

Application of Financial Time Series Techniques in Analysing the Volatility of Metical/dollar and Metical/rand Exchange Rates in Mozambique (2010-2020)

Fernando João Nhampossa^{a*}, Oclidio Francisco Tete^b, Américo José Fombe^c

^{a,b}*Universidade Complutense de Madrid, Madrid, Spain*

^c*Universidade Pedagógica de Maputo, Maputo, Mozambique*

^a*Email: f.nhampossa@gmail.com*

^b*Email: oclidesfrancisco@gmail.com*

^c*Email: americofombe@yahoo.com*

Abstract

Exchange rates play an important role in the economic and financial outlook of any country, making it interesting to evaluate and predict their fluctuations. Based on the combination of ARMA (Autoregressive Moving Average) models with ARCH (Autoregressive Conditional Heteroscedasticity) class models, a study was carried out to analyse and predict the volatility of the metical/dollar and metical/rand exchange rates in Mozambique for the period from January 2010 to December 2020. The use of the ARMA-ARCH combination is justified by the fact that ARMA models are not capable of modelling the variation in the variance of financial series over time. During the empirical study, several common stylized facts of financial series were verified, such as the non-stationarity of financial time series, the existence of volatility clusters, among others. It was possible to find three (03) models with good adjustment to model the volatility of exchange rate returns, two (02) for metical/dollar namely: AR(1)-GARCH(1,1) and AR(1)-EGARCH(1,1) and ; one (01) for metical/rand designated AR(1)-ARCH(1). Based on the selection criteria, the results obtained show that for metical/dollar exchange returns the model with the best performance in terms of forecasting is AR(1)-EGARCH(1,1) and for metical/rand exchange returns the AR(1)-ARCH(1) model stands out, being this the only candidate model found for the series. The volatility forecasts made for the two series based on the two (02) best models point to slightly low values for 2021, meaning that there will not be major fluctuations in the short term.

Keywords: Volatility; Return; Forecast; ARCH model.

Received: 11/15/2024

Accepted: 1/15/2025

Published: 1/25/2025

* Corresponding author.

1. Introduction

The growing need for economic and financial information that economic agents, groups of investors, governments, as well as individuals participating in financial markets seek, to support their decision-making, has proven to be a topic of study of greater importance. Currently, the forecast of economic and financial indicators has become vital for the success of those who are, directly or indirectly, related to local and international financial markets. Exchange rates represent a very important concept in the world economy, since, for any transaction involving international business, it is essential to be aware of the status and evolution of the respective rates. Therefore, predicting future values of exchange rates represents a challenge that, although not trivial, gives rise to numerous applications of great importance for all those involved in operations involving two or more currencies, referring to two or more countries. The fact that exchange rates fluctuate over time, in appreciations or depreciations or devaluations or depreciation, allows them to be exposed as time series. Time series modelling makes it possible to describe a stochastic process, using past values of the variable of interest (study), based on the idea that past observations contain information about the level and behaviour of the time series under analysis Reference [19]. Among the various methodologies used for this purpose, the Box & Jenkins models and the Holt and Winters Exponential Damping models stand out [19]. However, financial time series such as interest rates, stock prices, exchange rates, inflation rates, etc., often exhibit the phenomenon of “volatility clustering” [19] i.e. periods in which that their prices show large fluctuations over an extended period of time, followed by periods in which there is relative calm, suggesting that the variance of financial time series varies over time. As [10] points out, this characteristic can be described by the high auto-correlation in the square of returns. The auto-correlation present in the square of returns of financial series means that the conditional variance presents a temporal dependence on past shocks. However, there is now growing evidence to suggest that the use of forecast volatilities obtained through time series models has led to better and more accurate option valuations [6]. Among the volatility forecast models, the ARCH (Autoregressive Conditional Heteroscedasticity) model, originally developed by [7]. The ARCH model considers that the volatility of a time series is a random variable conditioned by the variability observed in past moments, that is, the conditioned variance observed over different periods can be auto-correlated Reference [11]. The study aims to show the applicability of the Auto-regressive Conditional Heteroscedasticity models in estimating and forecasting the volatility of the metical/dollar and metical/rand exchange rates in Mozambique from January 2010 to December 2020.

2. Literature Review

2.1. Basic Concepts of Return and Volatility

i) Historical Volatility

Historical volatility measures price fluctuations that occurred in the past and is generally used as a measure of the total risk of a financial asset. The simplest method of calculation is to calculate the standard deviation of the periodic return of assets, during a period prior to that for which volatility is to be predicted. The greater the volatility, the greater the uncertainty. It is important to note that volatility is not a direction, but rather an indicator, as variability may have occurred only in that

behaviour and not be repeated. It can be said that historical volatility only gives indications of what happened in the past, and it is not linear that they will happen in the future. It is said to be the starting point for estimating future volatility.

ii) Implied Volatility

Implied volatility is the volatility incorporated into the price of assets, that is, it is what the market thinks about a given asset at the moment. Implied volatility is a concept that applies only to options contracts, demonstrating the market's interests in relation to the volatility of financial assets. Implied volatility is constantly changing, and has a strong relationship with the price of financial assets, in the sense that, if it rises, the price of the financial asset also rises, just as happens vice versa. The model used in the financial market to measure implied volatility is the Black-Scholes model given by formulas (1) and (2), solving it in order to the variable that represents volatility, with the financial asset premium being an explanatory variable.

$$C = SN(d_1) - Xe^{-rT} N(d_2) \tag{1}$$

$$P = Xe^{-rT} N(-d_2) - SN(-d_1) \tag{2}$$

Whereby:

$$d_1 = \frac{\ln(S / X) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \quad e \quad d_2 = \frac{\ln(S / X) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Where: **C**: call option price; **P**: sell option price; **S**: current price; **X**: exercise or contract price; **r**: risk interest rate; **T**: due date; **σ**: financial asset price volatility; **N**: represents the accumulated normal distribution.

iii) Future or Forecast Volatility

Future or forecast volatility takes into account the uncertainty of the future and is therefore the most complex to estimate, given its difficulty in estimating the price of the underlying asset for the option period until its expiration. Efficient portfolio management requires good prediction of changes in asset prices in the market, as a more agitated market will require a greater forecast of volatility than a calm market. For a more approximate analysis, and to obtain a starting point for estimating future volatility, values of historical volatility and implied volatility can be used in order to arrive at values that are starting points for analyzes that attempt to project possible future scenarios. There are several methods proposed to determine the value of future or forecast volatility, and there is no more correct method to use. In general, variants of the ARCH/GARCH class models are used. Although volatility is not measured directly, it manifests itself in various ways in a financial series [17]:

- i) Volatility appears in groups, of greater or lesser variability;

- ii) Volatility evolves continuously over time and can be considered stationary;
- iii) It reacts differently to positive and negative values in the series.

