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Solution of Laplace`s Equation Using Numerical Methods

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Abstract

This paper is simple review of the solution of Laplace`s equation in rectangular coordinates system, cylindrical polar coordinates system and spherical polar coordinate system. It also covers numerical method in the solution of Laplace`s equation.

Keywords: Laplace equation; Numerical method; Laplacian difference; Uniform grid.

1. Introduction

During the last two centuries several methods have been advanced for solving partial differential equations. Among these we consider only two techniques known as the method of separation of variables and Laplace transformation , the method of separation of variables is perhaps the oldest systematic method for solving partial differential equations. It has been considerably refined and generalized in meantime and remains a method of great importance to day. In this study we will review how the method of separation of variables. In solving this problem it is eventually necessary to consider the equations of whether an essentially arbitrary function can be expressed as an infinite series of sine and cosine functions. Series of this kind are called Fourier series, then we will come across to in some typical problems as the Laplace`s equation solving it by separation of variables and Fourier series .By the other hand we will explain in following:

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In first we define rectangular coordinate system

$$\nabla^2 U = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

In second we transform the rectangular coordinates system to cylindrical coordinates system is which comes as:

$$U(x, y. z) = (\rho, \theta, z)$$

and its solution given by:

$$\frac{\partial^2 v}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial v}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 v}{\partial \theta^2} + \frac{\partial^2 v}{\partial z^2} = 0$$

In third we also transform the rectangular coordinates system to spherical polar coordinates system which comes as:

$$U = (x, y, z) = (r, \theta, \phi)$$

$$\nabla^2 U = \frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}(\sin\theta \frac{\partial u}{\partial \theta}) + \frac{1}{r^2}\frac{1}{\sin^2\theta}\frac{\partial^2 u}{\partial \theta^2} = 0$$

In fourth we explain computerized computation method to solve general solution of Laplace`s equation by finite different equation.

2. Two dimensional heat flow

Consider the flow of heat in a metal plate of uniform thickness $\alpha(cm)$**,** density $(\rho/gr/cm^3)$ **,**specific heat $S(cal./gr.$deg)and thermal conductivity $K(cal/cm.$deg) **.** Let $X \circ Y$plane be taken in face the point [1]. If temperature at any point is independent of the Zcoordinate and depends only on X, Y and time t. Then the flow is said to be two dimensional .In this case, the heat flow is in the XY-plane only and is zero along the normal to the XY-plane. Consider a rectangular element$ABCD$ of the plane with sides as shown in figure. By Aon the amount of heat entering the element, from the side

$$AB = -k\alpha\, \partial y \left[\frac{\partial u}{\partial x}\right]_x .$$

The quantity of heat flowing out through the side CD per second $= -k\alpha\partial x \left[\frac{\partial u}{\partial y}\right]_{y+\partial y}$ (1)

and the quantity of heat flowing out through the side BC per second $= -k\alpha\partial y \left[\frac{\partial u}{\partial x}\right]_{x+\partial x}$ (2)

Hence the total gain of heat by rectangular element per second

$$= -k\alpha\partial x\left[\frac{\partial u}{\partial y}\right]_y - k\alpha\partial y\left[\frac{\partial u}{\partial x}\right]_x + k\alpha\partial x\left[\frac{\partial u}{\partial y}\right]_{y+\partial y} + k\alpha\partial y\left[\frac{\partial u}{\partial x}\right]_{x+\partial x} = k\alpha\partial x\left[\frac{\left[\frac{\partial u}{\partial x}\right]_{x+\partial x} - \left[\frac{\partial u}{\partial x}\right]_x}{\partial x} + \frac{\left[\frac{\partial u}{\partial y}\right]_{y+\partial y} - \left[\frac{\partial u}{\partial y}\right]_y}{\partial y}\right]$$

Also the rate of gain of heat by the element $= \rho\partial x\partial y\alpha s\frac{\partial u}{\partial t}$

