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Analysis of India-Nepal Border Region Earthquake Wave for Studying the Layering Information of the Crust

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Abstract

Seismic surface wave group velocity dispersion has computed for India-Nepal border region earthquake of magnitude 5.0 occurred on 28 March 2012 of 23:40:14 UTC by graphical method. A model taking subsurface layer parameters is also constructed to compute the group velocity dispersion by modified Haskell matrix method. Group velocity dispersion by graphical method is then interpreted from model parameters. Sensitivity and the statistical errors of the model are studied and presented in this research. Interpreted crustal structure of the India-Nepal region shows that there are four major subsurface layers of thickness 4.0 km, 8.0 km, 11.0 km and 20.0 km.

Keywords: Layering information; period; group velocity; earthquake data; seismic wave; model parameter.

1. Introduction

There are different techniques for earthquake seismic surface wave analysis. Group velocity dispersion analysis is one of the most useful techniques.

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Seismic surface wave dispersion analysis of the local earthquakes can be used to study the layering information of the crust using simple models of continental or oceanic crust. Direct and indirect modeling techniques are commonly used in determination of the earth's interior from seismic surface wave dispersion. Direct modeling determines the layering information from observed surface wave dispersion [1, 2, 3, 4]. On other hand, the most widely used indirect modeling techniques deal with trial-and-error procedures. Dispersion of seismic surface wave is computed for different model parameters to see how the computed dispersion matches with observed dispersion [5]. Ewing in [6] first introduced such model for the oceanic crust using Rayleigh wave dispersion.

In this research, the group velocity dispersion has been computed and analyzed using graphical method [6] for the up-down component of the ground accelerated earthquake seismic wave of India-Nepal border region recorded at Dhaka Meteorological Department on 28 March 2012 of 23:40:14 UTC.

2. Material and Methods

2.1Earthquake Data

Magnitude 5.0 earthquake occurred on 28 March 2012 at India-Nepal border region. This region is 110 km far from Dinajpur city, Bangladesh and also not so far from the Himalayan frontal arch. Table 1 lists the source parameters of the selected event. The event was recorded at Dhaka Meteorological Department seismic station located at 23.78^0 N and 90.38^0E and the station is equipped with a three component digital broad–band sensor which can record up-down, north-south and east-west components. The recorded earthquake seismic wave is shown in Figure 1.

Table 1: Earthquake Source parameters

Date	Origin Time	Location	Depth (Km)	Distance of Epicenter (Km)	Mw
28th March 2012	23:40:14(UTC) 05:40:14 (BST)	26.09344^0N, 87.7513^0E	40	369	4.1

2.2 Methods

Seismic surface wave group velocity dispersion is considered as the factor, which has relationship with structure of the crust. Group velocity from recorded earthquake wave and multilayered crustal model can be obtained respectively by graphical method and modified Haskell matrix method as explained below.

- Graphical Method

Graphical method is basically a technique of group velocity dispersion determination. In this method the travel times (t) of some chosen phases along the surface wave train are measured and plotted on a graph versus the order number (n) of the chosen phases. Usually the travel times of the wave crests and troughs are read. The (n,t) curve built by these points is then approximated with linear segments. The period is determined by the slope of these lines and the corresponding travel times are read from the midpoints of the segments [6]. The group velocity, U_g of seismic surface wave can be obtained as:

