# Determination of Springback Values in Bending I-sections with Tresca Criteria 

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#### Abstract

The springback ratio for bending I-section beams is introduced analytically in the present study. A complete analysis using the strength of materials approach is carried out with Tresca yielding criteria. Analytical methods are given in the form of equations for two cases according to the positions of the yield point along the height of the beam. The results represent the effect of different parameters affecting the springback ratio.


Keywords: Bending sections; Springback; I sections; Tresca.

## 1. Introduction

Bending of different sections shape is one of the most widely applied metal forming operations. In the fabrication of metals, the elastic recoveries after unload causing the springback phenomenon in which the radius of curvature in bending increases after the bending moment is removed. Therefore, the precise prediction of springback is a key to assessing the accuracy of part geometry. Many analyses have been carried done to calculate and investigate the springback phenomenon in bending different sections. Moon and his colleagues [1] have investigated the effect of tool temperature on the reduction of springback amount of aluminum 1050 sheet. Emphases are placed on the reduction of springback amount in terms of combination of die and punch temperature. It was found that the combination of hot die and cold punch can reduce the amount of springback up to $20 \%$ when compared to conventional room temperature bending test. While T. Liu and his colleagues [2] have studied the springback behaviors of the age-hardened 2196-T8511 and 2099-T83 Al-Li alloys extrusions with Z-section and T-section under displacement controlled cold stretch bending (DCSB).

[^0]Analytical model as well as FEM were used. It was found that increasing pre-stretching or post-stretching strain can significantly reduce the radius springback and time-dependent springback. Moreover [3] have introduced a study to springback in sheet metal forming and discussed the parameters affecting the springback phenomenon such as punch angle, grain direction of sheet metal material, die opening, ratio of die radius to sheet thickness, sheet thickness, punch radius, punch height, coining force, pre bend condition of strip etc. reference [4] have evaluated the elastic springback and the residual stress after stretch bending of a rectangular section of 7075 aluminum sheet. Also [5] have introduced a theoretical analysis of the elastic-plastic bending of tubes and sections with different shapes. Analytical methods are given in the form of equations to provide a quantitative method for predicting the moment for forming the section of the tube to a specific radius of curvature. Reference [6] has studied the mechanism of the springback-free phenomenon observed at warm forming temperatures higher than 750 K . It was found that the abrupt decrease in springback at approximately 750 K is mainly caused by a momentary increase of high-temperature creep strain just after the forming process. Z. Damián-Noriega and his colleagues [7] have presented a new equation to predict the springback in the bending and rolling processes of metallic sheet. The resulted equation has been experimentally applied in the design of a wood truncated cone to form an aluminum sheet truncated cone. Wenjuan Liu and his colleagues [8] have introduced a process optimal control scheme of sheet metal forming springback based on evolutionary strategy. The experiment results indicated that the best process scheme can be achieved with the process optimal springback control method based on evolutionary strategy; the springback can be controlled and reduced effectively. Feng Ruan and his colleagues [9] have tried to predict and control the springback for sheet metal forming. In this paper, back propagation (BP) neural network and genetic algorithm (GA) was introduced to predict springback of complex sheet metal forming parts. The model can be used to predicate springback and provides a theoretical guide for complex sheet metal parts forming, tools designing and die modification. ZHAO Jun and his colleagues [10] have addressed an analytical method for springback of small curvature plane bending with unloading rule of classical elastic- plastic theory and principle of strain superposition. The springback equation of plane bending is considered to analyze the expanding and setting round process, and the results agree with the experimental data. Sanjay Kumar Patel and his colleagues [11] were interested in the springback analysis in sheet metal forming using modified Ludwik stress-strain relation. Using the deformation theory of plasticity, formulation of the problem and springback ratio is derived using modified Ludwik stress strain relationship with Tresca and von Mises yielding criteria. The mechanical behavior for beams has an important value for studying because of their widely used in engineering components. Moreover the use of I- beam sections in forming structural components for cars, trains, aircraft, boats and ships has grown exponentially. Thus, the aim of this paper is to present a theoretical analysis for springbackin I-section beams under bending. In addition, studying the affecting parameters is also one of the targets of this paper.