Thus equating (1) and (2) $= k\alpha\partial x\partial y\left[\frac{\left[\frac{\partial u}{\partial x}\right]_{x+\partial x} - \left[\frac{\partial u}{\partial y}\right]_x}{\partial x} + \frac{\left[\frac{\partial u}{\partial x}\right]_{y+\partial y} - \left[\frac{\partial u}{\partial y}\right]_y}{\partial y}\right] = \rho\partial x\partial y\alpha s\frac{\partial u}{\partial t}$ (3)

Dividing both sides by $\alpha\partial x\partial y$ and taking limit as $\rightarrow 0$, we get:

$$k\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] = \rho s\frac{\partial y}{\partial t}$$

$$\frac{\partial u}{\partial t} = C^2\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] \quad (4)$$

where $C^2 = k/\rho s$ is the diffusivity. Hence the equation (4) gives the temperature distribution of plane in the transit state [1].

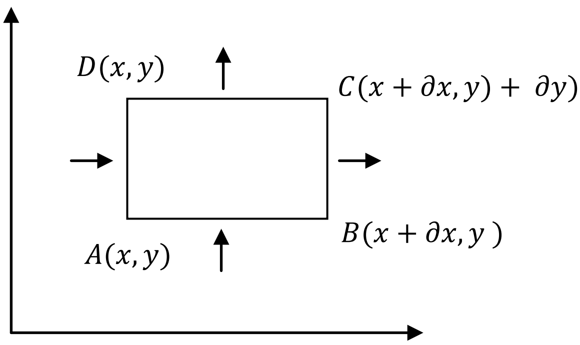


Figure1

3. The diffusion equation in two-dimensions

When cylindrical co-ordinates ρ, θ, z are used see [2],we have

$x = \rho\cos\theta$, $y = \rho\sin\theta$, $z = z$and the equation of the conduction of heat comes

$$\frac{\partial^2 v}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial v}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2 v}{\partial\theta^2} + \frac{\partial^2 v}{\partial z^2} = 0 \quad (5)$$

Let us first consider solutions which are independent of z

$$\frac{\partial^2 v}{\partial \rho^2}+\frac{1}{\rho}\frac{\partial v}{\partial \rho}+\frac{1}{\rho^2}\frac{\partial^2 v}{\partial \theta^2}=0 \qquad (6)$$

the equation for R is $\frac{\partial^2 R}{\partial \rho^2}+\frac{1}{\rho}\frac{\partial R}{\partial \rho}+\frac{1}{\rho^2}\frac{\partial^2 R}{\partial \theta^2}=0$ (7)

By separation of variables

$$R^{\backslash\backslash}\psi+\frac{1}{\rho}R^{\backslash}\psi+\frac{1}{\rho^2}R\psi^{\backslash\backslash}=0 \qquad (8)$$

$$R\psi\frac{R^{\backslash\backslash}}{R}+\frac{1}{\rho}\frac{R^{\backslash}}{R}+\frac{\psi^{\backslash\backslash}}{\psi}=-\lambda \qquad (9)$$

$$\rho^2\frac{R^{\backslash\backslash}}{R}+\rho\frac{R^{\backslash}}{R}+\lambda^2\rho^2-\frac{\psi^{\backslash\backslash}}{\psi}=\mu^2 \qquad (10)$$

from (10),we $\mu^2\psi=0$ (11)

$$\rho^2R^{\backslash\backslash}+\rho R^{\backslash}+(\lambda^2\rho^2-\mu^2)R=0 \qquad (12)$$

The solution of (11) and (12) given respectively by :

$$\psi(\theta)=A_1\cos\mu\theta+A_2\sin\mu\theta \qquad (13)$$

$$R(\rho)=B_1J_\mu(\lambda\rho)+B_2Y_\mu(\lambda\rho) \qquad (14)$$

The general solution is

$$V(\rho,\theta)=[A_1\cos\mu\theta+A_2\sin\mu\theta]\big[B_1J_\mu(\lambda\rho)+B_2Y_\mu(\lambda\rho)\big] \qquad (15)$$

Since V is bounded $\rho=0$ then $B_2=0$ gives

$$V(\rho,\theta)=[A_1\cos\mu\theta+A_2\sin\mu\theta]\big[B_1J_\mu(\lambda\rho)\big] \qquad (16)$$

