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Efficiency Analysis of Multi-Stage Systems in the Presence of Undesirable Outputs

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Abstract

Most traditional DEA models treat their reference technologies as black boxes. Moreover, they do not consider undesirable factors in the model. There exist some Network DEA models that consider intermediate products. The aim of this paper is to extend the available network DEA models by considering some undesirable outputs. A model is proposed to evaluate the performance of this type multistage system. The proposed approach is applied to a number of case studies from the literature and compared with existing approaches.

Keywords: Network DEA; Undesirable Outputs; Intermediate Products; Scale efficiency; MPSS.

1. Introduction

Data envelopment analysis (DEA) is a nonparametric method in operations research for evaluating relative efficiency and inefficiency score for each member of a set of decision making units (DMUs), which consumes multiple inputs to generate multiple outputs. Conventional DEA models consider the system as a single-process black box which consumes inputs and produces outputs. There are however a number of so-called network DEA approach that consider the system as composed by distinct stages, each one with its own inputs and outputs and with intermediate flows among the stages.

In traditional DEA models, the performance of inefficient DMUs is improved by either increasing current output level or decreasing current input level. However, both desirable and undesirable inputs and outputs may be present. For example suppose that one would evaluate efficiency scores of a set of airports.

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The total delay time of flights is an undesirable output. Some other examples can be found in the references [1, 2, 3, 4].

Recently Lozano and Gutierrez proposed an slack-based method for evaluating efficiency scores of airports with undesirable outputs [5]. Schaar and Sherry show that depending on the DEA model chosen, radically different results may be obtained [7]. Consequently, any study of airport efficiency needs to begin with a thorough examination of the models available and a motivation for why a particular model was selected. Without an upfront analysis of this kind, a study's final results may be called into question different DEA models. This implies that the slack-based model is not sufficient for analyzing the performance of multistage systems with undesirable factors and some other models are needed. In this paper, we follow the recent paper of Lozano [6] and answer to one of its open question by proposing a new network DEA model which considers undesirable outputs. The rest of the paper is organized as follows: In Section 2, the notations and the key PPS concept are presented. In Section 3, a radial input-oriented DEA model is proposed. Section 4 illustrates the proposed approach using a two-stage problem from the literature.

2. Notations and PPS

Consider that a sample of observed data of a set of n DMUs is available. Suppose that the process to produce final outputs consists of Q stages for each DMU. Let I(p) the set of exogenous inputs used in stage p and, for each $i \in I(p)$, let x_{ij}^p denotes the observed amount of exogenous input *i* consumed by stage *p* of DMU_j . Let $P_I(i)$ the set of stages that consume the exogenous input i and $x_{ij} = \sum_{p \in P_I(i)} x_{ij}^p$ the total amount of exogenous input consumed by all stages of DMU_i . Also, let $O^d(p)$ the set of final desirable outputs, and $O^u(p)$ the set of final undesirable outputs of stage p, for each $k \in O^d(p)$, $(or \ k \in O^u(p))$, let y_{ki}^p denotes the observed amount of final desirable (undesirable) output k produced by stage p of DMU_i . Let $P_{od}(k)$ and $P_{ou}(k)$ be the sets of stages that produce the final desirable and undesirable output k respectively. The total amount of final desirable (undesirable) output k produced by all stages of DMU_j is $y_{kj} = \sum_{p \in P_{Od}(k)} y_{kj}^p \left(\sum_{p \in P_{Ou}(k)} y_{kj}^p \right)$. Let $D = \sum_{p \in P_{Ou}(k)} y_{kj}^p \left(\sum_{p \in P_{Ou}(k)} y_{kj}^p \right)$. $\bigcup_p O^d(p)$ denotes the set of all desirable outputs and $U = \bigcup_p O^u(p)$ denotes the set of all undesirable outputs of the system. In addition to these exogenous inputs and outputs, there exist R intermediate products generated and consumed within the system. Thus, let $P^{out}(r)$ the set of stages that generate the intermediate product r and for each $p \in P^{out}(r)$ let z_{rj}^p the observed amount of intermediate product r generated by stage p of DMU_j . Analogously, let $P^{in}(r)$ the set of stages that consume the intermediate product r and for each $p \in P^{in}(r)$ let z_{ri}^p the observed amount of intermediate product r consumed by stage p of DMU_j . Let us assume that the intermediate products consumed by a DMU are completely generated in-house so that there is no need to acquire them externally (as it occurs with the exogenous inputs) nor to sell them (as it occurs with the final products). The sets $P^{out}(r)$ and $P^{in}(r)$ jointly determine the structure of intermediate flows within the system, which alternatively may be expressed through sets $R^{out}(p)$ and $R^{in}(p)$ corresponding to the intermediate products produced and consumed, respectively, by a certain stage p.