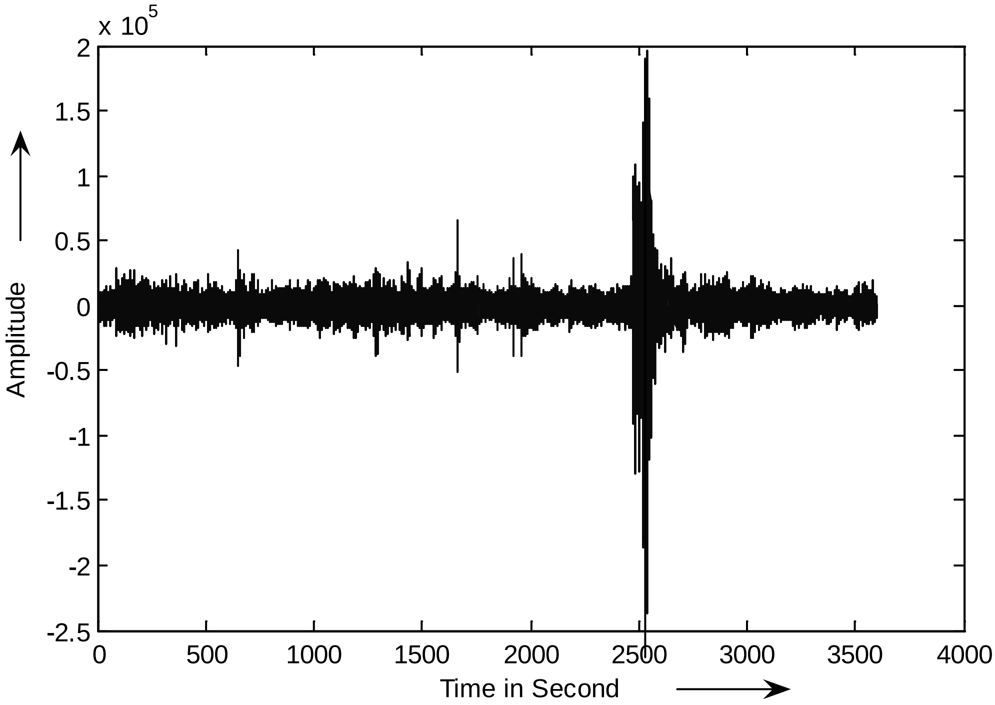


Figure 1: Up-down ground accelerated earthquake seismic wave recorded at Dhaka Meteorological Department of 5.0M earthquake on 28 March 2012 at 23:40:14 UTC that occurred at India-Nepal border region

$$U_g = \frac{\Delta}{t} \qquad (1)$$

Where Δ is epicenteral distance and t is the travel time.

- Modified Haskell Matrix Method

Modified Haskell matrix method for the case of $n-1$ homogeneous, isotopic elastic layers over a half-space matrix can be written as [7]:

$$J = \widehat{E}^n A^{n-1} \ldots . A^m \ldots A^2 . A^1 \qquad (2)$$

Where A^m is the $4X4$ Haskell matrix for the m'th layer and $\widehat{E}^n$ is the half-space inversion matrix. Then the secular function (dispersion relation) can be written as:

$$\Delta(T,C) = J\,|_{12}^{12} = J_{11}\,J_{22} - J_{12}\,J_{21} = 0 \qquad (3)$$

Hence two columns or rows of J are necessary for this result and it requires a $2X4$ matrix to store the product as each layer. It has seen that Haskell matrix poses a loss of significant figures in the secular function [8, 9, 10]. In order to minimize the losses a $6X6$ matrix is employed where the elements are second order sub-determinant of the Haskell matrix. The matrix can be written as:

$$R_i^m = \sum_{j=1}^{6} B_{ij}^m R_j^{m-1} \qquad (4)$$

Where $B_{ij}^m = A^m\,|_{kl}^{ij}$

Hence the secular function:

$$\Delta(T,c) = R^n B^n = 0 \qquad (5)$$

B^n is the matrix of sub-determinants of the half-space $\widehat{E}^n$.