The justification for working with returns is that they are scale-free and have interesting statistical properties, such as stationarity and ergodicity. Volatility is directly linked to the price of an asset, having as an important characteristic the leverage effect. In the case of the constant presence of price increases, this effect is identified by the low volatility in the return series. In the case of constant price drops, the return series presents high volatility. However, for the financial market, volatility is an important measure for describing the speed of market change. Markets that move slowly are markets with low volatility and those that move faster are markets with high volatility. Thus, according to [16] returns present the following characteristics below, also known as stylized facts: i) They have an average close to zero; ii) Its squares are auto correlated; iii) Its series show clustering of volatility over time; vi) They present an asymmetry effect; v) They have a leverage effect; vi) Their distributions have heavier tails than the normal ones; vii) Some return series are non-linear.

According to [23,8,15], the distribution of financial series has heavier tails than a normal distribution. For Reference [15,23], although the return series are approximately symmetric, they present excess kurtosis (they are leptokurtic series).

2.2. Exchange Rates

Commercial exchange between countries is carried out using an exchange rate, which represents the price that residents of these countries use in their commercial transactions. [21] defines the exchange rate as the price of one country's money in relation to another country's money. There are two types of exchange rates: nominal exchange rate and real exchange rate. The nominal exchange rate is the relative price of the currency of two countries. In turn, the real exchange rate represents the rate at which economic agents from different countries can transact goods and services between them. Sometimes this rate is called terms of trade [13]. The real exchange rate between two countries is calculated from the nominal rate and the price level between two countries. Thus, the real exchange rate can be defined as the product between the nominal exchange rate and the price level ratio. If “ e ” is considered as the nominal exchange rate, “ P ” as the domestic price and “ P^* ” as the rest of the world price, the concept of real exchange rate can be mathematically written as:

$$\varepsilon = e \frac{P}{P^*} \quad (3)$$

If the real exchange rate is high, it means that the product from abroad is relatively cheap, and the domestic product is relatively expensive. If the real exchange rate is low, it means that products abroad are relatively expensive and domestic products are relatively cheap.

2.3. Exchange Rate Regimes

Countries can adopt different types of exchange rate policies, namely:

Fixed exchange rate regime: the one which the exchange rates remain unchanged. In these systems, whenever there is an increase in exchange rates, which generally has been by definition, it is said that there has been an appreciation of the currency rather than appreciation and when the exchange rate decreases it is said that there has been a devaluation rather than depreciation. The government is the one who decides the level of exchange rate that will prevail in the market.

Floating exchange rate regime: the one which its determination depends on demand and supply, and government intervention is null or almost non-existent. In this regime, the increase in exchange rates is called appreciation, while the reduction in exchange a rate is called depreciation [14].

Within the floating regime, two main subtypes stand out, namely the pure floating exchange rate regime and the administered exchange rate regime. The first, which is also known as Free-float, is characterized by the determination of the value of the exchange rate being completely freely done by the market. In practical terms, no country has a pure fluctuation, however, according to some economists, countries such as the United States of America (USA), Switzerland and Japan present some characteristics very close to this regime [Cfr, Canuto, Octávio, Holland Márcio, op. ci]. The second is characterized by the existence of some infrequent interventions by the Central Bank in the exchange rate through changes in international foreign currency reserves. It is the regime most used by many of the economies that have adopted the floating exchange rate regime; this is the case, for example, in Mozambique. The adoption of a floating exchange rate regime entails some advantages, of which the lack of exchange rate distortions in the economy stands out. Currently, there has been considerable development in foreign exchange markets. The Bank of Mozambique is demonstrating the liberalization of the foreign exchange market, a fact that favours the increasing number of actors in the foreign exchange financial subsystem in Mozambique.

2.4. Conditional Volatility Models

i) ARCH Model

The ARCH model differs from the ARMA models, as it considers that the conditional variance may not be homoscedastic. Many series have very high variance in certain periods, while in others the variance is relatively low. For an investor who wants to make money through t period arbitrage operations, it is important to know the expected return and variance of the assets in his portfolio. In this sense, the conditional variance gains important weight, unlike the long-term variance (unconditional, to which the series converges). In the pioneering study for modelling volatility in time series, [7] introduced the ARCH (Autoregressive conditional heteroscedasticity) model, in which the conditional variance of errors (volatility) can be modelled by the lag of the square of returns distributed in the past. Thus, the ARCH(p) model can be defined as:

$$\varepsilon_t = \sigma_t \mu_t, \mu_t \sim i.i.d \quad (26)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 \quad (27)$$

Which p determines the number of lags of the necessary returns and the error (μ_t) are the random variables with average of zero (0) and variance of one (1). Furthermore, the estimation of α_i occurs through a linear regression,

$$\text{so that } \omega > 0 \quad \alpha_i \geq 0 \quad \sum_{i=1}^p \alpha_i < 1$$

The ARCH models (1) are the simplest and most used version in financial series. Assuming that the errors are normally distributed, the variance is given by: $\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2$

ii) GARCH Model

Later, [5], in response to the fact that ARCH (p) models need many parameters to be adjusted correctly, suggested a generalization of the ARCH model, the GARCH (Generalized Autoregressive Conditional Heteroscedasticity) model. In GARCH, the conditional variance of the error, in addition to being explained by the lag of returns in the past, should include the lag of the square of the conditional variance itself in previous periods. The author argued that the model was potentially more parsimonious than the previous one.

The (p, q) GARCH model can be summarized as follows:

$$\varepsilon_t = \sigma_t \mu_t, \quad \mu_t \sim i.i.d \tag{28}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \tag{29}$$

Where q is entered to determine the number of lags of the conditional variance itself. It is assumed that the error (μ_t) has the same characteristics as the previous modelling and that $\omega > 0$, $\alpha_i \geq 0$ e $\beta_j \geq 0$. In terms of analysis, α_i represents the reaction coefficient to shocks and β_j indicates how much of the volatility perceived from the previous period persists at the current moment. Although it appears that GARCH(p,q) has more parameters than ARCH(p), the truth $pARCH > pGARCH + qGARCH$ is in general [5].