An initial distribution of concentration, let $V=f(\rho,\theta)$

when $t=0$,we may try to satisfy the condition ,when $\mu=n$

and V must have period 2π in the variable θ.

$$V(\rho,\theta)=[A_1\cos\mu\theta+A_2\sin\mu\theta]\big[B_1J_\mu(\lambda\rho)\big] \qquad (17)$$

Now by substituting boundary condition $V(1,\theta)$ =0

$$V(\rho,\theta) = [A_1 \cos n\theta + A_2 \sin n\theta][BJ_n(\lambda\rho)] \quad (18)$$

Now equation (17) can be expressed us

$$V(\rho,\theta) = [A \quad \cos n\theta + B \sin n\theta][J_n(\lambda_{mn}\rho)] \quad (19)$$

By using super position

$$V(\rho,\theta) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} [A_{mn} \cos n\theta + B_{mn} \sin n\theta] J_n(\lambda_{mn}\rho) \quad (20)$$

We let $V = f(\rho,\theta)$ gives

$f(\rho,\theta) = \sum_{n=0}^{\infty} D_n \cos n\theta + E_n \sin n\theta$(21)Formula (21) represent Fourier Series of$f(\rho,\theta)$ with period 2π

$D_n = \frac{1}{\pi}\int_0^{2\pi} f(\rho,\theta) \cos n\theta \, d\theta$and$E_n = \frac{1}{\pi}\int_0^{2\pi} f(\rho,\theta) \sin n\theta \, d\theta$ for $n = 1,2,\ldots$

We have $A_{mn} = \frac{2}{J_{n+1}^2(\lambda_{mn})} \int_0^1 \rho J_n(\lambda_{mn}\rho) D_n d\rho$

And $B_{mn} = \frac{2}{J_{n+1}^2(\lambda_{mn})} \int_0^1 \rho J_n(\lambda_{mn}\rho) E_n d\rho$

4. The elementary solution

If we solve

$\nabla^2 U = 0$ in spherical coordinates (r,θ,ϕ)

$$\nabla^2 U = \frac{1}{h_1 h_2 h_3} [\frac{\partial}{\partial q_1} [\frac{h_2 h_3}{h_1} \frac{\partial u}{\partial q_1}] + \frac{\partial}{\partial q_2} [\frac{h_1 h_3}{h_2} \frac{\partial u}{\partial q_2}] + \frac{\partial}{\partial q_3} [\frac{h_2 h_1}{h_3} \frac{\partial u}{\partial q_3}]$$

where$h_1 = 1, h_2 = r$ and $h_3 = r \sin\theta$ and $q_1 = r, q_2 = \theta$, and $q_3 = \phi$.If U is independent ofϕ with boundary condition $U(1,\theta) = f(\theta)$

the solution of $\nabla^2 U = 0$ in spherical coordinates which is independent of ϕ expressed as:

$$r^2 \frac{\partial^2 u}{\partial r^2} + 2r \frac{\partial u}{\partial r} + \frac{1}{\sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial u}{\partial\theta}\right) = 0 \quad (22)$$

Take $u(r,\theta) = R(r)\psi(\theta)$(23)

We get

$$r^2 R^{\backslash\backslash}\psi + 2rR^{\backslash}\psi + \frac{R}{\sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta \, \psi^{\backslash}\right) = 0$$

Dividing the above equation by $R\psi$,then we get

$$r^2 \frac{R^{\backslash\backslash}}{R} + 2r\frac{R^{\backslash}}{R} = -\frac{1}{\psi \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\, \psi^{\backslash}\right) = \lambda^2$$

$$r^2 R^{\backslash\backslash} + 2rR^{\backslash} + \lambda^2 R = 0 \qquad (24)$$