## 2. Analysis

In the present work an analysis was made for I-beams under bending to evaluate the springback values. The stress- strain relationship is given by:
$\sigma=\left[\begin{array}{ll}E \varepsilon & \varepsilon \leq \frac{Y}{E} \\ K \varepsilon^{n} & \varepsilon \geq \frac{Y}{E}\end{array}\right]$

Where $\sigma$ is the bending stress, $\varepsilon$ is the bending strain, Y is the yield strength, E is the modulus of elasticity, K is the strength coefficient and n is the strain hardening exponent.

Figure 1 shows the schematic plot of applied bending moment versus curvature during the formation of a wide beam of metal around a portion of a cylindrical die. At point A, the material yields and plastic deformation continuous until the inside surface of the material conforms to die at point B when the applied moment is released elastic springback occurs from B to C . The change in curvature due to this elastic springback is $\left(1 / R_{0}\right)-$ $\left(1 / R_{f}\right)$, which is from figure 1 ,

$$
\frac{1}{R_{o}}-\frac{1}{R_{f}}=\frac{M_{\max }}{\partial M_{E} / \partial(1 / R)}
$$



Figure 1: Schematic representation of curvature versus applied moment during bending

The figure 2 shows the I-section considered in this study.

The following assumptions are considered to simplify the analysis of the bending process:
(1) The stress-strain characteristics of material are the same in tension and compression.
(2) The friction effect at the interface between the beam and the die is neglected.
(3) The cross section dimensions of the beam are such that the width to height ratio is large.
(4)The natural surface is always in the centre of the beam, and plane sections remain plane during bending.
(5) The cross section dimensions of the beam do not change significantly in bending.
(6) The radius of bending is large compared to the height of the beam so radial stresses are assumed to be negligible.
(7) The circumferential strains are sufficiently small so that the conventional strain and the true strain are approximately equivalent.
(8)The transverse strain is zero at any point in the beam.
(9) The circumferential strain for any fiber does not vary along the bent section.

The relationship between the principal stresses and strain for the elastic according to Hooke's law ,


Figure 2: The I-section
$\varepsilon_{x}=\frac{1}{E}\left(\sigma_{x}-\mathcal{V}\left(\sigma_{y}+\sigma_{z}\right)\right), \varepsilon_{y}=\frac{1}{E}\left(\sigma_{y}-\mathcal{V}\left(\sigma_{x}+\sigma_{z}\right)\right), \varepsilon_{z}=\frac{1}{E}\left(\sigma_{z}-\mathcal{V}\left(\sigma_{y}+\sigma_{x}\right)\right)$,

According to the assumptions that the bending radius is large compared to the height of the beam so radial stresses is zero at any point along the beam,
$\sigma_{y}=\varepsilon_{z}=\delta_{z}=0$

As well as the circumferential strain for any fiber does not vary along the bent section; then
$\varepsilon_{x}=\delta_{x}$

Substituting the values of Eq. (4) in Eq.(3); yields to
$\sigma_{z}=\mathcal{V} \sigma_{x}$

From Tresca yield criteria (the maximum shear stress theory)
$\mathrm{Y}=\sigma_{x}-\sigma_{z}$
or by using Eq. (6)
$\mathrm{Y}=\sigma_{x}(1-\mathcal{V})$

At yield point the stress in elastic and plastic region is equal, so
$\mathrm{E} \varepsilon_{o}=K \varepsilon_{\mathrm{o}}^{\mathrm{n}}$

Which yields to
$\varepsilon_{o}=\left(\frac{K}{E}\right)^{\frac{1}{1-n}}$

Therefore the stress at the yield point is
$\sigma_{o}=K \varepsilon_{o}^{n}=K\left(\frac{K}{E}\right)^{\frac{n}{1-n}}$

Substation with Eq. (8) in Eq. (11)
$\sigma_{o}=\sigma_{x}(1-\mathcal{V})=K\left(\frac{K}{E}\right)^{\frac{n}{1-n}}$
$\sigma_{x}=\frac{K\left(\frac{K}{E}\right)^{\frac{n}{(1-n)}}}{(1-\mathcal{V})}$

Thus,
$\varepsilon_{x}=\left(\frac{1}{E}\right) \sigma_{x}\left(1-\mathcal{V}^{2}\right)$

In addition the circumferential strain at the yield point is
$\varepsilon_{o x}=\left(\frac{1}{E}\right) \sigma_{o}\left(1-V^{2}\right)$

Substitute with Eq.(11), get
$\varepsilon_{o x}=\left(\frac{K}{E}\right)^{1 /(1-n)}(1+\mathcal{V})$