Stationarity is guaranteed if $\sum_{i=1}^p \alpha_i + \sum_{j=1}^q \beta_j < 1$

The GARCH (1,1) model is the simplest and most used version in financial series, assuming that the errors are normally distributed, the variance is given by:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{30}$$

The intuition of the GARCH model is that the persistence of volatility shocks is measured by the sum $(\alpha_i + \beta_j)$, the closer to 1, the longer the shock will take to dissipate

iii) EGARCH Model

Reference [17] point out that models in which conditional heteroscedasticity, whether governed by ARCH or GARCH models, assume symmetric effects on returns, since it is related to their square. However, this assumption is not consistent with empirical data, in which, normally, negative shocks increase volatility more than positive shocks, generating asymmetric effects of returns on volatility. In this sense, other variants of the GARCH class such as EGARCH, TGARCH, among others[In addition to EGARCH and TGARCH, there are other variants of the GARCH class, such as: GJRGARCH, PGARCH, GARCH-M, among others. The exponential generalized autoregressive conditional heteroscedasticity model (EGARCH), proposed by [18], and inserts the logarithmic specification in terms of the GARCH model, allowing the capture of asymmetric effects and the possibility that some coefficients are negative, considering the specification of the logarithm.

Thus, EGARCH (p, q, r) models the conditional variance according to equation (31) below:

$$\ln \sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| + \sum_{j=1}^q \beta_j \ln \sigma_{t-j}^2 + \sum_{k=1}^r \gamma_k \frac{\varepsilon_{t-k}}{\sigma_{t-k}} \quad (31)$$

In which the γ_k coefficients will adjust shock asymmetry. Therefore, if $\gamma_k = 0$, at all times k, a positive and negative shock have the same effect on volatility, that is, the impacts are symmetric and have no leverage effect; if $\gamma_k \neq 0$ the impacts are asymmetric. In these cases, if $\gamma_k < 0$ negative shocks increase volatility more than positive shocks (leverage effect), which would be expected in financial series, such as exchange rates.

The roots of the polynomial $\left(1 - \sum_{j=1}^q \beta_j L^j\right)$ must be outside the unit circle, so that the variance is stationary.

Where L is the polynomial operator. Strict stationarity is given, according to [18], if $\sum_{i=1}^p \alpha_i^2 < \infty$.

iv) TGARCH Model

The generalized conditional heteroscedasticity model with bounding (TGARCH) or Treshold GARCH, was raised by Zakoian (1994). In this modelling, a binary variable is inserted in the GARCH model and the coefficients must be positive to guarantee stationarity of the series. The definition of TGARCH can be summarized below:

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 + \gamma_t d_{t-i} \varepsilon_{t-i}^2 \quad (32)$$

Whereby d is the dummy variable that will be equal to one (1) if the error satisfies the imposed condition and will be equal to zero (0) otherwise. This means that, if there is negative news, $\varepsilon_{t-i} < 0$ the dummy will have a value of one (1) and its impact will be $\alpha_i + \gamma_i$, otherwise, there will be no effect on volatility. Furthermore, if $\gamma_i > 0$ there is evidence of leverage effect. In terms of analysis, α_i it represents the reaction coefficient to shocks and β_j indicates how much of the volatility perceived from the previous period persists at the current moment. In general, the model assumes an increase in volatility when there is a negative shock, $\varepsilon_{t-i} < 0$, since they are accompanied by a positive coefficient ($\gamma_i > 0$), therefore, signalling asymmetric effects on returns.

3. Material and Methods

3.1. Data

To carry out the research, monthly data on the exchange rates of Mozambique metical/dollar and metical/rand were used, referring to the period from January 2010 to December 2020. The database was obtained from the Bank of Mozambique (BM) through its website (www.bm.mz), as this is the entity that guides and controls monetary and exchange rate policies in Mozambique.

The data was divided into two subsamples, namely: i) In-sample: corresponding to the period from January 2010 to December 2019. This data was used to model the series and make the forecast; ii) Out-of-sample: relating to the remaining data in the series, which runs from January to December 2020. The data was compared to the results obtained in the forecast.

In the case of financial time series, the log-returns of exchange rates were used, expressed by the following formula: $R_t = \ln\left(\frac{X_t}{X_{t-1}}\right)$, whereby X_t and X_{t-1} represent the metical/dollar or metical/rand exchange rates at the instant t e $t-1$ respectively. Log-returns are also simply called returns.

The use of returns is justified by the fact that exchange rates are not stationary series. The logarithm helps to stabilize the variance and the difference helps to remove the trend from the series, in addition, the returns are scale-free and present attractive statistical properties such as ergodicity and stationarity already referenced previously. The data were processed using the statistical packages R version 4.0.3 and STATA version 14. All hypotheses of the study's statistical tests were validated at a significance level of 5%.

3.2. Estimation of ARCH Class Models

3.2.1. ARMA Model Estimation

To estimate the ARCH class models, it was first necessary to use the specifications of the ARMA model, that is, an auto-regressive process of moving averages in order to remove the temporal dependencies present in the series. The identification and definition of the most parsimonious ARMA model among those estimated was carried out

based on the analysis of the Auto-correlation Functions (FAC) and Partial Auto-correlation (FACP) and the use of model selection criteria commonly Criteria Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). After defining the ARMA model, the parameters were estimated using the Maximum Likelihood method. Then, diagnostic methods were applied to check whether the residues met the requirements of a white noise process (zero mean, constant variance and uncorrelated process).

Therefore, the Ljung-Box and Box and Pierce tests were applied, the hypotheses of which are as follows:

H_0 : Waste follows a white noise process

H_1 : Waste does not follow a white noise process

The test statistic for Ljung-Box is $LB = n(n+2) \sum_{i=1}^m \left(\frac{\hat{\rho}_k^2}{n-k} \right) \sim \chi_{(m)}^2$ and, the null hypothesis is rejected if the statistics $LB > \chi_{(m)}^2$

The test statistic for Box and Pierce is $Q = n \sum_{i=1}^n \hat{\rho}_k^2$ and, null hypothesis is rejected if $Q > \chi_{(m)}^2$

Although, in large samples, both the Q statistic and the LB statistic follow the chi-square distribution with “m” degrees of freedom, the LB statistic has better properties (more powerful in the statistical sense) for small samples than the Q statistic [11].