It is well-known that the PPS can be derived axiomatically from a set of assumptions using the minimum

extrapolation principle. The following axioms are considered in the each stage of the system.

- Envelopment,
- All inputs and desirable outputs are free disposable,
- Undesirable outputs are weakly disposable,
- Convexity

Following the conventional DEA approach at the individual stage, the PPS of stage p is defined as

$$T_{p} = \begin{cases} \exists \lambda_{j} \in \Lambda_{p} \quad \forall j \\ x_{i}^{p}, y_{k}^{p}, z_{r}^{p} \end{cases} : \begin{array}{c} x_{i}^{p} \geq \sum_{j} \lambda_{j}^{p} x_{ij}^{p} \quad \forall i \in I(P) \\ y_{k}^{p} \leq \sum_{j} \lambda_{j}^{p} x_{kj}^{p} \quad \forall k \in O^{d}(p) \\ y_{k}^{p} \geq \sum_{j} \lambda_{j}^{p} x_{kj}^{p} \quad \forall k \in O^{u}(p) \\ z_{r}^{p} \geq \sum_{j} \lambda_{j}^{p} z_{rj}^{p} \quad \forall r \in R^{in}(p) \\ z_{r}^{p} \leq \sum_{j} \lambda_{j}^{p} z_{rj}^{p} \quad \forall r \in R^{out}(p) \end{cases} ,$$
(2.1)

where Λ_p represent the return to scale assumption for stage p. Thus, $\Lambda_p^{CRS} = \{\lambda_j : \lambda_j \ge 0 \forall j\}$ corresponds to Constant Returns to Scale (CRS), $\Lambda_p^{IRS} = \{\lambda_j : \lambda_j \ge 0 \forall j, \sum_{j=1}^n \lambda_j = 1\}$ corresponds to Non-Increasing Returns to Scale (NIRS), and corresponds to Variable Returns to Scale (VRS).

The PPS of the multistage systems with undesirable outputs can be defined as the composition of the stage technologies in the following form:

$$T = \begin{cases} \exists \left(x_i^p, y_k^p, z_r^p\right) \in T_p \ \forall p \ , x_i \ge \sum_{p \in P_I(i)} x_{ij}^p \ \forall i \\ (x_i, y_k): \qquad y_k \le \sum_{p \in P_od(K)} y_{kj}^p \ \forall k \in D \\ y_k \le \sum_{p \in P_ou(k)} y_{kj}^p \ \forall k \in U \\ \sum_{p \in p^{out}(r)} z_{rj}^p - \sum_{p \in p^{in}(r)} z_{rj}^p \ge 0 \ \forall r \end{cases} \right\}, (2.2)$$

where similar axioms as the PPS of the each stage are considered.

