW limits	(1st-8th)SMW and (19th-27th)SMW	(1st -9th)SMW and (16th - 27th)SMW
		Percentages limits	(90-**10**) %	(13-63) %
	wet	SMW limits	(5th – 18th) SMW	(4th – 23rd) SMW
		Percentages limits	(10-80) %	(20-60) %
3 consecutive	dry	SMW limits	(1st-6th) SMW and (20th -27th) SMW	(1st-4th)SMW and (20th -27th) SMW
		Percentages limits	(71-14) %	(20-60) %
	wet	SMW limits	(7th – 18th) SMW	(5th – 21st)SMW
		Percentages limits	(14-57)%	(20-80)%

Table 4: The probability of a week being dry or wet after two weeks

Station	Starting	End season	Higher point
ElObied	8$_{th}$ SMW (11 – 17th June) 51%	24th SMW (23 – 31th October) 5%	94%at 12nd SMW
Kadugli	7$_{th}$ SMW (16 – 22th June) 55%	26th SMW (8 – 15th November) 13%	91%at 15nd SMW

4.4 Drought-proneness Index (DI) frequencies

Table (4-5) explain the stationary distribution and Index of Drought-proneness. The result reveal that the DI of areas as in the following table.

Table 5: Drought-proneness Index frequencies

Criteria	Degree of drought-proneness	ElObied station SMW	Kadugli Station SMW
0.00<DI<0.125	Chronic	10	7
0.125 < DI < 0.180	Severe	2	2
0.180 < DI < 0.235	Moderate	3	3
0.235 < DI < 0.310	Mild	1	2
0.310 < DI < 1.000	Occasional	11	14
General DI		0.33	0.38

The criteria of DI results reveal that there are 10 weeks in ElObied area with Chronic Drought-proneness and only 7 weeks with chronic degree in Kadugli area. On the anther hand, there are 14 SMW with Occasional degree in Kadugli area, but only 11 SMW with occasional degree in ElObied. That is means the Drought-proneness degree of ElObied (General DI = 0.33) and of Kadugli (General DI = 0.38).

5. Conclusions

This study applied the stochastic process , using the transition probability matrix (Markov Chain models), to estimate the rainfall sequences during the rainy season in Kurdufan area (ElObied an Kadugli stations), for 21 years (1990-2010). The transition matrices were calculated on a week period. This taken as the optimum length of time. The results tabulated in the previous tables represent the following:

1. The conditional probabilities of (28 SMW) indicated that the season starts effectively at 8^{th} SMW (11 – 17th June) with length of 18 weeks in ElObied area and 7th SMW (16 – 22th June) with a length of 18 weeks in Kadugli area.
2. The initial and conditional probabilities of dry and wet weeks revealed that during rainy season the probability of occurrence of wet week in ElObied area is more than 75% from 9^{th} week to the 18^{th} (9 weeks). This is a short period compared with Kadugli area 12 weeks.
3. The analysis of consecutive dry and wet spells during rainy season reveals the second and third order but it is not suitable for short autumn season.
4. The result reveals that is the Drought -proneness degree of ElObied (General DI = 0.33) and of Kadugli area (General DI = 0.38).

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