3.2.2. LM test (Lagrange Multiplier) of ARCH effect

To test the presence of the ARCH effect, the Lagrange Multiplier (LM) test proposed by [7] was used. [7] showed that the LM statistic can be calculated from $TR^2 \sim \chi_q^2$, where T represents the number of observations and is the multiple regression correlation coefficient of the following model:

$$\varepsilon_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \dots + \alpha_q \varepsilon_{t-q}^2 \quad (33)$$

The hypotheses for the LM test are as follows:

$$\begin{cases} H_0 : \alpha_1 = \alpha_2 = \dots = \alpha_q = 0 \\ H_1 : \exists \alpha_j \neq 0, \text{ com } j = 1, \dots, q \end{cases} \quad (34)$$

So that H_0 is rejected if $TR^2 > \chi_{q,\alpha}^2$

This way of proceeding also serves to test that the residuals (errors) follow a GARCH (p, q) process [9].

Reference [9] derived the modified LM statistic for:

$$H_0 : \alpha_i = \beta_j = 0 \quad (i = 1, 2, \dots, q; j = 1, 2, \dots, p) \quad (35)$$

$$H_1 : \text{There is at least one } \alpha_i \neq 0 \text{ e } \beta_j \neq 0$$

Reference [9] showed that this is a test equivalent to testing the non-existence of an ARCH(q). Thus, in the null hypothesis of homoscedasticity, the GARCH effect and the ARCH effect are equivalent alternatives.

The Average

The average was carried out using the maximum likelihood method, making it necessary to use numerical optimization methods given that the functions to be optimized are non-linear in the parameters.

Verification or Diagnosis

- i) Testing the Properties of Standardized Errors;
- ii) Testing for a lack of a higher order GARCH;
- iii) Misspecification test of linear GARCH models; and
- iv) Parameter stability test.

Model Selection Criteria

Choosing the best model consists of using an information criterion through a set of rules. Therefore, the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) were used to select the models. The equations for these criteria are:

$$AIC = \ln(\hat{\sigma}^2) + \frac{2(p+q)}{T} \quad (36)$$

$$BIC = \ln(\hat{\sigma}^2) + \frac{\ln T(p+q)}{T} \quad (37)$$

whereby $\hat{\sigma}^2$ is the variance estimated via maximum likelihood and T is the number of observations.

The ideal situation is that the lower the AIC and BIC value, the better the model fit. However, it is necessary to compare the AIC and BIC of alternative models, to know which model best explains the dynamics of the time series under study. Therefore, the best model will be the one that presents the lowest values of these criteria [Other aspects must be taken into consideration, placing emphasis on parsimony when choosing the best model]. The

selection of the best models in the ARCH class was complemented with the performance statistics presented below, which consisted of comparing the predicted volatility with the observed one, where the model that presented the lowest values of these statistics was subsequently chosen.

$$\text{Absolute mean error: } EAM = \frac{1}{n} \sum_{i=1}^n |\hat{\sigma}_i^2 - \sigma_i^2| \quad (38)$$

$$\text{Absolute mean error percentage: } EAMP = \frac{1}{n} \sum_{i=1}^n \left| \frac{\hat{\sigma}_i^2 - \sigma_i^2}{\sigma_i^2} \right| \quad (39)$$

$$\text{Mean squared error: } EQM = \frac{1}{n} \sum_{i=1}^n (\hat{\sigma}_i^2 - \sigma_i^2)^2$$

$$\text{Root mean square error: } REQM = \sqrt{\frac{1}{n} \sum_{i=1}^n (\hat{\sigma}_i^2 - \sigma_i^2)^2} \quad (40)$$

[20] Maintain that the squares of return rates are a proxy for realized volatility. Therefore, to determine the observed volatility values, the squares of the rates of returns from out-of-sample data were considered.

4. Analysis and Discussion of Results

4.1. Analysis of Results

4.1.1 Sample Description

Observing the behaviour of the metical/dollar exchange rate series (Figure 1a), it is clear that there are five (5) sub-periods. In the first sub-period the series stabilized around the average (January 2010 to May 2015) although with a slight increase in August 2010, in the second sub-period the series showed an increasing trend (July 2015 to November 2016), this The trend was explained by the strengthening of the US dollar in the international market, a shortage of foreign exchange in the domestic foreign exchange market, an increase in charges for servicing external public debt, speculation by some economic agents holding foreign exchange, in the face of uncertainties regarding the future behaviour of the exchange rate [1]. The third sub-period refers to the months of 2017 in which the metical appreciated against the dollar, having stabilized again around the average between January 2018 and September 2020 (fourth sub-period), from August to December 2020 the graph shows slight signs of depreciation of the metical against the dollar (fifth sub-period), this depreciation may be the result of several factors, among which the COVID-19 pandemic stands out, which hampered the movement of people, goods and services in 2020. In relation to the metical/rand exchange rate series (Figure 1b), there are also five (5) sub-periods. The first sub-period corresponds to the months of 2010, the series showed an increasing trend (depreciation of the metical in relation to the rand), the second sub-period which goes from January 2011 to December 2014, there is a decreasing trend, meaning that the metical appreciated in relation to the rand, the third sub-period corresponds to the months

from January 2015 to March 2017 in which the metical registered an increasing trend of depreciation in relation to the rand. The increasing behaviour of the metical/rand exchange rate recorded in the third sub-period can be explained by the financial crisis resulting from several factors, with emphasis on the Mozambican public debt triggered in 2015, which reflected in the currency deficit in the market, in a context of suspension of the direct support for the State Budget and the balance of payments from the country's official external partners, increased public external debt service [2]. The fourth sub-period starts from April 2017 to December 2019, which showed a decreasing trend in the metical/rand exchange rate, which means that throughout this period the metical appreciated in relation to the rand. The fifth sub-period refers to the months of 2020 in which the national currency registered depreciation trends, probably caused by the COVID-19 pandemic that hampered the movement of people, goods and services.

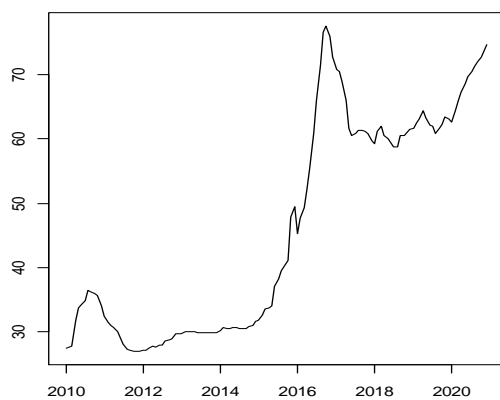


Figura 1a. Taxa de câmbio metical/dólar em Moçambique

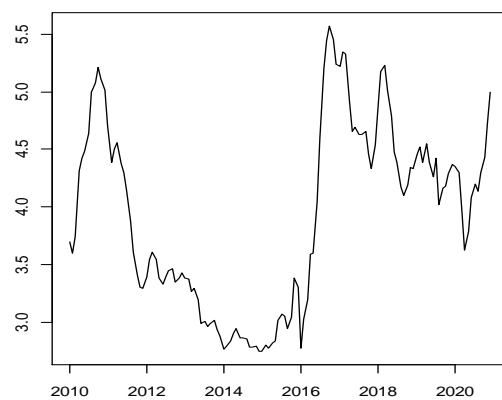


Figura 1b. Taxa de câmbio metical/Rand em Moçambique

Source: The authors

Figure 1: Evolution of metical/dollar and metical/rand exchange rates from January 2010 to December 2020

