Hysteresis Modeling of Amplified Piezoelectric Stack Actuator for the Control of the Microgripper

Royson D. D'Souza [a]*, Bineesh Benny [b], Anil Sequeira [c], Navin Karanth P [d]

[a,c]*School of Engineering and IT, Manipal University, Dubai Campus, UAE*

[b,d]*Department of Mechanical Engineering, National Institute of Technology Karnataka (NITK), India*

[a]*Email: roysonnitk@gmail.com*

Abstract

This paper presents Bouc-Wen hysteresis modelling and tracking control of piezoelectric stack APA120S. The actuator is used to control a microgripper. A modified Bouc-Wen non-symmetric model is applied to study the behaviour of the system in static and dynamic state. The good agreement between predicted and measured curve showed that the Bouc-Wen model is an effective mean for modelling the hysteresis of piezoelectric actuator system. Subsequently, the inverse Bouc-Wen model is formulated and applied to cancel the non-linear hysteresis. In perspective of a control design, it is desirable to linearize the non-linear Bouc-Wen model to produce a static system. Finally, in order to increase damping of the actuator system and to improve the control accuracy, a cascaded PID controller is designed with consideration of the dynamics and static behaviour of the actuator. Experiment result shows that error is of only 5% if PID is cascaded with hysteresis compensation. Therefore, hysteresis compensation with PID controller greatly improves the micromanipulation accuracy of the microgripper actuated by piezoelectric stack.

Keywords: Hysteresis modeling; amplified piezoelectric actuator (APA); Bouc-Wen model; tracking control.

1. Introduction

Piezoelectric stack actuators have advantages of high precision, solid state actuation and fast responses. It is used in many applications like, vibration control [1], hydraulic flow control [2], energy harvesting [3,4] and sonochemistry application [5]. Piezoelectric stack APA120S possess hysteresis nonlinearity. This nonlinearity considerably degrades tracking control, especially in micro-manipulators [6,7].

----

* Corresponding author.

Piezoelectric stack has been studied for actuation of microgripper. Hysteresis nonlinearity need to be cancelled to accurately control the microgripper. Hysteresis has to be mathematically modelled to apply compensation.

Various methods are Ishlinskii hysteresis model [8], generalized Maxwell resistive capacitor-based lumped parameter model [9], the variable time relay hysteresis model [10], the Jiles-Atherton model hysteresis [11], the Preisach model and Bouc-Wen model.

Applications of Bouc-Wen model to simulate the hysteresis of piezoelectric stack have been studied in [12,18]. Habineza, Rakotondrabe and Le Gorrec applied the Bouc-Wen model in multi degree freedom system piezoactuator. They extended Bouc-Wen model used for 1-DOF systems [12]. Kozlov explained finding the parameter using successive approximation using Fourier Transform [13]. Qiao Zhi, Gan Minggang andWang Chenyi explains about applying sliding mode control in sinusoidal application using Bouc-Wen model [14]. Zhi Liu, Guanyu Lai, Yun Zhang and Chen used Bouc-Wen model with adaptive neural output feedback control for hysteresis compensation [15]. Habineza, Rakotondrabe and Le Gorrec proposes a way to address multivariable cross coupled hysteresis by extending monovariable Bouc-Wen model [16]. Laudani and Fulginei used a new hybrid heuristic called metric-topological-evolutionary optimization, the Bouc-Wen identification is presented [17]. Zhi Liu, Guanyu Lai, Yun Zhang and Xin Chen explained about using adaptive neural network for unknown hysteresis application [18].

Modelling of hysteresis demands more computational complexity, but it will provide partial cancellation of hysteresis using feed-forward path. By efficient cancellation of nonlinearity, a linear feedback controller can be applied. This is important in position control devices like micro and nano positioning system.

This paper presents modified Bouc-Wen model to simulate the static hysteresis behaviour of piezoelectric stack APA120S. To implement this model, hysteresis curve is measured with voltage starting from 0-160V. Parameters are verified experimentally. The good agreement between experimental and predicted curve shows that Bouc-Wen model is an effective mean for hysteresis prediction.

Subsequently, the inverse Bouc-Wen model is applied to the piezostack to cancel its hysteresis for micro positioning of microgripper. Then inverse Bouc-Wen model is linearized for static application. A cascaded PID feedback controller is designed based on the linear model of the piezoactuator. Experimental result shows that error is of only 5% if hysteresis compensation with PID feedback controller is cascaded. Therefore accuracy in micro positioning is greatly improved compared to that without hysteresis compensation.

2. Modeling Hysteresis

In order to design a precise tracking controller for a piezoelectric actuator, an appropriate model which describes the behavior of the piezoelectric actuator is necessary. One of the critical fields for designing a robust controller is the modeling of the hysteresis phenomenon, as previously discussed. There are many models that have been proposed in order to capture the hysteretic characteristics for analysis of hysteresis behavior. Hysteretic models can be categorized into two categories [21]:

1. Physics based models
2. Phenomenology based models

In physics-based modes, the basic magnetizing modes are described by simulation of the basic processes. Some physics-based models are Jiles-Atherton model and homogenized energy model. In phenomenology-based models, the gross behavior of the material is mathematically described. Examples of such models include Preisach model, Prandtl-Ishlinkii model, Duhem model, and Bouc Wen model.

There exist a plethora of strategies for compensation of hysteretic error of piezoelectric actuators either using charge driven or voltage driven control. As aforementioned, voltage driven control proves to be the better of the two options. Hysteretic error compensation via voltage control may be accomplished by two control principles:

1. A feedback voltage control utilizing sensors to measure the output of the actuator
2. Feedforward voltage control scheme incorporating the actuator model. This model may be a direct or an inverse.

Their usage differs based on the model type. The feedback for the controller may be obtained from the direct actuator model. The inverse actuator, on the other hand, will estimate the input for the desired output which is to be obtained. In order to demonstrate with an example, the microgripper will be required to move by a certain value. This displacement value can be used in order to estimate the amount of voltage which is to be fed to the piezostack actuator. The system model may consist of a surfeit of sub-models which may depend on a large number of parameters. Optimization techniques must be incorporated in order to select the 'best' set of values for these parameters.

For piezoelectric actuators, the gross behavior of the material is mathematically described using phenomenology-based models. In voltage driven control, there exist two approached for modelling and control:

1. the Preisach
2. and the Prandtl–Ishlinskii

In both cases, complex hysteresis is broken down in to the sum of many elementary hysteresis, called hysterons. The hysteresis inverse model, or the compensator, is then computed using the identified model. The accuracy of these approaches is directly proportional to the number of hysterons. However, higher accuracy will correspond to higher compensator complexity and thus implementation may be compromised. A low number of elementary hysterons, on the other hand will compromise the accuracy. The additional computation of the inverse model poses an additional constraint for these methods.