And

$$\frac{\partial}{\partial\theta}\left(\sin\theta\, \psi^{\backslash}\right) - \lambda^2 \psi \sin\theta = 0$$

$$\sin\theta \frac{\partial^2\psi}{\partial\theta^2} + \cos\theta \frac{\partial\psi}{\partial\theta} - \lambda^2\psi \sin\theta = 0 \qquad (25)$$

the solution of (24) is given by

$$R(r) = Ar^n + \frac{B}{r^{n+1}} \qquad (26)$$

where$\lambda^2 = -n(n+1)$ (27)

from (27) and equation(25)

$$\sin\theta \frac{\partial^2\psi}{\partial\theta^2} + \cos\theta \frac{\partial\psi}{\partial\theta} + n(n+1)\sin\theta\, \psi = 0 \qquad (28)$$

Equation (28) is a Legendre ODE, with general solution

$$\psi(\theta) = CP_n(\cos\theta) + DQ_n(\cos\theta) \qquad (29)$$

$$u(r,\theta) = \left[Ar^n + \frac{B}{r^{n+1}}\right]\left[CP_n(\cos\theta) + DQ_n(\cos\theta)\right] \qquad (30)$$

If $u(r,\theta)$ represent temperature on sphere radius with center at origin, there the heat must be bounded. When $\theta = 0$ orπ (along Z - axis in spherical coordinates) when

$Q_n(1) \to \infty$ this provided that $D = 0$ (31)

To avoid infinitely temperature at center of sphere $r = 0$ we take $B = 0$ (32)

Now from (31) and (32) the solution is given by (30)

$$u(r,\theta) = Er^n P_n(\cos\theta) \qquad (33)$$

By the supper position (33) can be written as

$$u(r,\theta) = \sum_{n=0}^{\infty} E_n r^n P_n(\cos\theta) \tag{34}$$

By applying the boundary condition $u(1,\theta) = f(\theta)$ into (34).

$$f(\theta) = \sum_{n=0}^{\infty} \mathrm{E_n P_n}(\cos\theta) \tag{35}$$

Let $\mu = \cos\theta$,then$\theta = cos^{-1}\mu$

$$f(cos^{-1}\mu) = \sum_{n=0}^{\infty} \mathrm{E_n P_n}(\mu) \tag{36}$$

Now

$$E_n = \frac{(2n+1)}{2}\int_{-1}^{1} f(cos^{-1}\mu)P_n(\mu)d\mu \tag{37}$$

From (34), we have

$$u(r,\theta) = \sum_{n=0}^{\infty}\left[\frac{(2n+1)}{2}\int_{0}^{\pi} f(\theta)P_n(\cos\theta)\sin\theta d\theta\right][r^n P_n(\cos\theta)].$$

5. The Laplacian Difference Equation

Central differences based on the grid and scheme used for the finite - difference solution in two independent variables such as the Laplace equation [3].

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\nabla x^2} \tag{38}$$

And

$$\frac{\partial^2 T}{\partial y^2} = \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\nabla y^2} \tag{39}$$

respectively which have errors of $\mathrm{O}[\nabla(x)^2]$ and $\mathrm{O}[\nabla(y)^2]$

Substituting these expressions into equation into equation

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0$$

Gives

$$\frac{T_{i+1,j}-2T_{i,j}+T_{i-1,j}}{\nabla x^2}+\frac{T_{i,j+1}-2T_{i,j}+T_{i,j-1}}{\nabla y^2}=0 \qquad (40)$$

For the square grid, $\nabla x=\nabla y$ and by collection of terms ,the equation becomes

$$T_{i+1,j}+\mathrm{T}_{\mathrm{i-1,j}}+\mathrm{T}_{\mathrm{i,j+1}}+\mathrm{T}_{\mathrm{i,j-1}}-4\mathrm{T}_{\mathrm{i,j}}=0 \qquad (41)$$

This relationship, which holds for all interior points on the plate, is referred to as the Laplacian different equation.