4.2. Analysis of Returns

Analysing the return series of metical/dollar exchange rates (figure 2a) it is possible to verify the existence of periods of high variation (April 2010) and (November 2015) that are followed by periods of low variation, the same happens in series of metical/rand exchange rate returns (figure 2b) periods of high variation (April 2010), (November 2015) and (June to August 2016) are followed by periods of low variation, which suggests the presence of clusters volatility, which is a common characteristic of financial assets.

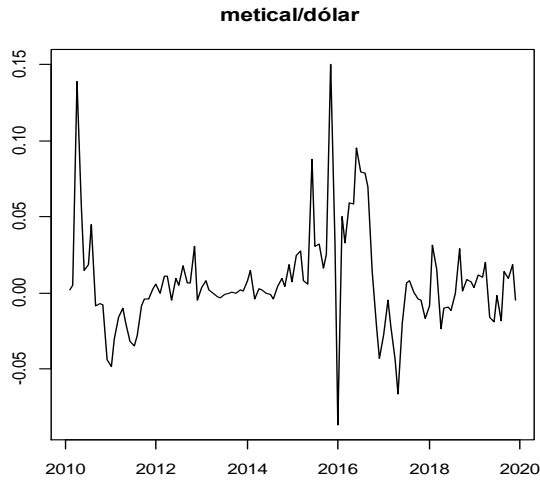


Figura 2a.Retornos de câmbio metical/dólar em Moçambique

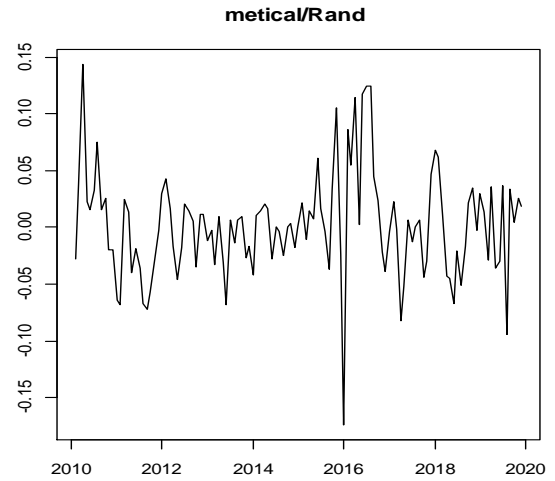


Figura 2b.Retornos de câmbio metical/Rand em Moçambique

Source: The authors

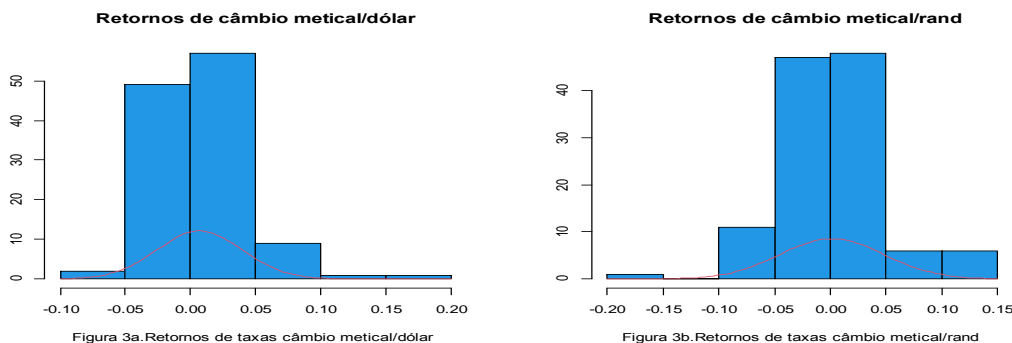
Figure 2: Evolution of metical/dollar and metical/rand exchange rate returns

4.3. Descriptive statistics of returns

Table 1: Descriptive statistics of metical/dollar and metical/rand exchange rate returns

Statistics	Metical/dollar return	Return Metical/rand
N	119	119
Minimum	-0,086975	-0,174497
Maximum	0,150293	0,144170
Average	0,006977	0,022221
Standard deviation	0,032909	0,046701
Asymmetry	1,371071	0,234986
Kurtosis	4,725040	1,968249
JB	155,1859	22,01790
JB P value	< 2,2e-16	0.000017

Source: Authors, processed in R 4.0.3



Source: Authors

Figure 3: Histograms of returns of the metical/dollar and metical/rand exchange rates

Analysing table 1 and the histograms (figure 3), for the metical/dollar exchange rate returns, it can be seen that the average value is very close to zero (0.006977) as well as the standard deviation (0.032909). It can also be seen that the series presents an asymmetry with a deviation to the right (the value of the asymmetry coefficient [Normally for ARCH processes, the asymmetry coefficient is very different from zero, suggesting the applicability of these models to series with this behaviour. In the case of asymmetry being null it is said that the series follows a normal distribution.] is equal to $1.371071 > 0$), which reveals that it is not symmetric as it is in the case of a normal distribution. Using the Jarque–Bera test, the null hypothesis that the series of metical/dollar exchange rate returns follows a normal distribution is rejected since the Jarque–Bera statistic is 155.1859 and the probability value is less than 0.05. In relation to the metical/rand exchange rate returns, similar to the other series, there is an average close to zero (0.022221) as well as the standard deviation (0.046701). The series presents a slight asymmetry with a deviation to the right (the value of the asymmetry coefficient is equal to $0.234986 > 0$), which reveals that it is not symmetric as it is in the case of a normal distribution. Using the Jarque–Bera test, the null hypothesis that the series of metical/rand exchange rate returns follows a normal distribution is rejected since the Jarque–Bera statistic is 22.01790 and the probability value is less than 0.05.

4.1.2. Empirical Results

Stationarity Test

Tables 2 and 3 present the unit root test for each of the return series. The Augmented Dickey Fuller statistics indicate values of -5.649 and -7.016 for metical/dollar and metical/rand respectively, and these values are higher in modulus than the critical values for all significance levels, this means that the hypothesis is rejected null that the series have a unit root, that is, the metical/dollar and metical/dollar exchange rate return series are stationary.