The proposed compensator scheme adapted to model hysteresis is based on the multiplicative inverse structure and is expressed by the Bouc-Wen model. The advantage Bouc-Wen model holds over its competitors is the lack of computation required for the compensator. To add to its advantage, the same direct model is used in the inverse model (compensator model). Bouc Wen model is also based on a set of equations, allowing for easy adaptation from a control-theory point of view.

Therefore, in the interest of simplicity in implementation, computation, and optimization, an approach to modelling hysteresis is taken dedicated to the Bouc-Wen model of hysteresis in piezoelectric actuators.

3. Bouc-wen Model

In order to design a precise tracking control system for the proposed microgripper using APA120S as the actuator, an appropriate model describing the behaviour of the actuator is required. A hysteretic semi-physical model was initially introduced by Subsequently, in 1976, it was generalized by Wen. Since then, the resultant model is known as the Bouc-Wen model been extensively used to describe devices and components with hysteretic behaviour . Essentially, the model consists of a second-order non-linear differential equation. It relates the input (displacement) to the output (restoring force) in a hysteretic way. The equation varies from one component to the next due to the inherent parameters. By choosing an appropriate set of parameters, it is possible to model the response of the model to real hysteresis loops. Thus, an important aspect of the control system design is the tuning of the parameters for the specific application and material.

Let us consider a piezoelectric actuator subject to hysteresis. As described by the Bouc-Wen model, it can be identified as a second-order linear model preceded by hysteretic non-linearity as follows:

$$m\ddot{x} + b\dot{x} + kx = k(du - h) \qquad\qquad (1)$$

$$\dot{h} = \alpha d\dot{u} - \beta|\dot{u}|h|h|^{n-1} - \gamma\dot{u}|h|^n \qquad\qquad (2)$$
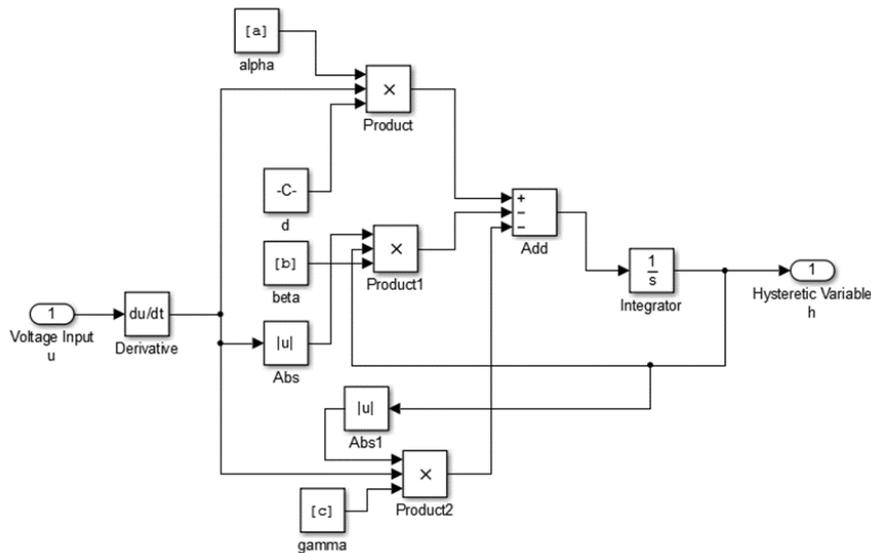
Figure 1: Simulink Block diagram representing BOUC-WEN model

Where m is the effective mass (kg), b is the effective damping (N-s/m), k is the mechanical stiffness (N/m), and d is the effective piezoelectric coefficient (m/V) of the piezoactuator. The input voltage to the piezoelectric actuator is represented by u (volts), x is the displacement (m) of the piezoelectric actuator, and his the hysteretic state variable (in m). α, β, and γ are parameters which control hysteretic loop's magnitude and shape. The 'n' controls the smoothness of transition from elastic to plastic region; however, for the piezoelectric actuator, assuming it follows elastic behaviour, n is considered as 1. The equation becomes:

$$\dot{h} = \alpha d\dot{u} - \beta|\dot{u}|h - \gamma\dot{u}|h| \qquad (3)$$
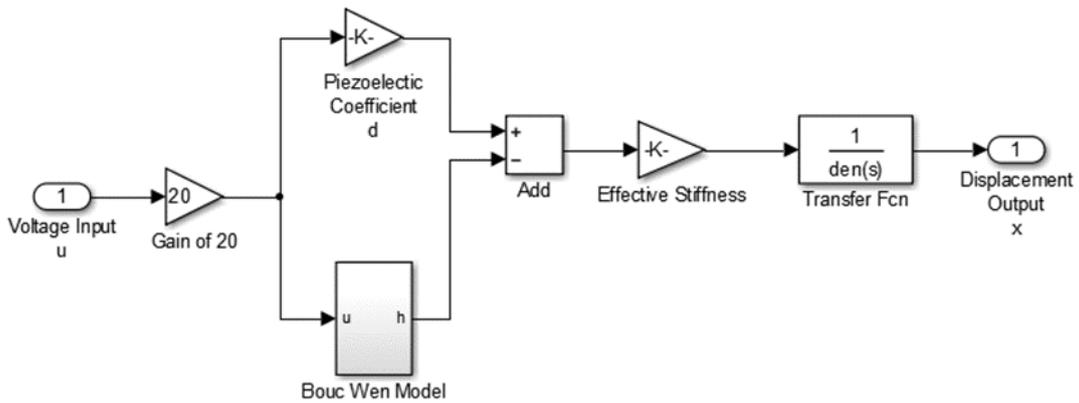
Figure 2: Simulink Block Diagram for Overall Dynamic Model for a Piezoelectric Actuator

4. Determination of stiffness of mechanical flexure APA 120-S

The external mechanical flexure of the APA120 was obtained and modified based on the dimensions provided by the manufacturer. The step model was imported into COMSOL to determine the stiffness value of the mechanical flexure of the APA120S. The modelled and meshed APA120S is shown in Figure 3a. The sides of the flexure were applied a force (as the piezostack would have exerted) and the resulting output at the head of the APA120S is recorded as shown in Figure 3b.

The graph of input force vs output displacement is plotted as shown in Figure 4. From the relation between force and displacement, the stiffness value can be determined by calculation of the slope from the graph of input force vs displacement. The stiffness value comes out to be 8.2e5 N/m. This stiffness value is used for the overall dynamic hysteresis modelling of the APA120S.