The previous value is the circumferential strain at elastic-plastic interface. Therefore the identical stress is
$\sigma_{x}=\frac{E}{\left(1-\mathcal{V}^{2}\right)} \varepsilon_{x}$

Where $_{x}=\frac{y}{R_{o}}$, then
$\sigma_{x}=\frac{E}{\left(1-\mathcal{V}^{2}\right)}\left(\frac{y}{R_{o}}\right)$

From the deformation theory of plasticity
$\delta_{x}=\frac{1}{K^{\frac{1}{n}}}\left(\sigma_{x}{ }^{2}+\sigma_{y}{ }^{2}+\sigma_{z}^{2}-\sigma_{x} \sigma_{y}-\sigma_{y} \sigma_{z}-\sigma_{z} \sigma_{x}\right)^{(1-n) / 2 n} *\left(\sigma_{x}-\frac{\sigma_{y}}{2}-\frac{\sigma_{z}}{2}\right)$
$\delta_{y}=\frac{1}{K^{\frac{1}{n}}}\left(\sigma_{x}{ }^{2}+{\sigma_{y}}^{2}+\sigma_{z}{ }^{2}-\sigma_{x} \sigma_{y}-\sigma_{y} \sigma_{z}-\sigma_{z} \sigma_{x}\right)^{(1-n) / 2 n} *\left(\sigma_{y}-\frac{\sigma_{x}}{2}-\frac{\sigma_{z}}{2}\right)$
$\delta_{z}=\frac{1}{K^{\frac{1}{n}}}\left(\sigma_{x}{ }^{2}+{\sigma_{y}}^{2}+\sigma_{z}^{2}-\sigma_{x} \sigma_{y}-\sigma_{y} \sigma_{z}-\sigma_{z} \sigma_{x}\right)^{(1-n) / 2 n} *\left(\sigma_{z}-\frac{\sigma_{x}}{2}-\frac{\sigma_{y}}{2}\right)$

Substituting with Eqs. (4) and (5) in Eqs. (21)yields to;
$\sigma_{z}=\frac{\sigma_{x}}{2}$

Replacing the previous value of $\sigma_{z}$ in Eq. (19), yields to the following
$\sigma_{x}=\frac{K}{\left(\frac{3}{4}\right)^{(1+n) / 2}} \delta_{x}{ }^{n}$

Since the initial assumption state that the circumferential strain is sufficiently small so that the axial strain and true strain are approximately the same,
$\varepsilon_{x}=\frac{y}{R_{o}}=\delta_{x}$
$\sigma_{x}=\frac{K}{\left(\frac{3}{4}\right)^{\frac{1+n}{2}}}\left(\frac{y}{R_{o}}\right)^{n}$

Then, the maximum bending moment for I-section could be introduced as
$M_{\max }=2 \int_{0}^{\frac{H}{2}} \sigma_{x} d A$

The present analysis went through two cases according to the position of the yield point ( $\mathrm{h}^{*}$ ) which is the distance from the neutral surface up to the layer at which yielding occurs. For that reason, The stress and strain distributions across the I-section beam loaded by bending moment M will be derived for the following two cases
according to the position of yielding along the beam height:
case (1): when yielding occur in the flange height
case (2): when yielding occur in the web height
2.1. case: $1 \quad \frac{h}{2} \leq h^{*} \leq \frac{H}{2}$

The first case is when yielding occurs in the flange i.e. $\frac{h}{2} \leq h^{*} \leq \frac{H}{2}$, thus, for this case, Eq.(26) could rewritten as follows:
$M_{\max }=2\left[\int_{0}^{\frac{h}{2}} \sigma_{x_{\text {elastic }}}(B-b) y d y+\int_{\frac{h}{2}}^{h^{*}} \sigma_{x_{\text {elastic }}} B y d y+\int_{h^{*}}^{\frac{H}{2}} \sigma_{x_{\text {plastic }}} B y d y\right]$

Replacing the value of $\sigma_{x}$ in the previous equation with the equivalent values given in Eqs. (18) and (25)
$M_{\max }=2\left[\int_{0}^{\frac{h}{2}} \frac{E}{\left(1-\mathcal{V}^{2}\right)} * \frac{y^{2}}{R_{o}}(B-b) d y+\int_{\frac{h}{2}}^{h^{*}} \frac{E}{\left(1-\mathcal{V}^{2}\right)} * \frac{y^{2}}{R_{o}} B d y+\int_{h^{*}}^{\frac{H}{2}} \frac{K}{\left(\frac{3}{4}\right)^{\frac{1+n}{2}}} \frac{y}{\mathrm{R}_{\mathrm{o}}^{\mathrm{n}}}{ }^{n+1} B d y\right]$