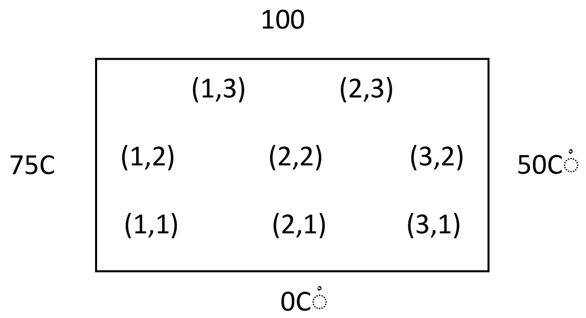


Figure 2

A heated plate where boundary temperature are hold at const balance

ant levels. This called adirichlet boundary condition. Where the edges are hold at constant temperature for the case illustrated in figure 2 balance figurec1 is ,according to equation (41)

$$T_{21}+\mathrm{T}_{01}+\mathrm{T}_{12}+\mathrm{T}_{10}-4\mathrm{T}_{11}=0 \qquad (42)$$

and $\mathrm{T}_{10}=0$However, $\mathrm{T}_{01}=75$

therefore equation $\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0$, can be expressed as $-4T_{11}+T_{12}+T_{21}=-75$

Similar equation can be developed for the other interior points. the result is the following set of nine simultaneous equations with nine unknowns.

$$4T_{11} \quad -T_{21} \quad -T_{12} \qquad = 75$$

$$-T_{11}4T_{21}-T_{11}-T_{22} \qquad = 0$$

$$-T_{21}4T_{31}-T_{32} \qquad = 50$$

$-T_{11}\ 4T_{12} - T_{22} - T_{13} \qquad = 75$

$-T_{21} - T_{12} 4T_{22} - T_{32} - T_{23} \qquad = 0$

$-T_{31} - T_{22} 4T_{32} T_{33} \qquad = 50$

$-T_{12} 4T_{13} - T_{23} = 175$

$-T_{22} - T_{13} 4T_{23} - T_{33} \qquad = 100$

$-T_{32} - T_{23} 4T_{33} = 150$

5.1 Example

Temperature of heated plate with fixed boundary conditions problem statement [4]. Use Lobmann's method (Gauss-Seidel) to solve for temperature of the heated in figure (4.2).Employ over relaxation with a value of 1.5 for the weighting factor and iterate $\varepsilon_a = 1\%$.

solution

From equation $T_{i,j} = \frac{T_{i+1,j}+T_{i-1,j}+T_{i,j+1}+T_{i,j-1}}{4}$

at $i = 1, j = 1$

is $T_{11} = \frac{0+75+0+0}{4}$ =18.75 and applying relaxation yield

$T_{11} = 1.5(18.75) + (1 - 1.5)0 = 28.125$

$$T_{21} = \frac{0 + 28.125 + 0 + 0}{4} = 7.03125$$

$T_{21} = 1.5(7.03125) + (1 - 1.5)0 = 10.54688$

$$T_{31} = \frac{50 + 10.54688 + 0 + 0}{4} = 15.13672$$

$T_{31} = 1.5(15.13672) + (1 - 1.5)0 = 22.70508$

The computation is repeated for other rows to give

$T_{12} = 38.67188 \qquad T_{22} = 18.45703 \qquad T_{32} = 34.18579$

$T_{13} = 80.12696 \qquad T_{23} = 74.46900 \qquad T_{33} = 96.99554$

Because all the $T_{i,j}$`s are initially zero, all ε_a for the first iteration will be 100%.

For the second iteration the results are:

$T_{11} = 32.51953$ $T_{21} = 22.35718$ $T_{31} = 28.60108$

$T_{12} = 57.95288$ $T_{22} = 61.63333$ $T_{32} = 71.86833$

$T_{13} = 75.21973$ $T_{23} = 87.95872$ $T_{33} = 67.68736$

The error forT_{11} can be estimated as equation $-\frac{\partial q}{\partial x} - \frac{\partial q}{\partial q} = 0$

$$\left|(\varepsilon_a)_{i,j}\right| = \left|\frac{32 - 1953 - 28 - 12500}{32 - 51953}\right| 100\% = 13.5\%$$

Because this value is the stopping criterion of 1% the computation is continued.

The ninth iteration gives the result:

$T_{11} = 43.0061$ $T_{21} = 33.29755$ $T_{31} = 33.88506$

$T_{12} = 63.21152$ $T_{22} = 56.11238$ $T_{32} = 52.33999$

$T_{13} = 78.58718$ $T_{23} = 76.06402$ $T_{33} = 69.71050$

where the maximum error is 0.71%.