5. Hysteresis modeling of APA-120s using BOUC-WEN model

APAs are solid-state, long-stroke linear actuators. They are based on the expansion of the active material, which is coupled with an external mechanism for amplification of the displacement. This amplified displacement is proportional to the voltage within a 170V range. When deriving the Bouc-Wen model for the APA120S, the stiffness of the mechanical flexure (ks N-s/m) must be taken into account as it affects the displacement of the

piezoelectric actuator. The mechanical stiffness of the external flexure was determined from COMSOL simulations. The equation now becomes:
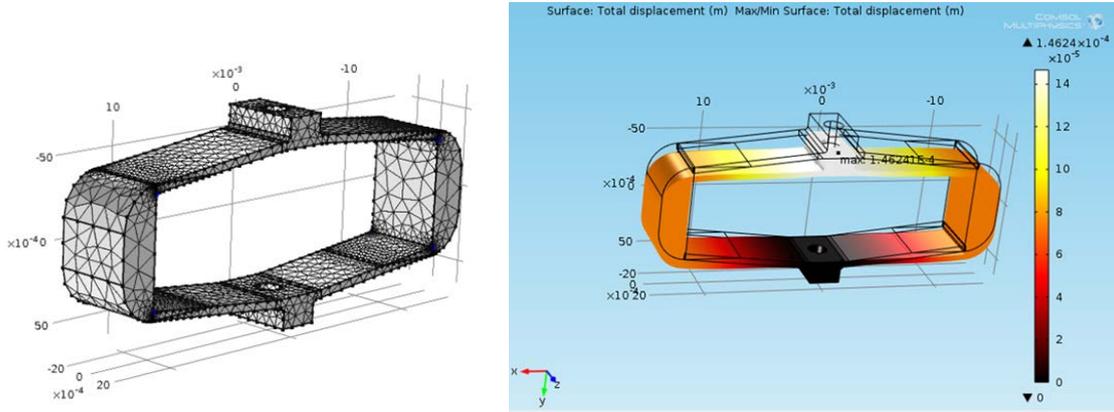
Figure 3: (a) Meshed APA120S Outer Mechanical Flexure **(b)** COMSOL Simulation: Displacement Output for 120 N of Input Force
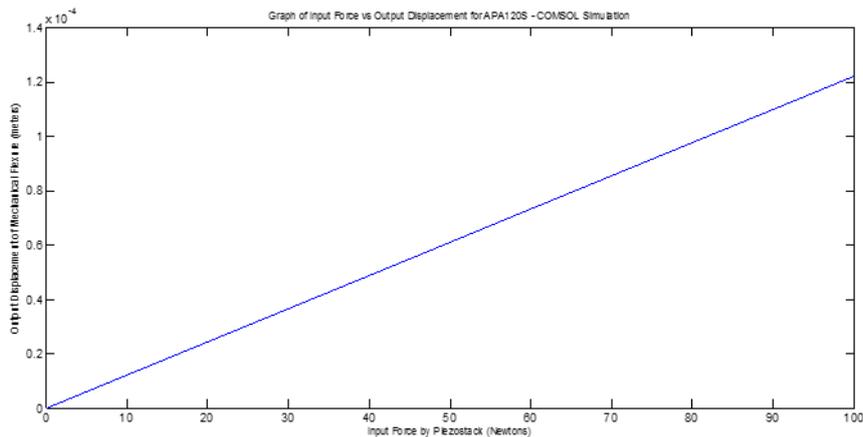
Figure 4: Graph of Input Force vs Output Displacement for APA120S Mechanical Flexure

$$m\ddot{x} + b\dot{x} + kx = k(du - h) - k_s x \qquad\qquad (4)$$

The overall dynamic model for the APA120S actuator is altered to accommodate the stiffness of the mechanical flexure by introduction of the k_s gain through a feedback path. The Simulink block diagrams for the Bouc-Wen model as well as the overall dynamic model are shown in Figure 5. The Parameter Estimation Toolkit in MATLAB is used to determine the three parameters of the Bouc-Wen model for the APA120S. The values obtained from the experimental characterization of the amplified piezostack actuator are used. The voltage values ranging from 0 to 160 V are fed as the input and the displacements obtained from the experiment are fed as the outputs required. Figure 6 details the experimental setup. Experimental setup consists of a APA-120S Piezostack actuator which is driven by a LC-75, LA-75 Amplifier rack by CEDRAT Technologies having a gain of 20. The input signal is given to the amplifier using NI-9264 DAQ by National Instruments. The displacement is measured using a laser pickup OPTO NCDT-1402 by microepsilon having a resolution of 1µm. Using the

simplex method followed by trust-region reflective nonlinear least squares method, the values of the parameter are obtained as shown in Table 1. Figure 7 shows the graph of simulated and measured displacement for the overall dynamic model of APA 120S. Figure 8 shows the simulated results of input voltage and output displacement for the hysteresis model using BOUC WEN.
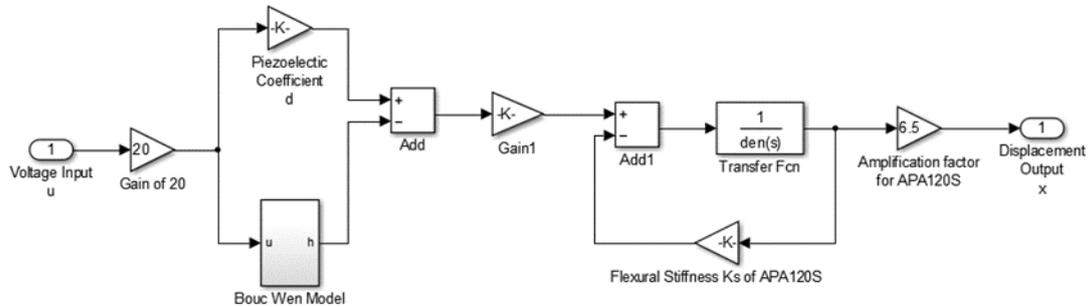
Figure 5: Simulink Block Diagram for Overall Dynamic Model for APA120S
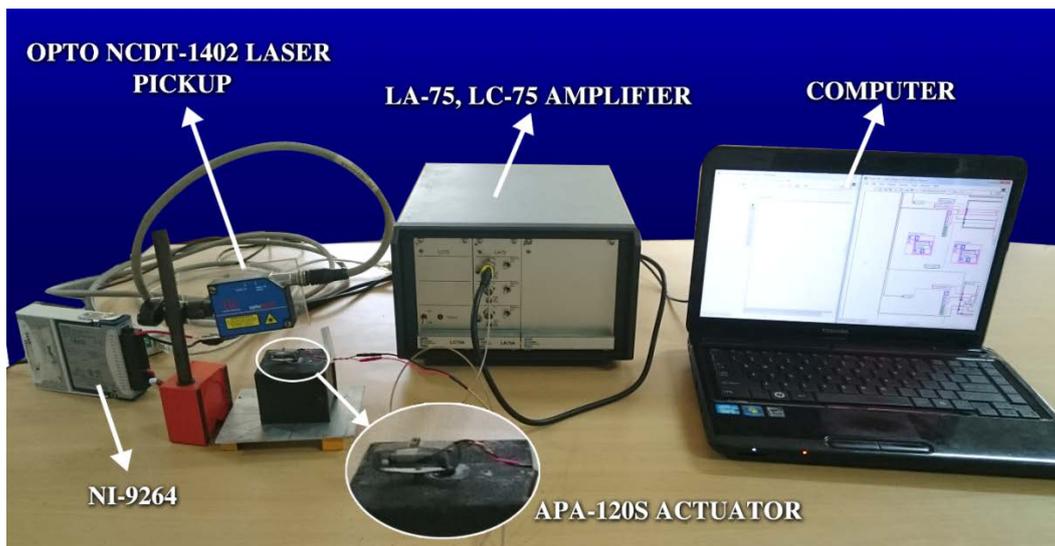
Figure 6: Experimental setup for the investigation of APA 120S