3. Technical efficiency

In order to compute the technical efficiency of the system we will formulate a relational network DEA model taking into account the PPS of each individual stage. To that end it is first necessary to assume the returns to scale of each stage. Thus, let P_{CRS} , P_{VRS} and P_{NIRS} the sets of stages with constant, variable and non-increasing returns to scale sub-technologies, respectively. The following model is proposed to evaluate the efficiency of multistage systems with undesirable outputs.

$$TE_0 = min\theta \tag{3.1}$$

$$\sum_{p \in P_I(i)} \sum_j \lambda_j^p x_{ij}^p \le \theta x_{i0} \qquad \forall i$$
(3.2)

$$\sum_{p \in P_{od}(k)} \sum_{j} \lambda_{j}^{p} y_{ij}^{p} \ge y_{i0} \qquad \forall k \in D$$
(3.3)

$$\sum_{p \in P_{ou}(k)} \sum_{j} \lambda_{j}^{p} y_{ij}^{p} \ge y_{i0} \qquad \forall k \in U$$
(3.4)

$$\sum_{p \in p^{out}(r)} \sum_{j} \lambda_j^p z_{rj}^p - \sum_{p \in p^{in}(r)} \sum_{j} \lambda_j^p z_{rj}^p \ge 0 \quad \forall r \quad (3.5)$$

$$\sum_{j} \lambda_{j}^{p} = 1 \quad \forall p \in P_{VRS} , \sum_{j} \lambda_{j}^{p} \le 1 \quad \forall p \in P_{NIRS} \quad (3.6)$$

$$\lambda_i^p \ge 0 \quad \forall j \forall p \ \theta \ free \tag{3.7}$$

Solving the above model without the constraints (3.6), i.e. assuming all the stages with constant return to scale, a Global Efficiency score of $DMU_o(GE_o)$ can be computed for the multistage system. The network Scale Efficiency of $DMU_o(SE_o)$ can be computed as

$$SE_O = \frac{TE_O}{GE_O},$$
(3.8)

for more details see [5].

4. Numerical results and discussion

In this section, we present and discuss the results of the application of the proposed model to airport benchmarking. We use Lozano and Gutierrez [6] data set involving 39 Spanish airports in years 2006 and 2007. All the airports are managed by the Spanish Airport and Air Navigation Agency(AENA). The inputs considered are related to the existing infrastructure at the airports, namely total runway area, apron capacity, number of baggage belts, number of check-incounters and number of boarding gates. These inputs are considered non-discretionary and have been extracted from AENA [6]. Annual passenger movements and aircraft traffic movements as well as cargo handled are considered as desirable output. The undesirable outputs considered include the percentage of delayed flights and average conditional delay of delayed flights at each airport. The Global Efficiency (GE), Technical Efficiency (TE), Scale Efficiency (SE) and NIRS Efficiency (NE) score of each DMU (airport) are shown in Tables 1 and 2 for years 2006 and 2007 respectively. Moreover, It is shown that which DMUs have the Most Productive Scale Size (MPSS) property in the Tables 1 and 2. The DMUs that its productive scale size is greater that 0.95 are indicated with " $\approx MPSS$ ".

Table 1 shows that the airports Girona Costa, Gran Canaria, Granada Jaen, Jerez, La Palma, Leon, Malagas, Melilla, Murcia, Palma de Mallorca, Pamplona, Reus, Salamanca and Tenerife North are technically inefficient while they are efficient in slack-based method [6]. In the same manner, it can be seen that some airports are inefficient in Table 2 while they are technically efficient in Slack-based method [6]. Thus, study of the properties of each model is necessary to understand which is the most appropriate in a given multistage system.

Table 1: Global, technical and scale efficiency scores With RTS for the year 2006. The sixth and seventh columns of the table show the reference set for technical efficiency in stages 1 and 2 respectively.