Table 2: ADF unit root test for metical/dollar exchange rate returns

Null Hypothesis: The series of metical/dollar exchange rate returns has a unit root		
	Statistic	Prob.
Augmented Dickey Fuller Statistical Test	-5,649	0,000
Critical values	1%	-2,504
	5%	-2,889
	10%	-2,579

Source: Authors, processed in STATA.14

Table 3: ADF unit root test for metical/rand exchange rate returns

Null Hypothesis: The series of metical/rand exchange rate returns has a unit root		
	Statistic	Prob.
Augmented Dickey Fuller Statistical Test	-7,016	0,000
Critical values	1%	-3,504
	5%	-2,889
	10%	-2,579

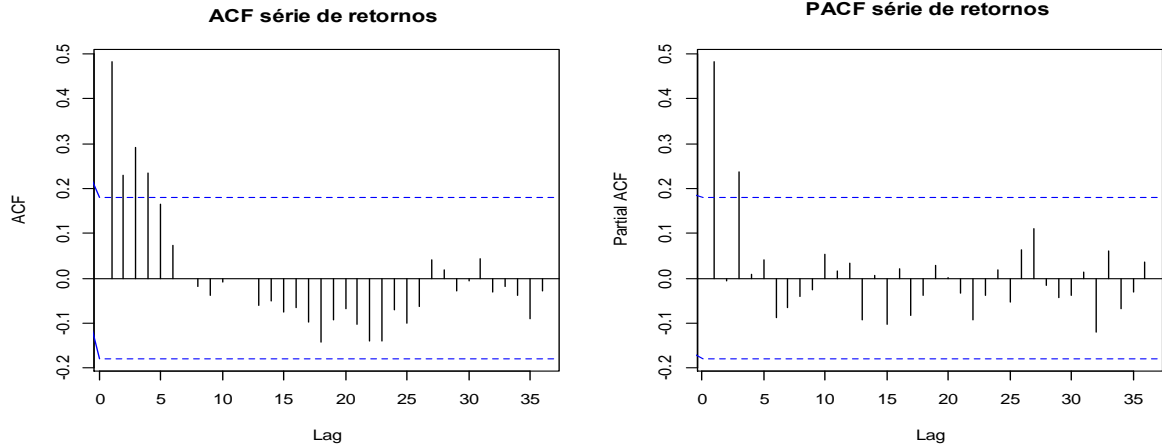
Source: Authors, processed in STATA.14

4.1.2.1 Model Estimation

ARMA Model

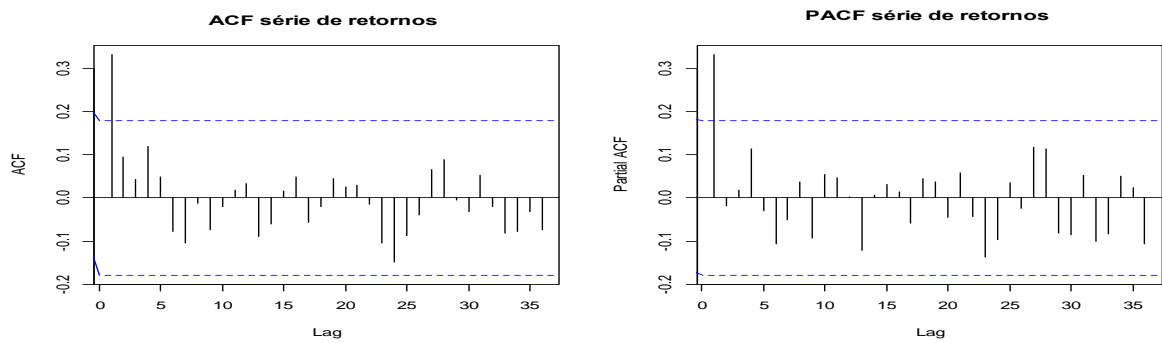
Based on figures 4 and 5, when analysing the auto-correlation function (FAC) and the partial auto-correlation function (FACP) of each of the metical/dollar and metical/rand exchange rate return series, one can verify that the patterns of these are similar. For metical/dollar exchange returns, the autocorrelation functions up to lag 4 appear to be quite different from zero in statistical terms, but all other lags appear to be not different from zero and in the partial autocorrelation function they appear to be only lags 1 and 3 are significant. For metical/rand exchange returns, only the first lag is significant for both FAC and FACP. Therefore, different ARMA models were tested based on significant lags of FAC and FACP in order to find in each series of returns the model that better describes the average equation, for this we will assume that the process generated series of return for both metical/dollar exchange rate and metical/rand exchange rate is AR (1). dos^a

^a De acordo com Morettin e Tolo (2006), dada a forma complicada da FAC e FACP de um modelo ARMA, estas funções muitas das vezes não são muito úteis para identificar os modelos, o que se recomenda neste caso é ajustar modelos de baixa ordem, por exemplo: ARMA(1,0); ARMA(0,1); ARMA(1,1); ARMA(1,2); ARMA(2,1).



Source: Authors

Figure 4: FAC and FACP correlogram of the metical/dollar exchange rate return series



Source: Authors

Figure 5: FAC and FACP correlogram of the series of metical/rand exchange rate returns

Tables 4 and 5 present the statistical values of the estimated models for each of the return series. For both models, the intercepts (0.0068481 and 0.0013501) of the metical/dollar exchange and metical/rand exchange return series respectively are not statistically significant at the 5% significance level, as the probabilities for the respective statistics are 0.1723 and 0.821652. Therefore, the null hypothesis that the true population parameters are equal to zero cannot be rejected. Thus, for the auto-regressive parameters, the null hypothesis that the true population parameters are equal to zero is rejected.

Table 4: Estimation of the AR (1) model with intercept for metical/dollar exchange returns

	Average	Standard error	Statistic	Prob.
ar1	0,4797964	0,0797288	6,0179	1,767e-09
Intercept	0,0068481	0,0050173	1,3649	0,1723
log likelihood	253,63			
AIC	-503,2			

Source: Authors, processed in R 4.0.3

Table 5: Estimation of the AR (1) model with intercept for metical/rand exchange returns

	Average	Standard error	Statistics	Prob.
ar1	0,3317243	0,0861224	3,8518	0,0001173
Intercept	0,0013501	0,0059892	0,2254	0,821652
log likelihood	203,23			
AIC	-402,46			

Source: Authors, processed in R 4.0.3

Once the non-significance of the intercepts in the previous models was verified, the models were estimated again without taking into account the intercept parameters. The results of the model statistics are found in tables 6 and 7. However, the auto-regressive parameters estimates are statistically different from zero (Prob. <0.05). Furthermore, the models improved fit measures since the Akaike information criterion and log likelihood values reduced.