Table 1: APA 120S Parameters (BOUC-WEN Model)

Parameter	Value	Unit
m	0.0038	kg
b	150	Ns/m
k	5E7	N/m
d	1.4447E-7	m/V
α	0.0801	-
β	0.0152	-
γ	-0.0227	-
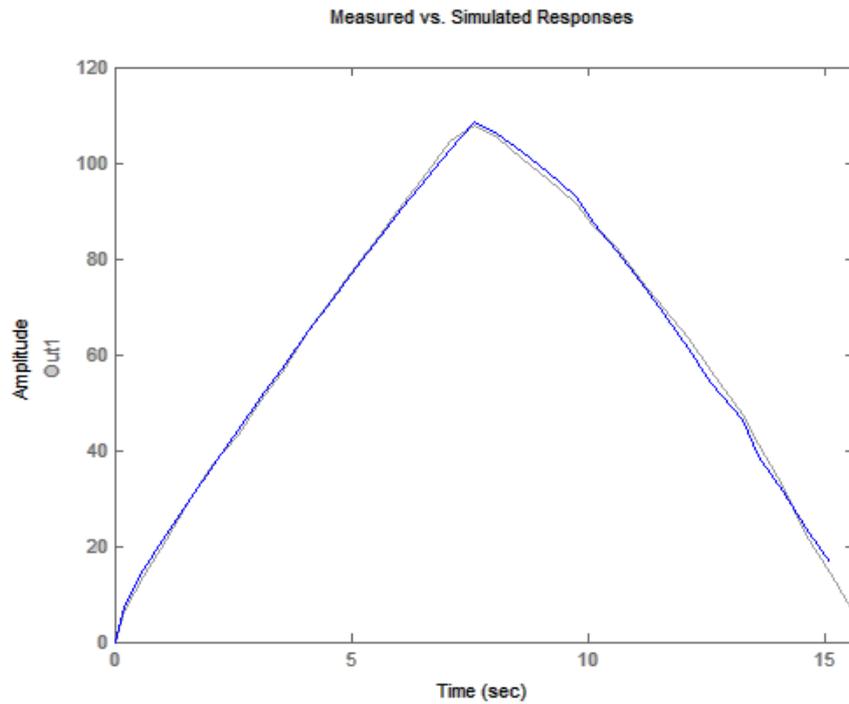
Figure 7: Measured (blue line) vs Simulated Displacement (µm) Response (grey line) for the overall dynamic model for APA120S
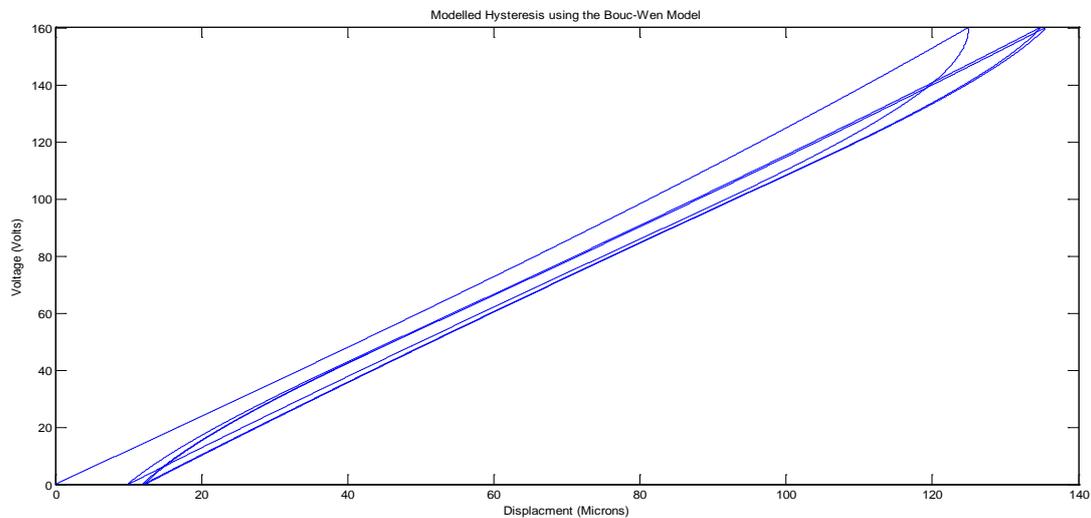
Figure 8: Hysteresis Modelling using Bouc-Wen Model: Graph of Input Voltage vs Output Displacement (Simulated)

6. Inverse BOUC-WEN model

The Bouc-Wen model relates the output displacement with the input excitation (voltage); however, in order to control the position of the actuator, an inverse model has to be created. The advantage Bouc-Wen model holds over its competitors is the lack of computation required for the compensator. The same direct model is used to

model the inverse (compensator).

$$m\ddot{x} + b\dot{x} + kx = k(du - h) - k_s x \qquad (5)$$

Rearranging the above equation, we get:

$$u = \left[\frac{m}{kd}\right]\ddot{x} + \left[\frac{b}{kd}\right]\dot{x} + \left[\frac{1}{d} + \frac{k_s}{kd}\right]x + \left[\frac{h}{d}\right] \qquad (6)$$