However, B matrix shows that

$$R_i^1 = B_{i1}^1 \qquad (6)$$

and $R_3^m = R_4^m$ [Using Eqn. (4)]

This phenomenon leads to define a $5X5$ matrix $\widehat{B}$ rather than $6X6$ matrix and the modified $\widehat{B}$ matrix can be expressed as:

$$B_{11}$$

$$\widehat{B} = \begin{bmatrix} B_{12} & 2B_{13} & B_{15} & B_{16} & \\ B_{21} & B_{22} & 2B_{23} & B_{25} & B_{26} \\ B_{31} & B_{32} & (2B_{33}-1) & B_{35} & B_{36} \\ B_{51} & B_{52} & 2B_{53} & B_{55} & B_{56} \\ B_{61} & B_{62} & 2B_{63} & B_{65} & B_{66} \end{bmatrix} \qquad (7)$$

and similarly R_i^m is thus reduced from a six dimensional vector to five.

Dispersion relation (Equation. 5) can be solved numerically according to the model parameters (Vp, Vs, ρ and thickness) in the form of group velocity versus time period plot. On other hand same plot can also be obtained from the recorded earthquake data using graphical method. Hence crustal interpretations are now possible in an indirect way by matching both dispersion relations.

3. Sensitivity of Earth Model Parameter

Rayleigh wave phase velocity and group velocity (dispersion data) are the function of four parameters: S-wave velocity, P-wave velocity, density, and layer thickness [11]. Each of the parameters contributes to the dispersion curve. Using above parameters an initial earth model is constructed (Table 2). The group velocity with period is also computed and shown in Figure 2. It is observed that the variations in S-wave velocities have a dramatic effect on Rayleigh wave group velocities (Figure 2). S-wave velocities are changed by 1% in the model (Table 2), an average change of 2.81% in group velocity. After changing the S-wave velocity, the group velocity is represented by the diamond. Effects of the 1% changes in S-wave velocity are quite dramatic in comparison to similar changes in P-wave velocity and density (Figure 2).Group velocities are influenced much less by changes in density than P-wave velocity. A 1% increase in P-wave velocities (Table 2) represents an average group velocities change 0.48% and average group velocity changes 0.0067% for changing the density. This significant change in density has very subtle effect on group velocity (Figure 2).The effect of layer thicknesses on Rayleigh wave group velocities can be minimized by dividing the sub-surface into thinner layers within each unique and constant S-wave interval velocity. When the model (Table 2) defines a thickness increase of 1% the average change in group velocities is approximately 0.012% (Figure 2).

According to inspection (Figure 2) it can be said that the group velocity increases with S-wave, P-wave velocity and density, but decreases with increasing thickness. The S-wave velocity is the dominant parameter influencing the changes in group velocity for this particular type model.

Table1 2: Initial Earth model parameters.

Layer number	Vp (km/s)	Vs (km/s)	ρ (gm/cc)	h (km)
1	5.54	3.20	2.54	5.0
2	5.63	3.25	2.57	12.0
3	5.89	3.40	2.65	15.0
Half-space	6.10	3.53	2.72	Infinite

4. Model Error Estimation

The aim of the current research is to study the layering information of the crust / shallow depth hence the depth of 46.0 km is being considered here.

The S-wave velocities of the layers are free to change during the inversion. Consequently, the P-wave velocities are estimated using the Vp/Vs ratio 1.732. Poisson's ratio (σ) in each layer was assumed to be .25 and the densities (ρ) are calculated from the P-wave velocities (Vp) using the relation 0.32Vp+0.77 [12]. Starting from initial estimates, the model parameters are iteratively improved until a good fit between the theoretical and observed dispersion curves is obtained. During the inversion, a number of criteria were adapted to calculate the goodness of fit. These criteria are the standard error of estimate (SE), mean residual (MR), average absolute residual (AR), weighted root mean square error (RMS) and the percent of signal power fit (SPF). These criteria are computed by [13]:

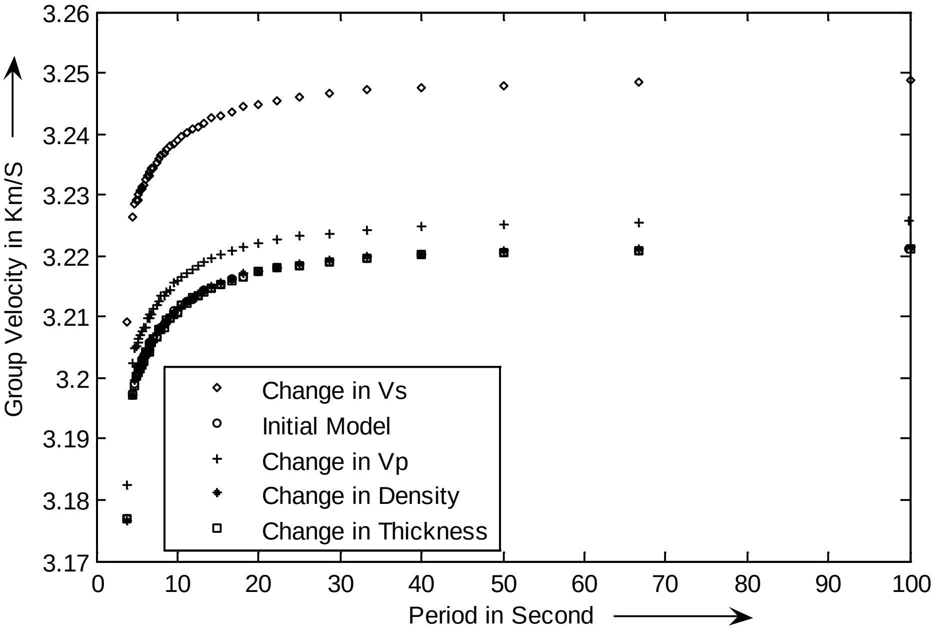


Figure 2: Sensitivity of group velocity dispersion obtained by 1% changes in each model parameter of the initial earth model as shown in Table 1.

$$SE = \sqrt{\frac{\sum_{i=1}^{N} (obsi - mean)^2}{N-1}} \qquad (8)$$

$$MR = \frac{\sum_{i=1}^{N} (obsi - predi)}{N} \qquad (9)$$

$$AR = \frac{1}{N} \left| \sum_{i=1}^{N} \frac{(obsi - predi)}{SE} \right| \qquad (10)$$

$$RMS = \sqrt{\frac{\sum_{i=1}^{N} \frac{(obsi - predi)^2}{mean}}{N(N-1)}} \qquad (11)$$

Where ‘obs’ is the observed group velocity at each period, ‘mean’ is the mean of the observed group velocities, N is the number of observations at each period and ‘pred’ is the predicted group velocity of the current model. Estimated errors of the models are shown in table 3.

$$SPF = \left(1 - \frac{\sum_{i=1}^{n} \left(obs_i - pred_i \right)^2}{\sum_{i=1}^{N} \left(obs_i \right)^2} \right) X\,100 \qquad (12)$$

5. Crustal Thickness Measurement

Group velocity dispersions are estimated in this section using graphical method (Eqn.1) and Haskell modified matrix method (Eqn. 5) as discussed below:

5.1. Group Velocity Estimation from Earthquake data

Group velocity is computed for the earthquake data recorded at Dhaka Meteorological Department seismic station, Bangladesh (located at 23.78^0 N and 9038^0 E) equipped with a three component digital broad–band sensor (Figure 1) and earthquake source parameters are shown in table 1.

Figure 3 shows order number (n) versus travel time (t) plot and Figure 4 shows group velocity variation with time period. This dispersion relation is computed by graphical method (Eqn. 1).