Table 6: Estimation of the AR (1) model without intercept for metical/dollar exchange returns

	Average	Standard error	Statistics	Prob.
ar1	0,502013	0,078485	6,3963	1,592e-10
log likelihood	252,74			
AIC	-503,48			

Source: Authors, processed in R 4.0.3

Table 7: Estimation of the AR (1) model without intercept for metical/rand exchange returns

	Average	Standard error	Statistics	Prob.
ar1	0,332340	0,086096	3,8601	0,0001133
log likelihood	203,21			
AIC				
-404,41				

Source: Authors, processed in R 4.0.3

According to the Ljung-Box and Box-Pierce tests, it can be seen that the null hypothesis that the residues are white noise cannot be rejected, since their probabilities are greater than 0.05, as shown in tables 8 and 9 below. In this case, there is no need to look for other ARMA models.

Table 8: Diagnosis of the residuals of the AR Model (1) without intercept for metical/dollar exchange returns

Box-Pierce test		
Qui-square Statistics	df	Prob.
7,5219	12	0,8213
Box-Ljung test		
Qui-square Statistics	df	Prob.
7,8829	12	0,7942

Source: Authors, processed in R 4.0.3

Table 9: Diagnosis of the residuals of the AR Model (1) without intercept for metical/rand exchange returns

Box-Pierce test		
Qui-square Statistics	df	Prob.
5,3618	12	0,9448
Box-Ljung test		
Qui-square Statistics	df	Prob.
5,7775	12	0,9269

Source: Authors, processed in R 4.0.3

4.1.3. ARCH test for the residuals of the estimated AR(1) models without intercept

From the results in table 10 it can be stated that it is feasible to build models of the ARCH class to correctly describe the process of formation of conditional variance of the errors generated by each model estimated in tables

6 and 7.

Table 10: LM test of ARCH effect for the variance of errors of AR models (1) without intercept

ARCH Effect ARCH LM Test				
Serie	Model	Qui-square Statistics	df	Prob.
Metical/dollar exchange returns	AR(1)	28,846	12	0,004153
Metical/rand exchange returns	AR(1)	28,225	12	0.005128

Source: Authors, processed in R 4.0.3

4.1.4. AR (1) – ARCH(1) Model Estimation

Table 11 presents the results of the AR (1) – ARCH (1) model of the series of metical/rand exchange rate returns, estimated in the most robust form of the parameters and assuming the normal distribution of errors. All coefficients are statistically different from zero to 5% significance. The estimated model captures the reaction to shocks, as the reaction coefficient to volatility shocks is $\alpha_1=0.512228$ (greater than 0.2) which represents a high reaction, that is, the volatility of the series presents sharp peaks. The residual analysis of the model revealed that it was able to conveniently capture the heteroscedastic structure of the conditional error variance, with the residual series being white noise^b.

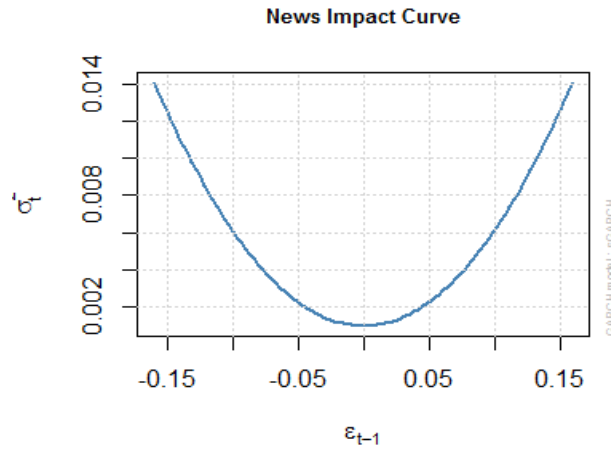
Table 11: AR (1)-ARCH (1) model for the metical/rand exchange rate return series

	Average	Standard error	Statistic	Prob.
ar1	0,502661	0,088298	5,6928	0,000000
omega	0,000969	0,000172	5,6303	0,000000
alpha1	0,512228	0,175199	2,9237	0,003459

Source: Authors, processed in R 4.0.3

As expected, through the impact curve (figure 8), the limitation of this model (the symmetric effect) can be confirmed, that is, positive and negative shocks have the same impact on the conditional variance.

^b Ver anexo1 (Tabela1. Análise residual do modelo AR(1)-ARCH(1) de retorno de taxa de câmbio metical/rand, estimação para o período de Janeiro de 2010 a Dezembro de 2019)



Source: The authors

Figure 6: Volatility impact graph of the AR(1) - ARCH (1) model

For the metical/dollar exchange rate return series, it was not possible to find the ARCH class model at the tested lags (1, 2 and 3) to model the variance equation using the AR mean equation (1), as the parameters of the tested models were not statistically different from zero to 5% significance.

4.1.5. AR (1) - GARCH (1,1) Model Estimation

Table 12 presents the AR (1) - GARCH (1,1) model of the metical/dollar exchange rate return series, estimated in the most robust form of the parameters and the normal distribution of errors was assumed. All coefficients are statistically different from zero to 5% significance, with a stationary process of volatility as the sum of their coefficients is less than unity ($0.1472298+0.774278 = 0.9549 < 1$). The model presents $\alpha_1=0.1472298$ (less than 0.2), this means that the series has a low reaction to shocks. The persistence component was equal to 0.774278, around what was expected (0.8), showing that a shock in volatility takes a while to dissipate.

Table 12: AR (1)-GARCH (1,1) model for the metical/dollar exchange rate return series

	Average	Standard error	Statistics	Prob.
ar1	0,622343	0,122549	5,078300	0,000000
omega	0,000044	0,000017	2,536700	0,011190
alpha1	0,147298	0,054845	2,685700	0,007238
beta1	0,774278	0,042846	18,071400	0,000000

Source: The authors, processado in R 4.0.3

According to the Ljung-Box test weighted on standardized residuals (table 13), at a significance level of 5%, there

is sufficient evidence to affirm that the residuals of the AR(1) - GARCH (1,1) model are purely random.