DMUs	AirPorts	GE	TE	SE	NE	RTS	Reference set	Reference set of
							of stage 1	stage 2
1	A Coruna	0.4792		0.6474	0.4792	IRS	DMU 6 DMU	DMU 2 DMU 9
			0.7402				10 DMU 16 DMU 27	DMU 12 DMU 39
2	Albacete	0.0207	0.9977	0.0207	0.0207	IRS	DMU 6 DMU 9 DMU 17	DMU 2 DMU 9
3	Alicante	1.0000	1.0000	1.0000	1.0000	MPSS	DMU 7 DMU	DMU 3
							16 DMU 22	
4	Almeria	0.2932	0.4327	0.6776	0.2932	IRS	DMU 9 DMU	DMU 2 DMU 9
							10 DMU 16 DMU 23	DMU 12
5	Asturias	0.5809	0.6748	0.8608	0.5809	IRS	DMU 10 DMU 16 DMU 27	DMU 2 DMU 9 DMU 12 DMU
							10 DMO 27	39
6	Badajoz	0.1283	1.0000	0.1283	0.1283	IRS	DMU 6	DMU 2 DMU 9
								DMU 21
7	Barcelona	1.0000	1.0000	1.0000	1.0000	MPSS	DMU 7	DMU 7
8	Bilbao	0.5096	0.5247	0.9712	0.5096	IRS ≈	DMU 10 DMU 16 DMU 27	DMU 3 DMU 7 DMU 9
						MPSS	10 DMU 27	DWU 9

9	Cordoba	1.0000	1.0000	1.0000	1.0000	MPSS	DMU 9	DMU 9
10	El Hierro	0.1622	1.0000	0.1622	0.1622	IRS	DMU 10	DMU 2 DMU 9 DMU 21
11	Fuerteventura	0.6150	0.6290	0.9776	0.6150	IRS ≈ MPSS	DMU 7 DMU 10 DMU 16 DMU 22	DMU 3 DMU 7 DMU 9
12	Girona -Costa Brava	0.9532	0.9896	0.9633	0.9532	IRS ≈ MPSS	DMU 7 DMU 9 DMU 10 DMU 16	DMU 3 DMU 7 DMU 9
13	Gran Canaria	0.8250	0.9574	0.9574	0.8325	DRS ≈ MPSS	DMU 10 DMU 13 DMU 22	DMU 3 DMU 21 DMU 35 DMU 39
14	Granada -Jaen	0.4275	0.6574	0.6503	0.4275	IRS	DMU 9 DMU 16 DMU 23	DMU 2 DMU 9 DMU 12
15	Ibiza	0.5338	0.6205	0.8603	0.5338	IRS	DMU 10 DMU 16 DMU 22	DMU 2 DMU 3 DMU 21 DMU 25 DMU 35
16	Jerez	0.5019	0.5832	0.8605	0.5019	IRS	DMU 9 DMU 10 DMU 16 DMU 23	DMU 2 DMU 9 DMU 12 DMU 39
17	La Gomera	0.0367	1.0000	0.0367	0.0367	IRS	DMU 17	DMU 2 DMU 9 DMU 21
18	La Palma	0.4468	0.7699	0.5803	0.4468	IRS	DMU 6 DMU 10 DMU 18 DMU 27	DMU 2 DMU 9 DMU 12 DMU 25 DMU 39
19	Lanzarote	0.6433	0.7291	0.8823	0.6433	IRS	DMU 10 DMU 16 DMU 22	DMU 2 DMU 3 DMU 12 DMU 21 DMU 25