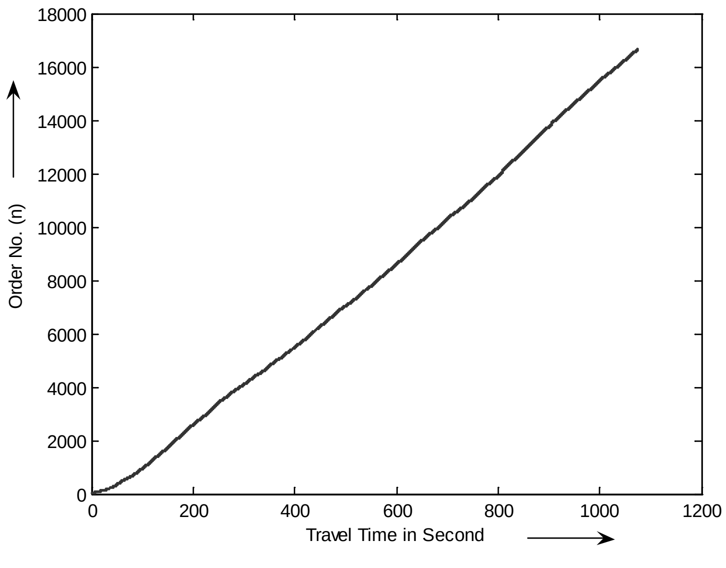


Figure 3: order number versus travel time curve for India-Nepal border region earthquake data

5.2 Group Velocity Estimation from Model

There are eight models (D1-D8) are considered in this work. Model based group velocity is computed using modified Haskell matrix method (Eqns. 2-7). The computed group velocity according to model parameters are shown in Figs. 5-12 also show the group velocity computed by graphical method.

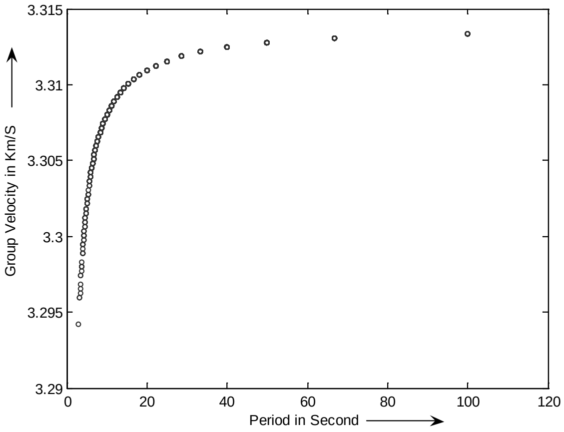
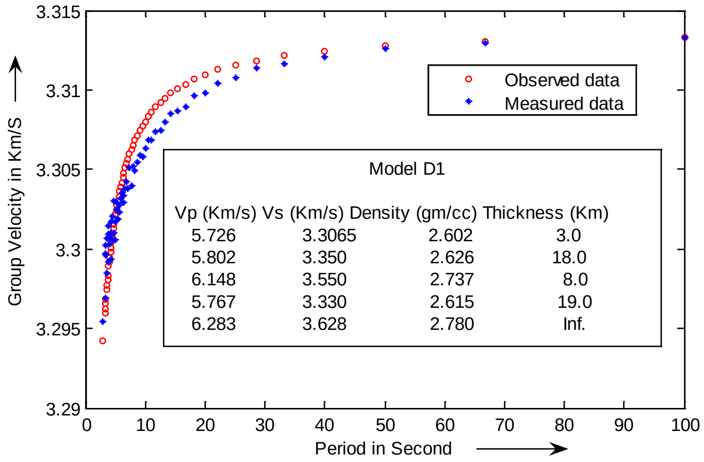


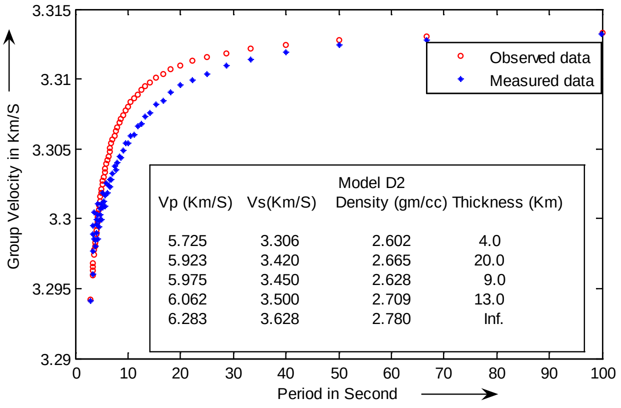
Figure 4: Group velocity dispersion curve for India-Nepal border region earthquake data