The Simulink block diagrams for the Inverse Bouc-Wen model as well as the overall dynamic model are shown in Figure 9. The Parameter Estimation Toolkit in MATLAB is used to determine the three parameters of the
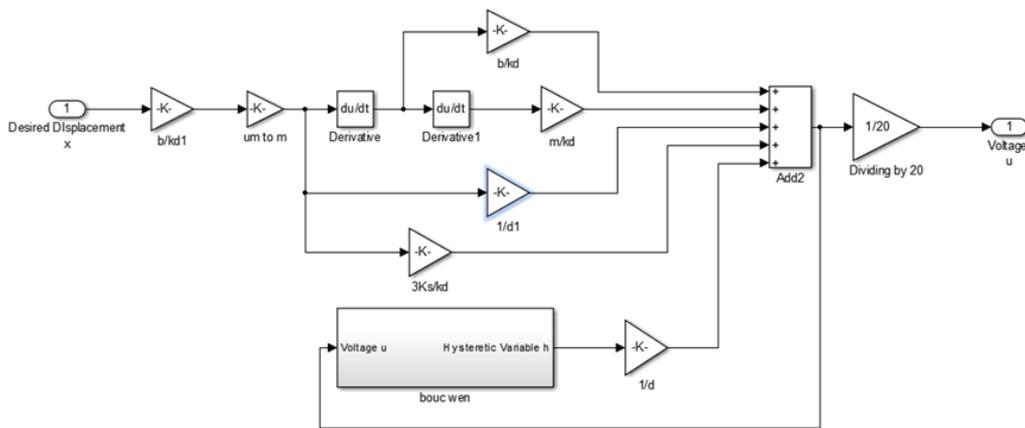
Figure 9: Simulink Block Diagram for Inverse Bouc-Wen Compensator

Bouc-Wen model for the APA120S. The values obtained from the experimental characterization of the amplified piezostack actuator are used. The displacements obtained from the experiment are fed as the inputs and the voltage values ranging from 0 to 160 V are fed as the output. Using the simplex method followed by trust-region reflective nonlinear least squares method, the values of the parameter are obtained as shown in Table 2.

Table 2: APA120S Parameters (Inverse Bouc-Wen Model)

Parameter	Value	Unit
M	0.0038	kg
B	150	Ns/m
K	5E7	N/m
D	1.4447E-7	m/V
A	0.0877	-
B	0.0147	-
Γ	-0.0205	-
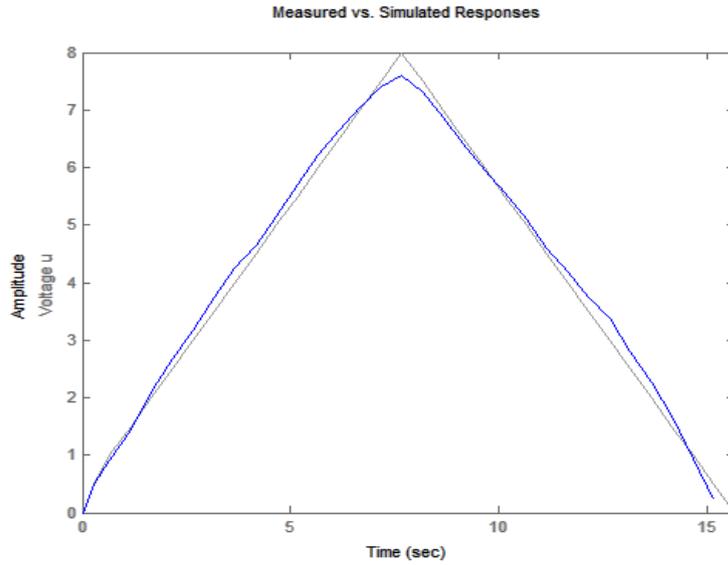
Figure 10: Measured (blue line) vs Simulated Voltage (V/20) Response (grey line) for the compensator model for APA120S

7. Linearization of non linear hysteresis

In perspective of a control design, it is desirable to linearize the non-linear Bouc-Wen model to produce a static system. In literature, there exist three approaches to linearize hysteretic type non-linear systems: 1.) the stochastic linearization, 2.) the Fokker-Planck equation approach, and 3.) perturbation techniques. The equivalent linear equation is written in the form:

$$\dot{h} = k_1 \dot{u} + k_2 h \qquad (7)$$

where k_1 and k_2 are the linearization coefficients. These coefficients can be calculated by minimization of the difference between the original non-linear system and the equivalent linearized system. The equation for the error can be written as:

$$\tilde{e} = \alpha d\dot{u} - \beta|\dot{u}|h - \gamma\dot{u}|h| - (k_1\dot{u} + k_2 h) \qquad (8)$$
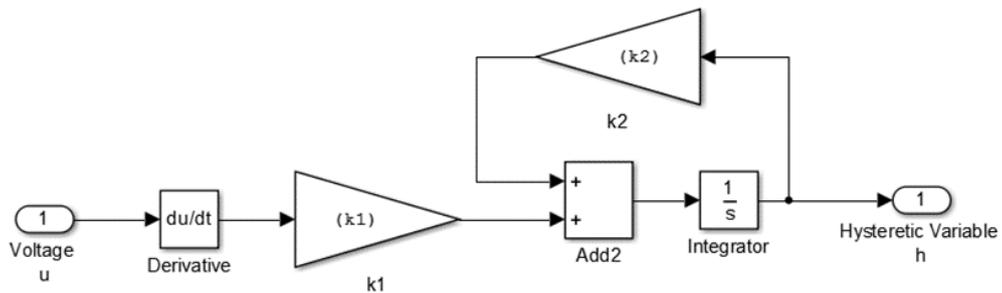
Figure 11: Simulink Block Diagram for Linearized Bouc-Wen Diagram for APA120S

Using Simulink Parameter Estimation Software (SPES), the Control and Estimation Tools Manager is used to estimate the parameters. Figure 11 shows the simulink Block Diagram for Linearized Bouc-Wen Diagram for APA120S. In order to set up the estimation data, the input voltage data is fed to the overall dynamic model of the APA120S (with tuned parameters), and the hysteretic variable data is recorded. The linearized system is modelled in Simulink and the input voltage data as well as the hysteretic data obtained are used as the input and output data. The simplex method is used for estimation of the parameters. The obtained coefficients were for the APA120S are:

$k_1 = 2.3E\text{-}8$

$k_2 = -0.533$

Thus, the following linear equation may be used to govern the hysteresis model for the APA120S:

$$\dot{h} = 2.3x10^{-8}\dot{u} - 0.533h \tag{9}$$
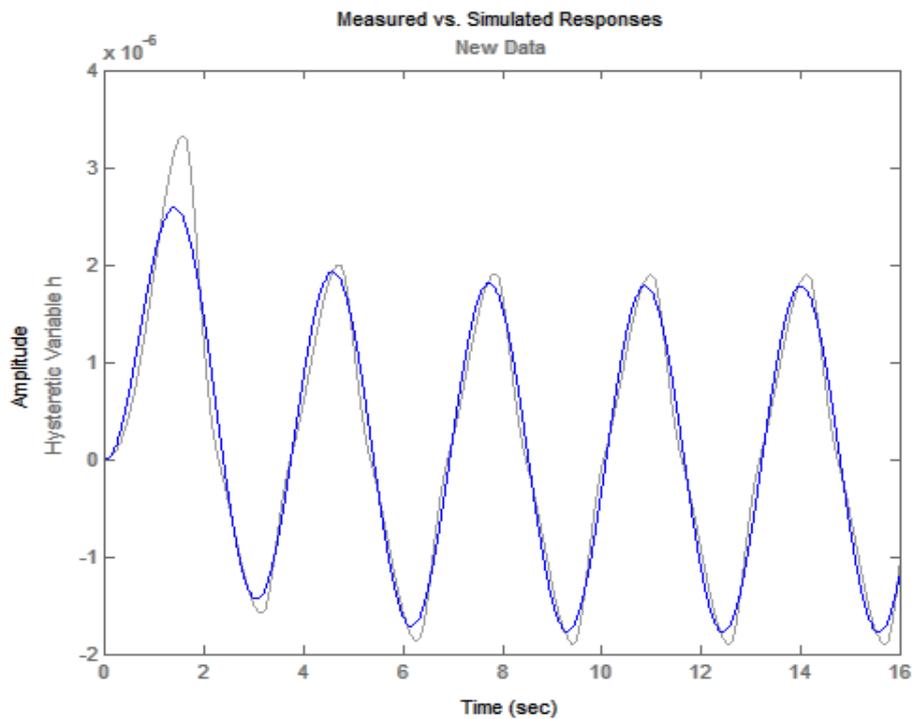
Figure 12: Measured (blue line) vs Simulated Hysteretic Variable Response (grey line) for the linear compensator model for APA120S

As the model is now linearized, it may be represented in the state space form. Let us first define a new state variable (v):

$$v = h - k_1 u \tag{10}$$

Then from Eq. (*10*), we get:

$$\dot{v} = \dot{h} - k_1\dot{u} = k_2h = k_1k_2u + k_2v \qquad\qquad (11)$$
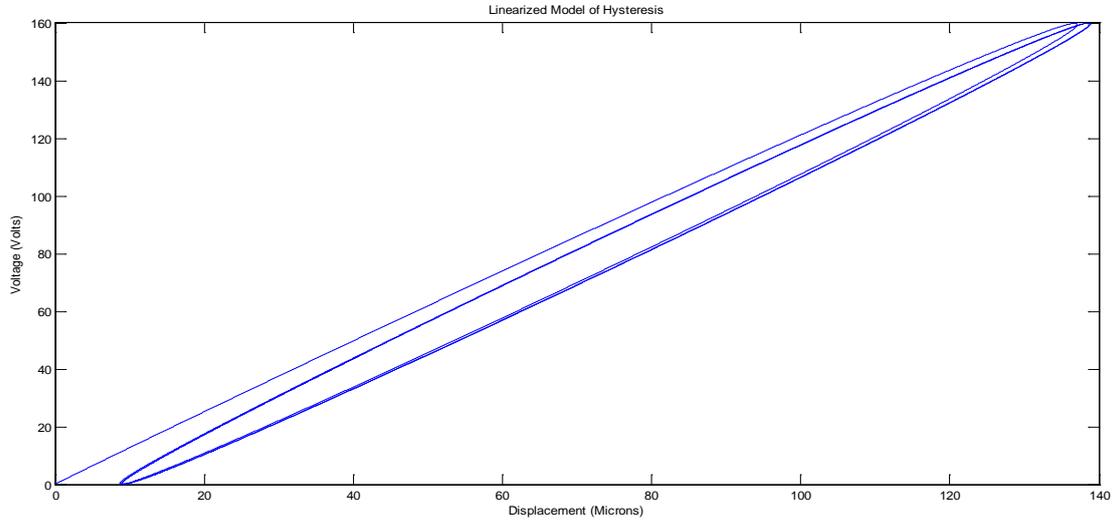
Figure 13: Hysteresis Modelled using Linear Bouc-Wen Model

Substituting Eq. (*11*) into Eq. (*1*), we obtain:

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x + \frac{k}{m}v = \frac{k(d-k_1)}{m}u \qquad\qquad (12)$$

The state vector is $X = [x\ \dot{x}\ v]^T$. Thus, the entire dynamic system (incorporating the hysteresis effect) can be expressed in state space form:

$$\dot{X} = AX + BU \qquad\qquad (12)$$

$$Y = CX \qquad\qquad (13)$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & -\frac{k}{m} \\ 0 & 0 & k_2 \end{bmatrix}, = \begin{bmatrix} 0 \\ \frac{k(d-k_1)}{m} \\ k_1k_2 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \qquad (14)$$

By substituting the values of piezoelectric actuator (APA120S) parameters, and converting the linear state-space model into its transfer function form, we get:

$$\frac{x}{u} = \frac{1.598\times10^3 s + 1.045\times10^3}{s^3 + 3.947\times10^4 s^2 + 1.316\times10^{10} s + 7.276\times10^9} \qquad\qquad (15)$$

The transfer function has a zero at -0.653 and three poles at -0.55288 and $(-19.73 \pm 113.006i) \times 10^3$.
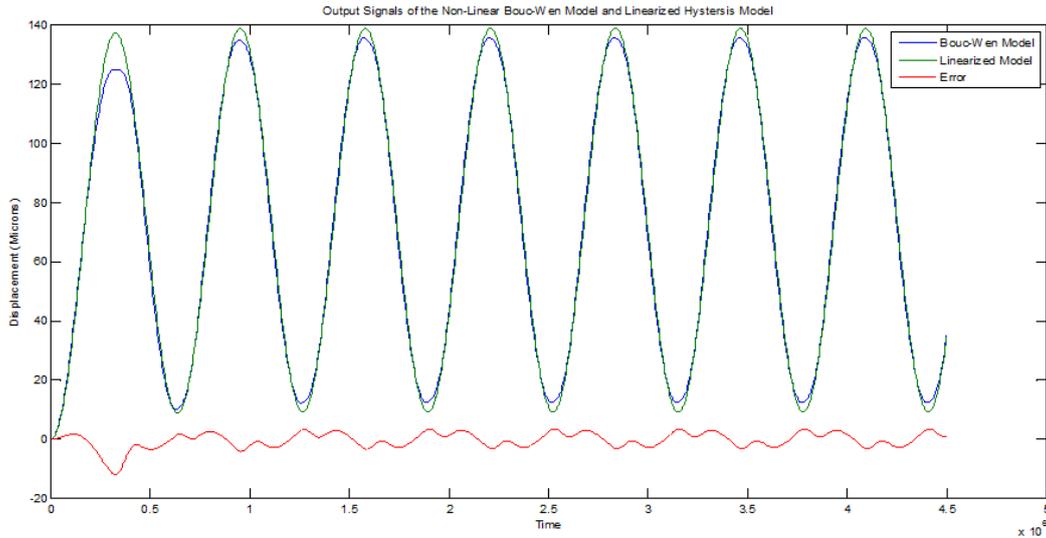
Figure 14: Graph of Displacement Outputs for full Voltage Input to APA120S using non-linear and linearized Bouc-Wen Model