20	Leon	0.1994	0.2030	0.1994	0.1994	IRS	DMU 9 DMU	DMU 2 DMU 9
							10 DMU 26	DMU 12
21	Madrid Barajas	1.0000	1.0000	1.0000	1.0000	MPSS	DMU 21	DMU 21
22	Malaga	0.8679	0.9404	0.9229	0.9266	DRS	DMU 10 DMU 22	DMU 3 DMU 7 DMU 25
23	Melilla	0.3651	0.9703	0.3763	0.3651	IRS	DMU 6 DMU 9 DMU 10 DMU 23	DMU 2 DMU 9 DMU 12 DMU 39
24	Murcia	0.5333	0.7231	0.7375	0.5333	IRS	DMU 6 DMU 10 DMU 27	DMU 2 DMU 3 DMU 35
25	Palma de Mallorca	0.9598	0.9673	0.9923	0.9598	IRS ≈ MPSS	DMU 7 DMU 10 DMU 13	DMU 7 DMU 9 DMU 25
26	Pamplona	0.4431	0.9726	0.4556	0.4431	IRS	DMU 9 DMU 10 DMU 26	DMU 2 DMU 9 DMU 12 DMU 39
27	Reus	0.8177	0.9137	0.8949	0.8177	IRS	DMU 6 DMU 10 DMU 27	DMU 3 DMU 9
28	Salamanca	0.0346	0.9264	0.0373	0.0346	IRS	DMU 6 DMU 9 DMU 17	DMU 1 DMU 2 DMU 9
29	San Sebastian	0.2899	0.6747	0.4296	0.2899	IRS	DMU 6 DMU 10 DMU 16 DMU 23	DMU 2 DMU 9 DMU 12 DMU 39
30	Santander	0.3834	0.5201	0.7372	0.3834	IRS	DMU 6 DMU 10 DMU 16 DMU 23	DMU 2 DMU 9 DMU 12
31	Santiago	0.4970	0.5364	0.9266	0.4970	IRS	DMU 10 DMU 16 DMU 22	DMU 3 DMU 7 DMU 9

32	Saragossa	0.5041	0.6686	0.7539	0.5041	IRS	DMU 9 DMU	DMU 2 DMU 9
							16 DMU 23	DMU 12 DMU
							DMU 26	39
33	Seville	0.5196	0.5765	0.9013	0.5196	IRS	DMU 10 DMU	DMU 7 DMU 9
							16 DMU 22	DMU 12 DMU 25 DMU 39
24	The second to Marcel	0 (02)	0 7 4 2 0	0.0100	0 (92)	IDC		
34	Tenerife North	0.6836	0.7439	0.9190	0.6836	IRS	DMU 10 DMU 16 DMU 22	DMU 7 DMU 9 DMU 12 DMU
							DMU 34	25 DMU 39
35	Tenerife South	0.7001	0.7286	0.9609	0.7001	IRS ≈	DMU 10 DMU	DMU 2 DMU 3
						MPSS	16 DMU 22	DMU 21 DMU
								25 DMU 35
36	Valencia	0.5628	0.6009	0.9367	0.5628	IRS	DMU 10 DMU	DMU 3 DMU 7
							16 DMU 22	DMU 9 DMU 12
								DMU 39
37	Valladolid	0.2701	0.4761	0.5674	0.2701	IRS	DMU 6 DMU	DMU 2 DMU 9
							10 DMU 16 DMU 23	DMU 12 DMU 39
38	Vigo	0.4682	0.5756	0.8135	0.4682	IRS	DMU 10 DMU	DMU 2 DMU 9
50	150	0.1002	0.5750	0.0155	0.1002	into	16 DMU 27	DMU 12 DMU
								39
20	Vitavia	1 0000	1 0000	1 0000	1 0000	MDgg	DMIL & DMIL	DMIL 20
39	Vitoria	1.0000	1.0000	1.0000	1.0000	MPSS	DMU 9 DMU 16 DMU 23	DMU 39
							DMU 26	

Table 2: Global, technical and scale efficiency scores With RTS for the year 2007. The sixth and seventh columns of the table show the reference set for technical efficiency in stages 1 and 2 respectively.