6. Explicit Solution for uniform grid increments

Consider a rectangular domain where the increments in both x and y are uniform [5]. The appropriate equation to use for an explicit solution to the Laplace equation is

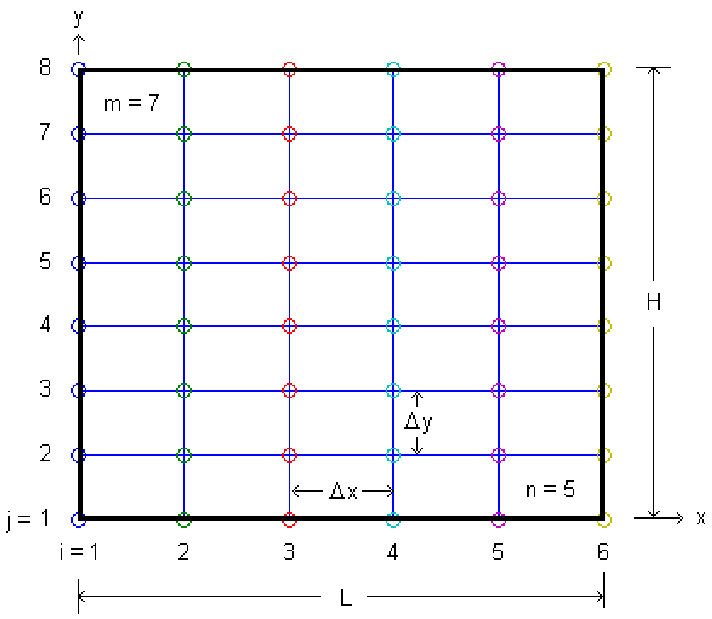
$$T_{ij} = \frac{T_{i-1,j} + T_{i+1,j} + \beta^2(T_{i,j+1} + T_{i,j+1})}{2(1+\beta^2)} \quad (43)$$

Where $\beta = \frac{\Delta x}{\Delta y}$

The solution will start by loading the boundary conditions, and then calculating the values of T_{ij} in the interior points of domain. While we are initially temped to calculateT_{ij} only once with equation (43), it should be mentioned that these values are only a fist approximation to the solution .We should, therefore , add additional index ,k, representing the current iteration, to each solution value .The solution values will now be referred to as T_{ij}^k , and equation (43) will be modified to read:

$$T_{i,j}^{k+1} = \frac{T^k{}_{i-1,j}+T^k{}_{i+1,j}+\beta^2(T^k{}_{i,j-1}+T^k{}_{i,j+1})}{2(1+\beta^2)} \quad (44)$$

The iterative process should be repeated until convergence is achieved in every interior point of the domain , or until a maximum number of iterations , say 100 , have been performed . Convergence can be achieved ,for example ,if ,given a tolerance value ε maximum difference two consecutive iterations is less than the tolerance , ,i.e., if



$$\max_{i,j}\left|T_{i,j}^{k+1} - T^k{}_{i,j}\right| \leq \varepsilon.$$

Consider , as an example ,a rectangular domain of length $L = 5cm$, and height $H = 3.5cm$, with increments $\Delta x = 1cm, and\ \Delta y = 0.5cm$, as illustrated in the figure blow.

There will be $n = {}^{L}/_{\Delta x}$ sub-intervals in x , and$m = {}^{H}/_{\Delta y}$ sub-intervals in , with

$$x_i = (i-1)\Delta x, for\ i = 1,2, \dots, n+1,$$

and

$$y_j = (j-1)\Delta y\ , for\ j = 1,2, \dots, m+1$$

The boundary conditions are given as follows: $T_{ij} = 5$ along the left and right sides of the domain , while the temperature are given by the function $T_b(x) = 5.x.(1-x)$ for the top and bottom sides of the domain ,respectively [5].