Where,
$h^{*}=R_{o}\left(\frac{K}{E}\right)^{\frac{1}{1-n}}(1+\mathcal{V})$

Thus,
$M_{\max }=2\left[\frac{E B R_{o}^{2}}{3\left(1-\mathcal{V}^{2}\right)}\left(\frac{K}{E}\right)^{\frac{3}{1-n}}(1+\mathcal{V})^{3}-\frac{E b}{3\left(1-\mathcal{V}^{2}\right) R_{o}}\left(\frac{h}{2}\right)^{3}+\frac{K}{(3 / 4)^{(1+n) / 2}(n+2)}\left(\frac{1}{R_{o}}\right)\left(\frac{H}{2}\right)^{n+2}-R_{o}^{2}\left(\frac{K}{E}\right)^{\frac{n+2}{1-n}}(1+\mathcal{V})^{n+2}\right]$

While the elastic bending moment could be expressed as;
$M_{E}=\frac{\sigma_{x} \cdot I_{N . A}}{y}$

Where $\mathrm{I}_{\mathrm{N} . \mathrm{A}}$ is the moment of inertia for the I-section about the neutral axis, which is equal to
$I_{N . A}=\frac{B H^{3}}{12}-\frac{b h^{3}}{12}$

Therefore Eq. (3) could be rewritten as
$M_{E}=\frac{E}{\left(1-\mathcal{V}^{2}\right) \cdot R_{o}}\left(\frac{1}{12}\right)\left(B H^{3}-b h^{3}\right)$

Then,
$\frac{\partial M_{E}}{\partial(1 / R)}=\frac{E}{\left(1-\mathcal{V}^{2}\right)}\left(\frac{1}{12}\right)\left(B H^{3}-b h^{3}\right)$

Referring to Eq. 2 which could be rewritten as
$\frac{R_{o}}{R_{f}}=1-\frac{M_{\max }}{\frac{\partial M_{E}}{\partial(1 / R)}} * R_{o}$

Replacing in the previous equation the value of bending moment in both Eqs. (30)and Eq. (34), gives
$\frac{R_{o}}{R_{f}}=1-\frac{1}{1-\beta \alpha^{3}}\left[\frac{3\left(1-v^{2}\right)}{\left(\frac{3}{4}\right)^{\frac{1+n}{2}}(n+2)}\left(\frac{\sigma_{o}}{E}\right)^{1-n}\left(\frac{2 R_{o}}{H}\right)^{1-n}-\beta \alpha^{3}+\left(\frac{2 R_{o}}{H}\right)^{3}\left(\frac{\sigma_{o}}{E}\right)^{3}(1+v)^{3}\left\{1-\frac{3(1-v)(1+v)^{n}}{\left(\frac{3}{4}\right)^{\frac{1+n}{2}}(n+2)}\right\}\right]$

Where,
$\alpha=\frac{h}{H}, \quad \beta=\frac{b}{B}$

Which is nothing but the equation for springback ratio derived for the first case

## 2.2. case: $2 \quad 0 \leq h^{*} \leq \frac{h}{2}$

The second case is when yielding occurs in the web i.e. when $0 \leq h^{*} \leq \frac{h}{2}$, thus Eq.(26) could be rewritten as follows for this case
$M_{\max }=2\left[\int_{0}^{h^{*}} \sigma_{x_{\text {elastic }}}(B-b) y d y+\int_{h^{*}}^{\frac{h}{2}} \sigma_{x_{\text {plastic }}}(B-b) y d y+\int_{\frac{h}{2}}^{\frac{H}{2}} \sigma_{x_{\text {plastic }}} B y d y\right]$