Model D1

Vp (Km/s)	Vs (Km/s)	Density (gm/cc)	Thickness (Km)
5.726	3.3065	2.602	3.0
5.802	3.350	2.626	18.0
6.148	3.550	2.737	8.0
5.767	3.330	2.615	19.0
6.283	3.628	2.780	Inf.

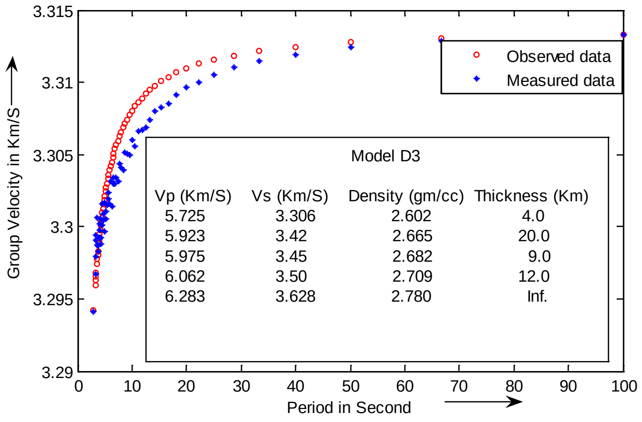
Figure 5: group velocity dispersion obtained from India-Nepal border region earthquake data and from modeling D1. Rectangular box contained the model parameters.



Model D2

Vp (Km/S)	Vs(Km/S)	Density (gm/cc)	Thickness (Km)
5.725	3.306	2.602	4.0
5.923	3.420	2.665	20.0
5.975	3.450	2.628	9.0
6.062	3.500	2.709	13.0
6.283	3.628	2.780	Inf.

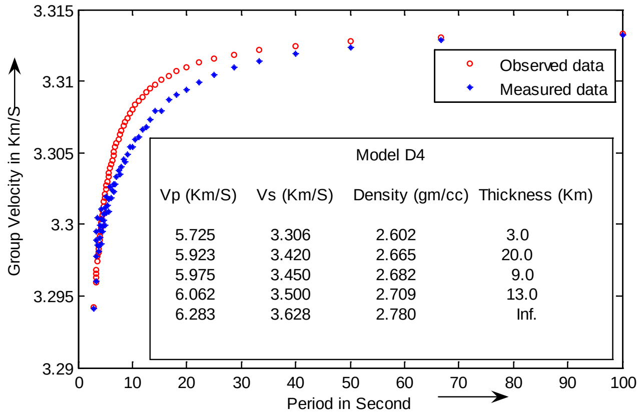
Figure 6: Group velocity dispersion obtained from India-Nepal border region data and from modeling D2. Rectangular box contained the model parameters.



Model D3

Vp (Km/S)	Vs (Km/S)	Density (gm/cc)	Thickness (Km)
5.725	3.306	2.602	4.0
5.923	3.42	2.665	20.0
5.975	3.45	2.682	9.0
6.062	3.50	2.709	12.0
6.283	3.628	2.780	Inf.