8. Control system using PID

Control design has centered mainly on simple linear, proportional-integral-derivative (PID) controller. A mathematical description of the PID controller is [21],

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{de(t)}{t} \qquad (16)$$

Where u(t) is the input signal to the plant model and e(t) = r(t) – y(t) is the error signal (difference between the reference signal r(t) and the output signal y(t)). K_p, Ki, and K_d are the proportional, integral, and derivative gains, respectively.

The MATLAB SISO Design Tool allows for tuning of controllers to facilitate the controller design process. Applying automated Ziegler-Nichols' open loop controller design method for the linear model in the SISO Design Tool, the initial parameter values for the PID controller are computed. The Simulink block diagram for optimizing the PID gains for minimum tracking error is shown in Figure 15. The subsystem is the overall dynamic model of the amplified piezostack actuator.
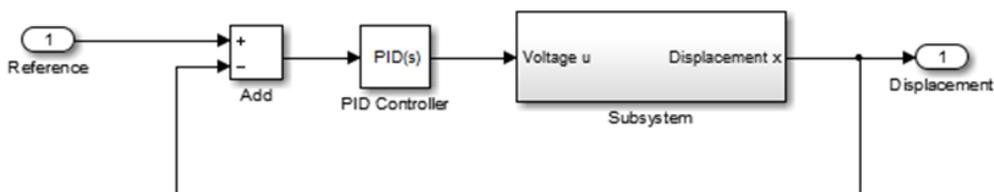
Figure 15: Simulink Model for the PID Controller

277

9. Testing of developed linear and non-linear models for the APA-120S

The Figure 16 shows the simulated voltage for different values of displacement using the linearized Bouc-Wen model. We can see that even in the estimated values there is hysteresis. The voltage hysteresis is such that it overcomes or negates the hysteresis of the piezo-actuator.
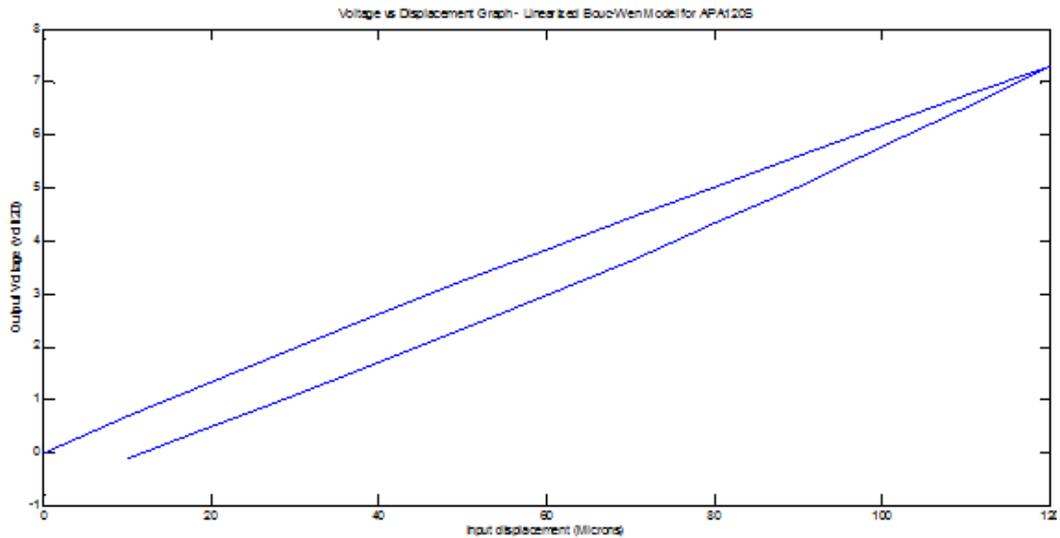
Figure 16: Linearized Bouc-Wen Model: Graph of Resulting Voltage (V/20) for Required Displacement (um)

In Figure 17 we can see the effect of the hysteresis in the input voltage. The output displacement faithfully follows the input or required displacement value with only slight deviations.
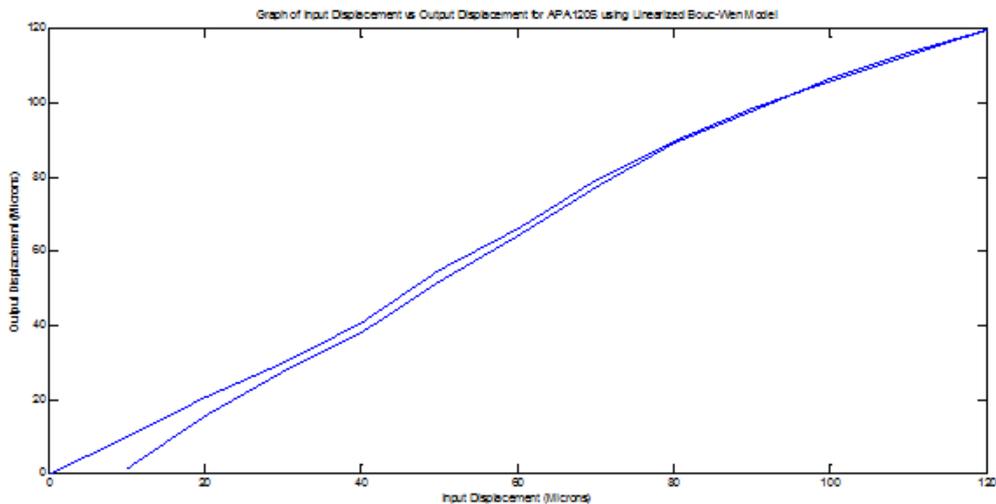
Figure 17: Linearized Bouc-Wen Model: Graph of Resulting Displacement (um) for Required Displacement (um)

The slope of the above graph is one proving that the even the linearized Bouc-Wen model is efficient for designing an open loop controller for the piezo actuator.