Replacing the value of $\sigma_{x}$ in the previous equation with the equivalent values given in Eqs. (18) and (25)
$M_{\max }=2\left[\int_{0}^{h^{*}} \frac{E}{\left(1-\mathcal{V}^{2}\right)} * \frac{y^{2}}{R_{o}}(B-b) d y+\int_{h^{*}}^{\frac{h}{2}} \frac{K}{\left(\frac{3}{4}\right)^{\frac{1+n}{2}}} \frac{y^{n+1}}{\mathrm{R}_{\mathrm{o}}^{\mathrm{n}}}(B-b) d y+\int_{\frac{h}{2}}^{\frac{H}{2}} \frac{K}{\left(\frac{3}{4}\right)^{\frac{1+n}{2}}} \frac{y^{n+1}}{\mathrm{R}_{\mathrm{o}}^{\mathrm{n}}} \quad B d y\right]$

Therefore the springback ratio could be introduced in the following form
$\frac{R_{o}}{R_{f}}=1-\frac{1}{1-\beta \alpha^{3}}\left[\frac{3\left(1-v^{2}\right)}{\left(\frac{3}{4}\right)^{\frac{1+n}{2}}(n+2)}\left(\frac{\sigma_{o}}{E}\right)^{1-n}\left(\frac{2 R_{o}}{H}\right)^{1-n}\left(1-\beta \alpha^{n+2}\right)+\left(\frac{2 R_{o}}{H}\right)^{3}\left(\frac{\sigma_{o}}{E}\right)^{3}(1+v)^{3}(1-\beta)\left\{1-\frac{3(1-v)(1+v)^{n}}{\left(\frac{3}{4}\right)^{\frac{1+n}{2}}(n+2)}\right\}\right]$

This is the equation for springback ratio derived for the second case

## 3. Results and discussion

For bending I-sections the springback ratio is calculated for the previous two cases in Eqs. (36) and (40). It is noticed that both equations have a final term that is very negligible because the $\mathrm{Y} / \mathrm{E}=5.5 \times 10^{-4}$. Therefore, the last term of both equations could be neglected. Thus the springback ratio will be introduced, for the first and second case respectively; as the following equations:
$\frac{R_{o}}{R_{f}}=1-\frac{1}{1-\beta \alpha^{3}}\left[\frac{3\left(1-v^{2}\right)}{\left(\frac{3}{4}\right)^{\frac{1+n}{2}}(n+2)}\left(\frac{\sigma_{o}}{E}\right)^{1-n}\left(\frac{2 R_{o}}{H}\right)^{1-n}-\beta \alpha^{3}\right]$
$\frac{R_{o}}{R_{f}}=1-\frac{1}{1-\beta \alpha^{3}}\left[\frac{3\left(1-v^{2}\right)}{\left(\frac{3}{4}\right)^{\frac{1+n}{2}}(n+2)}\left(\frac{\sigma_{o}}{E}\right)^{1-n}\left(\frac{2 R_{o}}{H}\right)^{1-n}\left(1-\beta \alpha^{n+2}\right)\right]$

This equations (Eqs. (41) and (42)) are depending on the ratio $\mathrm{Ro} / \mathrm{H}, \mathrm{Y} / \mathrm{E}$, the strain exponent (n), the geometrical coefficients $\alpha, \beta$ and the Poisson's ratio (v). Figures 3-5 shows the variation of springback ratio with $(\mathrm{Ro} / \mathrm{H})$ and different values of $(\mathrm{n})$ at constant value of $\alpha=0.9$ and $\beta=0.8$, with different values of $(\mathrm{Y} / \mathrm{E})$ : $5.5 \times 10^{-}$ ${ }^{4}, 1.522 \times 10^{-3}$ and $2.4 \times 10^{-3}$ (for different material c10100 copper, 1100al, and 1065steel) [11].


Figure 3:Thespringback ratio with $R_{o} / H$ With different values of n at $\mathrm{Y} / \mathrm{E}=5.5^{*} 10^{-4}, \mathcal{V}=0.33$