DMUs	AirPorts	GE	TE	SE	NE	RTS	Reference set of stage 1	Reference set of stage 2
1	A Coruna	0.4750		0.6226	0.4750	IRS	DMU 6 DMU	DMU 2 DMU 9
			0.7628				10 DMU 16	DMU 32 DMU
							DMU 27	39

2	Albacete	0.0185	1.0000	0.0185	0.0185	IRS	DMU 2	DMU 2
3	Alicante	0.8995	0.9499	0.9469	0.9336	DRS	DMU7	DMU3
5	Anounce	0.0775	0.9499	0.9409	0.9350		DMU16 DMU22 DMU36	DMU7 DMU12
4	Almeria	0.2635	0.4297	0.6132	0.2635	IRS	DMU 9 DMU 10 DMU 16 DMU 23	DMU2 DMU12 DMU20
5	Asturias	0.5290	0.6731	0.7860	0.5290	IRS	DMU10 DMU16 DMU27	DMU9 DMU12 DMU20 DMU32
6	Badajoz	0.1382	1.0000	0.1382	0.1382	IRS	DMU 6	DMU2 DMU7 DMU9 DMU21
7	Barcelona	1.0000	1.0000	1.0000	1.0000	MPSS	DMU 7	DMU 7
8	Bilbao	0.4581	0.4582	0.9999	0.4581	IRS ≈ MPSS	DMU 10 DMU 16 DMU 22	DMU 12 DMU 32 DMU 39
9	Cordoba	1.0000	1.0000	1.0000	1.0000	MPSS	DMU 9	DMU 9
10	El Hierro	0.1438	1.0000	0.1438	0.1438	IRS	DMU 10	DMU 2 DMU 7 DMU 9 DMU 21
11	Fuerteventura	0.5349	0.5781	0.9252	0.5349	IRS	DMU10 DMU16 DMU22	DMU2 DMU3 DMU12 DMU39
12	Girona -Costa Brava	1.000	1.000	1.000	1.000	MPSS	DMU 9 DMU 16	DMU 12

13	Gran Canaria	0.8166	0.8575	0.9523	0.8166	IRS ≈	DMU10	DMU 3 DMU 21
						MPSS	DMU13	DMU 35 DMU
							DMU22	39
14	Granada -Jaen	0.4540	0.6976	0.6509	0.4540	IRS	DMU9	DMU2 DMU3
							DMU16	DMU12
							DMU23	
							DMU26	
15	Ibiza	0.5363	0.6319	0.8488	0.5363	IRS	DMU10	DMU2 DMU3
							DMU16	DMU12 DMU21
							DMU22	DMU35
16	Jerez	0.4593	0.5807	0.7910	0.4593	IRS	DMU9	DMU9 DMU12
10	30102	0.1575	0.2007	0.7910	0.1575	nus	DMU10	DMU20 DMU32
							DMU10 DMU16	DM020 DM032
							DMU23	
17	La Gomera	0.0301	1.0000	0.0301	0.0301	IRS	DMU 17	DMU 2 DMU 9
17	La Gomera	0.0501	1.0000	0.0501	0.0501	ind		DMU 12
								Divid 12
18	La Palma	0.4100	0.7608	0.5389	0.4100	IRS	DMU6	DMU2 DMU9
							DMU10	DMU12 DMU39
							DMU16	
10		0.6105	0.510.6	0.0.404	0.6105	ID C	DMU18	
19	Lanzarote	0.6197	0.7186	0.8624	0.6197	IRS	DMU 10 DMU	DMU 2 DMU 3
							16 DMU 22	DMU 12 DMU
								21 DMU 35
20	Leon	0.2001	0.9603	0.2084	0.2001	IRS	DMU6 DMU9	DMU9 DMU12
							DMU10	DMU20
							DMU23	
21	Madrid	1.0000	1.0000	1.0000	1.0000	MPSS	DMU 21	DMU 21
	Barajas							
	·							
22	Malaga	0.8971	0.9563	0.9381	0.9461	DRS	DMU 10 DMU	DMU 3 DMU 7
<i>44</i>	iviaiaga	0.07/1	0.7503	0.7301	0.7401		22	DMU 3 DMU 7 DMU 25
							<i>LL</i>	DINIO 23
23	Melilla	0.3355	1.0000	0.3355	0.3355	IRS	DMU23	DMU9 DMU12
								DMU20 DMU32