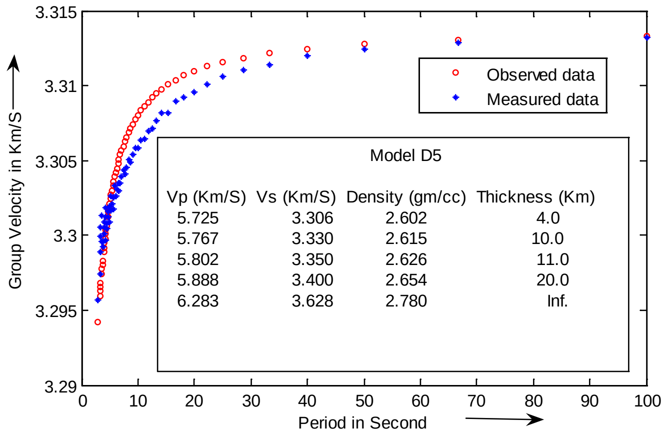
Figure 7: Group velocity dispersion obtained from India-Nepal border region earthquake data and from modeling D3. Rectangular box contained the model parameters.



Model D4

Vp (Km/S)	Vs (Km/S)	Density (gm/cc)	Thickness (Km)
5.725	3.306	2.602	3.0
5.923	3.420	2.665	20.0
5.975	3.450	2.682	9.0
6.062	3.500	2.709	13.0
6.283	3.628	2.780	Inf.

Figure 8: Group velocity dispersion obtained from India-Nepal border region earthquake data and from modeling D4. Rectangular box contained the model parameters.



Model D5

Vp (Km/S)	Vs (Km/S)	Density (gm/cc)	Thickness (Km)
5.725	3.306	2.602	4.0
5.767	3.330	2.615	10.0
5.802	3.350	2.626	11.0
5.888	3.400	2.654	20.0
6.283	3.628	2.780	Inf.

Figure 9: Group velocity dispersion obtained from India-Nepal border region earthquake data and from modeling D5. Rectangular box contained the model parameters.

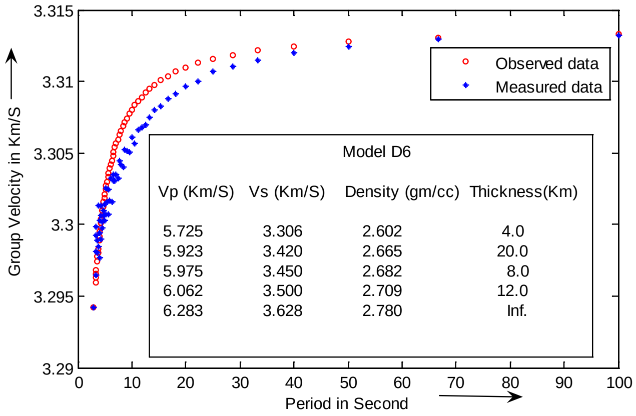


Figure10: Group velocity dispersion obtained from India-Nepal border region earthquake data and from modeling D6. Rectangular box contained the model parameters.

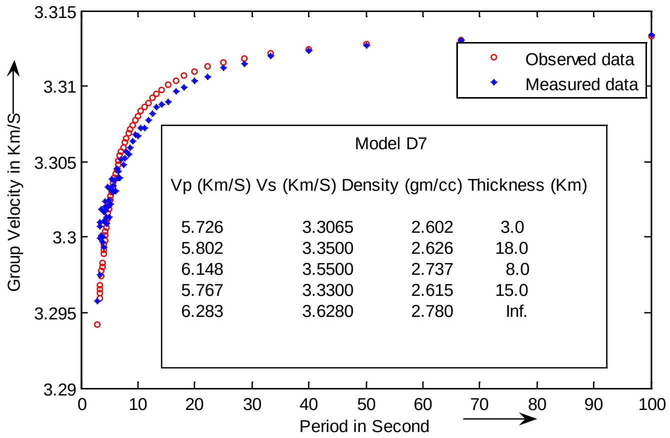


Figure 11: Group velocity dispersion obtained from India-Nepal border region earthquake data and from modeling D7. Rectangular box contained the model parameters.