The preceding figures showed that: decreasing of the springback ratio $\left(\mathrm{R}_{0} / \mathrm{R}_{\mathrm{f}}\right)$ occurs with increasing values of (n) (strain hardening exponent) as well as increasing of the values of (Y/E). Figures 6-9 show the variation of springback ratio with $\left(\mathrm{R}_{0} / \mathrm{H}\right)$ and different values of $(\mathrm{Y} / \mathrm{E})$ at constant value of $\alpha=0.9$ and $\beta=0.8$ with different values of $n$ (strain hardening exponent).It addition, It was noticed that the springback ratio ( $\mathrm{R}_{0} / \mathrm{R}_{\mathrm{f}}$ ) decreases with the increasing of the value of $(\mathrm{Y} / \mathrm{E})$ as well as the value of ( n ) (strain hardening exponent). Whereas figures $10-12$ show the variation of the springback ratio $\left(R_{0} / R_{f}\right)$ at constant value of $\alpha=0.9$ and $\beta=0.8$ with different
values of Poisson's ratio $(v)$ and at fixed value of (Y/E) $\left(5.5 \times 10^{-4}, 1.522 \times 10^{-3}\right.$ and $\left.2.4 \times 10^{-3}\right)$. It was confirmed that the springback ratio $\left(\mathrm{R}_{0} / \mathrm{R}_{\mathrm{f}}\right)$ increases with the increasing of the value of (v). Figures 12-15 show the variation of the springback ratio $\left(\mathrm{R}_{0} / \mathrm{R}_{\mathrm{f}}\right)$ with different values of $(\mathrm{H})$ (the height of the beam) as well as different values of $n=0.1,0.2,0.3$ and 0.4 and $R_{0}=40, \beta=0.8, \nu=0$.33.It could be noticed that the rate of the springback ratio is increasing rapidly from range 1 to 2 , then it becomes gradually less after wards.


Figure 4: The springback ratio with $R_{o} / H$ With different values of n at $\mathrm{Y} / \mathrm{E}=1.5^{*} 10^{-3}, \mathcal{V}=0.33$


Figure 5: The springback ratio with $R_{o} / H$ With different values of n at $\mathrm{Y} / \mathrm{E}=2.4^{*} 10^{-3}, \mathcal{V}=0.33$


Figure 6: The springback ratio with $R_{o} / H$ With different values of $\mathrm{Y} / \mathrm{E}$ at $\mathrm{n}=0.1, \mathcal{V}=0.33$


Figure 7: The springback ratio with $R_{o} / H$ With different values of $\mathrm{Y} / \mathrm{E}$ at $\mathrm{n}=0.2, \mathcal{V}=0.33$


Figure 8: The springback ratio with $R_{o} / H$ With different values of $\mathrm{Y} / \mathrm{E}$ at $\mathrm{n}=0.3, \mathcal{V}=0.33$


Figure 9: The springback ratio with $R_{o} / H$ With different values of $\mathrm{Y} / \mathrm{E}$ at $\mathrm{n}=0.4, \mathcal{V}=0.33$


Figure 10: The springback ratio with $R_{o} / H \mathrm{~W}$ ith different values of $v$ at $\mathrm{Y} / \mathrm{E}=5.5^{*} 10^{-4}$


Figure 11: The springback ratio with $R_{o} / H$ With different values of $v$ at $\mathrm{Y} / \mathrm{E}=1.5^{*} 10^{-3}$


Figure 12: The springback ratio with $R_{o} / H$ With different values of $v$ at $\mathrm{Y} / \mathrm{E}=2.4^{*} 10^{-3}$


Figure 13: The springback ratio with $R_{o} / H \mathrm{~W}$ ith different values of n at $\mathrm{Y} / \mathrm{E}=5.5 * 10^{-4}$


Figure 14: The springback ratio with $R_{o} / H \mathrm{With}$ different values of n at $\mathrm{Y} / \mathrm{E}=1.5^{*} 10^{-3}$


Figure 15: The springback ratio with $R_{o} / H \mathrm{~W}$ ith different values of n at $\mathrm{Y} / \mathrm{E}=2.4 * 10^{-3}$

## 4. Conclusions

The springback equations were derived for I-section beams taking into consideration the Tresca criteria. The Springback ratio for Ro/H less than 20, is assumed to be slightly insignificant but for high values of Ro/tor more than 20 , the differences in springback ratio is significant. Moreover, the following conclusions about the springback analysis in I-section beams using nonlinear constitutive equation are discovered:
(i) The theoretical analysis for the I-sections under pure bending has been done, and it was found that the prediction of springback was quite successful.
(ii) Springback ratio increases with decreasing the ratio of the yield point stress to Young's modulus of elasticity
(iii) Springback ratio increases with increasing Possion's ratio.
(iv) Springback ratio increases with increasing beam height
(v) The elastic recovery is found to be more effective with decreasing values of work hardening coefficient. While with lower values of $n$, the material will approach an elasto-ideal plastic behavior.

It is also recommended to investigate the springback behavior in bending I-sections with another criteria such as von Mises criteria.

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