Solution is achieved by using function *LaplaceExplicit.m*in Matlab :

```
function [x,y,T]= LaplaceExplicit(n,m,Dx,Dy)

echo off;

numgrid(n,m);

R = 5.0;

T = R*ones(n+1,m+1); % All T(i,j) = 1 includes all boundary conditions

x = [0:Dx:n*Dx];y=[0:Dy:m*Dy]; % x and y vectors

for i = 1:n % Boundary conditions at j = m+1 and j = 1

6

T(i,m+1) = T(i,m+1)+ R*x(i)*(1-x(i));

T(i,1) = T(i,1) + R*x(i)*(x(i)-1);

end;

TN = T; % TN = new iteration for solution

err = TN-T;

% Parameters in the solution

beta = Dx/Dy;

denom = 2*(1+beta^2);

% Iterative procedure

epsilon = 1e-5; % tolerance for convergence

imax = 1000; % maximum number of iterations allowed

k = 1; % initial index value for iteration

% Calculation loop

while k<= imax
```

```
for i = 2:n

for j = 2:m

TN(i,j)=(T(i-1,j)+T(i+1,j)+beta^2*(T(i,j-1)+T(i,j+1)))/denom;

err(i,j) = abs(TN(i,j)-T(i,j));

end;

end;

T = TN; k = k + 1;

errmax = max(max(err));

if errmax< epsilon

[X,Y] = meshgrid(x,y);

figure(2);contour(X,Y,T',20);xlabel('x');ylabel('y');

title('Laplace equation solution - Dirichlet boundary conditions

- Explicit');

figure(3);surfc(X,Y,T');xlabel('x');ylabel('y');zlabel('T(x,y)');

title('Laplace equation solution - Dirichlet boundary conditions

- Explicit');

fprintf('Convergence achieved after %i iterations.\n',k);

fprintf('See the following figures:\n');

fprintf('==========================\n');

fprintf('Figure 1 - sketch of computational grid \n');

fprintf('Figure 2 - contour plot of temperature \n');

fprintf('Figure 3 - surface plot of temperature \n');
```

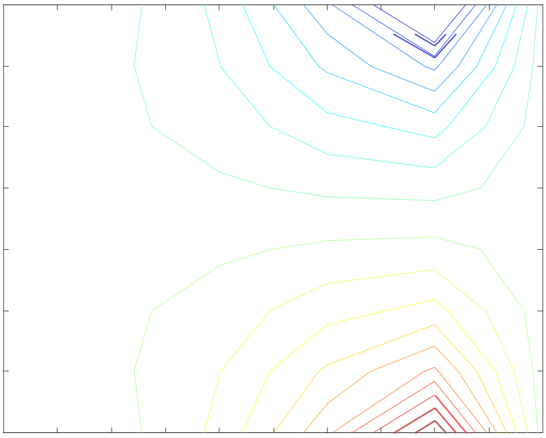
```
return

end;

end;

fprintf('\n No convergence after %i iterations.',k);
```

To activate the function for the case illustrated in the figure above we use:

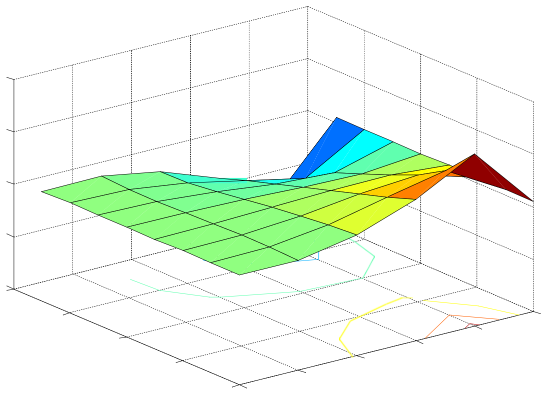
$\gg [X,Y,T] = LaplaceExplicit(5,7,1,0.5)$

The solution is returned in the vectors x and y ,and in matrix T .The function produces three plots: a sketch of the grid (similar to the figure above),the solution as a contours, and the solution as a surface .The last two figures are shown next:

Laplace equation solution –Dirichletboundary conditions-Explicit



Laplace equation solution - Dirichletboundary conditions - Explicit



7. Conclusion and Recommendation

This paper showed that Numerical methods are usually easier to use in the solution of Laplace's equation. We notice that our results agree with other works cited in article. It is aim to continue higher order for two dimensional.

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