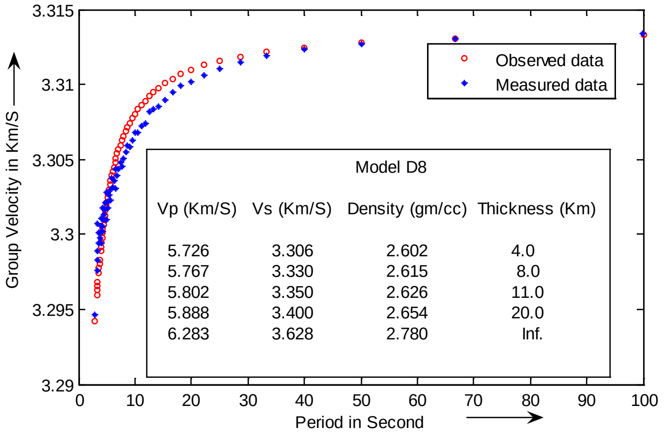


Figure 12: Group velocity dispersion obtained from India-Nepal border region earthquake data and from modeling D8. Rectangular box contained the model parameters.

6. Interpretation

From the modeling **s**ubsurface layers are estimated I th as seen that (Figs. 5-12) group velocity obtained from earthquake data and from models have the similar characteristics as both are varying with period, and to a reasonable maximum velocity. Therefore, interpretations are made from model parameters as shown in the rectangular box in Figs. 5-12. None of the plots is found matched exactly and it should not match as the models consider only four variables, in fact there should be few more variables. Hence, statistical errors are analyzed as explained in section (4). Using Eqns. 8-12 the computed errors are shown in Table 3.

Table 3: Data fit criteria

Model	SE	MR	AR	RMS	SPF
D1	0.0173109	0.0002477	0.0143114	0.0003304	99.99997829
D2	0.0173109	0.0011323	0.0654135	0.0003305	99.99997000
D3	0.0173109	0.0009765	0.0564137	0.0003252	99.99997234
D4	0.0173109	0.0011079	0.0640002	0.0003299	99.99997057
D5	0.0173109	0.0004663	0.0269418	0.0003499	99.99997487
D6	0.0173109	0.0009186	0.0530661	0.0003240	99.99997317
D7	0.0173109	-0.0002366	0.0136731	0.0002824	99.99997831
D8	0.0173109	0.0001649	0.0095266	0.0003144	99.99998440

According to estimated statistical errors (Table 3) the model E is found more acceptable. Hence it can be said that the India-Nepal earthquake wave is indicated that there are four major subsurface layers and layer thicknesses are shown in Figure 12.

7. Conclusion

There are few challenges to set up the model parameters. Most critical constraint is to consider the Poisson’s ratio of 0.25. In real cases the ratio might be different for different subsurface layers and hence the interpreted crustal structure from model might not be appropriate. However for the computational advantages Vp/Vs ratio or Poisson’s ratio were kept fixed as it has seen in many contributions to use the value of 1.732 or 0.25 respectively

On other hand from the sensitivity of the models, it has shown that the thickness of the layers is a vital factor therefore, thickness setting in the model is also found to be difficult [12]. However, from the investigations it is revealed that the setting of total depth rather than individual thicknesses of the subsurface layers can provide better interpretations that are more acceptable. Hence, total depth of 48.0 km is considered in our models. Instead of above limitations interpretation made from the four models are seemed good enough with the group velocities obtained by graphical methods as shown in Figs 5-12. Group velocity dispersion from the eight models (D1-D8) (Figs. 5-12) and considering statistical error analysis (Table 3), it can be said that all the models are very nearer to an acceptable matching level though the statistical confidence level SPF should be 91.5% but our results are around 99.99998440%. Considering all errors studying in this research (Table. 3) model D8 is seemed more acceptable of India-Nepal border region. Hence the interpreted subsurface layers of the studied this earthquake data shows that there are four major subsurface layers having respectively the thickness and density of 4.0 km, 2.602 gm/cc; 8.0 km, 2.615 gm/cc; 11.0 km, 2.626 gm/cc; 20.0 km, 2.654 gm/cc.

Acknowledgements

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