10. Conclusion

Through several different experiments it was proved that the Bouc-Wen model is a very efficient model for modelling the hysteresis in the piezo actuator. The open loop controller designed using the Bouc-Wen model performed very well. Although it was difficult to implement the non-linear open loop controller in real time, the simulated values are close to that of the linearized model which estimated the voltage accurately to generate desired output of the piezo actuator. The proposed model took into consideration the stiffness and displacement amplification of even the external flexure of the APA120S. Hence we can confidently claim that the model can be further extended to incorporate the dynamics of the micro gripper with significant changes. The error in the system can be reduced further using the PID controller. The increase in complexity with PID controller can be justified with the estimated error of less than 5%.

Acknowledgements

References

[1] Kui Yao; Uchino, Kenji; Yuan Xu; Shuxiang Dong; Leong Chew Lim "Compact piezoelectric stacked actuators for high power applications", IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, 2000, 47(4), Pages: 819 – 825.

[2] Pluta, J.; Sibielak, M., "The application of a piezoelectric stack for control of small flow intensity hydraulic fluid Carpathian", 13th International Control Conference (ICCC), 2012, 573 – 577.

[3]. S. Zhao, A. Erturk "Deterministic and band-limited stochastic energy harvesting from uniaxial excitation of a multilayer piezoelectric stack", Sensors and Actuators A: Physical, Volume 214, 1 August 2014, Pages 58-65.

[4] G. M'boungui, K. Adendorff, R. Naidoo, A.A. Jimoh, D.E. Okojie, "A hybrid piezoelectric micro-power generator for use in low power applications", Renewable and Sustainable Energy Reviews, Volume 49, September 2015, Pages 1136-1144.

[5] Wang, C.; Gachagan, A.; O'Leary, R.; Mackersie, J. "High inensity focused ultrasound array treansducers using a 2-2 stacked piezoelectric composite appropriate for sonochemistry applications", IEEE International Ultrasonics Symposium (IUS), 2012, Pages: 2497 – 2500.

[6]. M. Goldfarb and N. Celanovic, "Modeling piezoelectric stack actuators for control of micromanipulation," IEEE Contr. Syst. Mag., vol. 17, no. 3, Mar. 1997, pp. 69–79.

[7]. S. Salapaka, A. Sebastian, J. P. Cleveland, and M. V. Salapaka, "High bandwidth nano-positioner: A robust control approach," Rev. Sci. Instr., vol. 73, no. 9, pp. 3232–41, 2002.

[8]. P. Ge and M. Jouaneh, "Tracking control of a piezoceramic actuator," IEEE Trans. Control Syst. Technol., May 1996, vol. 4, no. 3, pp. 209–216.

[9]. M. Goldfard and N. Celanovic, "A lumped parameter electromechanical model for describing the nonlinear behavior of piezoelectric actuators," ASME J. Dyn. Syst., Meas. Contr., vol. 119, pp. 478–485, 1997.

[10]. M.-S. Tsai and J.-S. Chen, "Robust tracking control of a piezoactuator using a new approximate hysteresis model," ASME J. Dyn. Syst., Meas. Contr., vol. 125, pp. 96–102, 2003.

[11]. L. Dupre, R. van Keer, and J. A. A. Melkebeek, "Identification of the relation between the material parameters in the Presach model and in the Jiles-Atherton hysteresis model," J. Appl. Phys., vol. 85, pp. 4376–4378, au- 1999.

[12]. Habineza, D.; Rakotondrabe, M.; Le Gorrec, Y. "Modeling, identification and feedforward control of multivariable hysteresisby combining Bouc-Wen equations and the inverse multiplicative structure", American Control Conference (ACC), 2014, Pages: 4771 - 4777, IEEE Conference Publications.

[13]. Kozlov, D.V., "Approximate analytical solution of the Bouc-Wen hysteresis model by the Fourier transform", International Siberian Conference on Control and Communications (SIBCON), 2011, Pages: 76 – 80, IEEE Conference Publications.

[14]. Qiao Zhi; Gan Minggang; Wang Chenyi, "Sliding Mode Control Using linear extended state observer(LESO) andhysteresis compensator based on Bouc-Wen model in sinusoidal position control of a piezoelectric actuator" 33rd Chinese Control Conference (CCC), 2014 , Pages: 3840 – 3845.

 [15]. Zhi Liu; Guanyu Lai; Yun Zhang; Chen, C.L.P. "Adaptive Neural Output Feedback Control of Output-Constrained Nonlinear Systems With Unknown Output Nonlinearity", IEEE Transactions on Neural Networks and Learning Systems, 2015, Vol. 26(8), Pages: 1789 – 1802, IEEE Journals & Magazines.

[16].   Habineza, D.; Rakotondrabe, M.; Le Gorrec, Y. "Bouc–Wen Modeling and Feedforward Control of Multivariable Hysteresis in Piezoelectric Systems: Application to a 3-DoF Piezotube Scanner", IEEE Transactions on Control Systems Technology, 2015, Volume: 23, Issue: 5 Pages: 1797 – 1806.

[17]. Laudani, A.; Fulginei, F.R.; Salvini, A. "Bouc–Wen Hysteresis Model Identification by the Metric-Topological Evolutionary Optimization" IEEE Transactions on Magnetics, Year: 2014, Volume: 50, Issue: 2, IEEE Journals & Magazines.

[18]. Zhi Liu; Guanyu Lai; Yun Zhang; Xin Chen; Chen, C.L.P., "Adaptive Neural Control for a Class of Nonlinear Time-Varying Delay Systems With Unknown Hysteresis", IEEE Transactions on Neural Networks and Learning Systems, Year: 2014, Volume: 25, Issue: 12 Pages: 2129 – 2140, IEEE Journals & Magazines.

[19]. Rakotondrabe, M., "Bouc–Wen Modeling and Inverse multiplicative structure to Compensate Hysteresis Nonlinearity in Piezoelectric Actuators, IEEE Transactions on Automation Science and Engineering, Year: 2011, Volume: 8, Issue: 2, Pages: 428 – 431.

[20] Wei Zhu, Dai-hua Wang "Non-symmetrical Bouc–Wen model for piezoelectric ceramic actuators" Sensors and Actuators A: Physical, Volume 181, July 2012, Pages 51–60.

[21] Jayawardhana, Bayu, Hartmut Logemann, and Eugene P. Ryan. "PID control of second-order systems with hysteresis." International Journal of Control 81.8 (2008): 1331-1342.