24	Murcia	0.6093	0.7896	0.7717	0.6093	IRS	DMU6	DMU2 DMU3
24	Mulcia	0.0095	0.7890	0.7717	0.0095	IKS	DMU10 DMU16 DMU27	DMU35
25	Palma de Mallorca	0.9234	0.9346	0.9880	0.9346	DRS ≈ MPSS	DMU7 DMU13 DMU22	DMU7 DMU12 DMU25
26	Pamplona	0.4638	0.9506	0.4879	0.4638	IRS	DMU6 DMU9 DMU23	DMU2 DMU9 DMU12 DMU20 DMU32
27	Reus	0.6060	0.8104	0.7477	0.6060	IRS	DMU6 DMU10 DMU27	DMU9 DMU12 DMU20
28	Salamanca	0.0605	0.9105	0.0665	0.0605	IRS	DMU2 DMU6 DMU9 DMU10	DMU2 DMU9 DMU21
29	San Sebastian	0.2962	0.6950	0.4262	0.2962	IRS	DMU6 DMU10 DMU16 DMU23	DMU2 DMU12 DMU20 DMU32
30	Santander	0.3535	0.5259	0.6722	0.3535	IRS	DMU6 DMU10 DMU16 DMU23	DMU9 DMU12 DMU20
31	Santiago	0.4289	0.4513	0.9504	0.4289	IRS ≈ MPSS	DMU10 DMU16 DMU22	DMU9 DMU12 DMU32
32	Saragossa	0.9968	1.0000	0.9968	0.9968	IRS ≈ MPSS	DMU 9 DMU 16 DMU 23 DMU 26	DMU32
33	Seville	0.5259	0.5874	0.8952	0.5259	IRS	DMU7 DMU10 DMU16 DMU36	DMU2 DMU12 DMU21 DMU25 DMU39

	-						B 1 1 1 1 0	
34	Tenerife North	0.6806	0.7331	0.9283	0.6806	IRS	DMU10	DMU7 DMU12
							DMU16	DMU25 DMU39
							DMU22	
							DMU34	
35	Tenerife South	0.6566	0.7232	0.9079	0.6566	IRS	DMU7	DMU3 DMU12
							DMU10	DMU21 DMU25
							DMU16	DMU35
							DMU22	
36	Valencia	0.5873	0.6412	0.9159	0.5946	DRS	DMU10	DMU3 DMU7
							DMU16	DMU12 DMU21
							DMU22	DMU39
37	Valladolid	0.2382	0.4791	0.4971	0.2382	IRS	DMU6	DMU2 DMU9
							DMU10	DMU12 DMU20
							DMU16	DMU32
							DMU23	
38	Vigo	0.4668	0.5839	0.7994	0.4668	IRS	DMU10	DMU9 DMU12
	6						DMU16	DMU20 DMU32
							DMU27	211020211002
39	Vitoria	1.0000	1.0000	1.0000	1.0000	MPSS	DMU 9 DMU	DMU 39
							21 DMU 23	
							DMU 26	

5. Conclusion and future work

In this paper a way of modeling the internal flows within multistage system in presence of undesirable outputs has been proposed which allows for a simple and convenient way of defining the PPS of individual stages as well as the system PPS for any combination of RTS assumptions. The input-oriented relational network DEA model have been proposed to compute network technical and scale efficiencies and to estimate RTS.

The proposed approach has been illustrated with an airport efficiency assessment problem from the literature, showing the usefulness of a more detailed problem assessment both in terms of technical and scale efficiency and RTS.

The results show that the efficiency assessment of the airports is different and can therefore be misleading when different DEA models are used. Thus, further study needs to examine the implications and interpretations of each model in a network system since the characteristics of each model may take on different meanings depending on the application area. We recommend that future work should concentrate on the case that the intermediate products allow to be partially or totally acquired or sold externally. Moreover, one can consider the

case that data include fuzzy integer numbers.

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