Table 13: Ljung-Box test weighted on standardized residuals

	Statistic	Prob.
Lag [1]	1,253	0,26297
Lag [2]	2,505	0,09019
Lag [5]	4,665	0,13802

Source: The Authors, processed in R 4.0.3

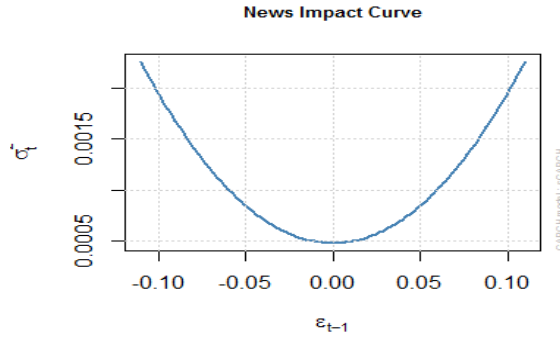
Based on the weighted ARCH-LM and weighted Ljung-Box tests on standardized squared residuals (table 14), the null hypothesis that the squared residuals are white noise cannot be rejected, since the probability values associated with the two tests are greater than 0.05. In this way, the AR (1) - GARCH (1,1) model correctly allows the correlation of squares of residuals to be removed.

Table 14: Weighted ARCH-LM and weighted Ljung-Box tests on standardized squared residuals

Weighted Ljung-Box Test on Standardized Square Residuals				
	Statistics	prob.		
Lag[1]	0,1197	0,7293		
Lag[5]	2,0871	0,5980		
Lag[9]	5,5884	0,3483		
Weighted ARCH-LM Tests				
	Statistics	Form	Scale	prob.
ARCH Lag[3]	0,01608	0,500	2,000	0,89909
ARCH Lag[5]	5,51221	1,440	1,667	0,07814
ARCH Lag[7]	6,59809	2,315	1,543	0,10584

Source: The authors, processed in R 4.0.3

As expected, through the impact curve (figure 9), the limitation of this model (the symmetric effect) can be confirmed, that is, positive and negative shocks have the same impact on the conditional variance.



Source: The authors

Figure 7: Volatility impact graph of the AR(1) - GARCH(1,1) model

For the series of metical/rand exchange rate returns, it was not possible to find the appropriate GARCH model to model the variance equation in the lags tested using the AR (1) mean equation, as the parameters of the tested models were not statistically different from zero to 5% significance.

4.1.6. AR (1) - EGARCH(1,1) Model Estimation

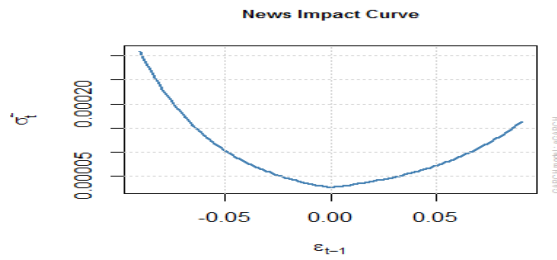
Due to the limitation of the GARCH model, which does not capture the asymmetric effect, which means that positive and negative shocks have the same impact on the conditional variance, the AR (1) - EGARCH (1,1) model was estimated. of the metical/dollar exchange rate return series considering the most robust form and assuming the normal distribution of errors. Table 15 below shows that the AR (1) - EGARCH (1,1) model presents all significant parameters at the 5% significance level, in addition, the value of beta1=0.97479 is quite high, which means presence of high persistence of volatility, indicating that a shock in the series of metical/dollar exchange rate returns will have an effect for several periods on the volatility of these returns. The model provides evidence of asymmetry in the volatility of returns in the metical/dollar exchange rate series, as the “gamma1” coefficient proved to be significant. In this way, positive and negative shocks have different impacts on volatility, and with evidence of the leverage effect[See Figure 10. Volatility impact graph of the AR(1)-EGARCH(1,1) model.

Table 15: AR (1)-EGARCH (1,1) model for the metical/dollar exchange rate return series

	Average	Standard error	Statistics	Prob.
ar1	0,40591	0,002856	142,13	0,000
Omega	-0,30928	0,000342	-903,98	0,000
alpha1	0,34312	0,000632	543,17	0,000
beta1	0,97479	0,004047	240,89	0,000
gamma1	-0,36255	0,001307	-277,32	0,000

Source: Authors, processed in R 4.0.3

As expected, through the impact curve (figure 10), the presence of the leverage effect can be confirmed, that is, that volatility increases with negative shocks than with positive shocks of the same magnitude.



Source: The authors

Figure 8: Volatility impact graph of the AR (1) - EGARCH (1,1) model

In relation to the metical/rand exchange rate return series, it was not possible to find a suitable asymmetric model to estimate the variance equation in the tested lags, when considering the average AR (1) equation, since the asymmetric models tested (EGARCH, TGARCH, GJRGARCH) presented parameters not statistically different from zero to 5% significance.

4.1.2.2. Models Comparison

In-sample Fit

To find out which of the models best fits the returns and volatility of metical exchange rates against the dollar in Mozambique, the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) were used. Based on the goodness-of-fit measures, it appears that the AR(1)-EGARCH (1,1) model is the one that best adjusts to the returns and volatility of metical/dollar exchange rates, as it presents lower values of information criteria (table 16).

Table 16: Statistics for model selection, return on metical/dollar exchange rates

Statistics	AR(1)-GARCH (1,1)	AR(1)-EGARCH (1,1)
AIC	-4,4847	-4,9977
BIC	-4,3913	-4,8809

Source: The authors, processed in R 4.0.3

5. Out-of-Sample Performance

According to Bacchi and Hoffman (1995), an econometric model can present all the statistical evidence that makes it consistent, but it is of no use if it does not make good predictions. This means that models with great statistical evidence can often have poor predictive performance, and others that are not as statistically fit can perform well

in their out-of-sample predictions. Therefore, for the purpose of comparing performances, forecasts of the two out-of-sample models were made for the period from January to December 2020 and based on the results of the forecasts, performance statistics were calculated (table 17), the results favour the AR(1)-EGARCH(1,1) model since this one presents all the calculated statistics smaller in relation to the other.

Table 17: Out-of-sample performance statistics of models, metical/dollar exchange rate return

Statistics	AR (1)-GARCH (1,1)	AR(1)-EGARCH (1,1)
EAM	0,00810432511	0,00571377008
EQM	0,00007838373	0,00005771180
REQM	0,008853458802	0,007596839368

Source: The authors, processedo in R 4.0.3

6. Estimation and Comparison of Models Using the Complete Series

In order to verify the consistency of the selected models, the model estimates were tested again, but already covering the entire study period (January 2010 to December 2020). The results (table 18) show that the AR(1)-EGARCH (1,1) model of metical/dollar exchange rate returns remains the best, as it presents the Akaike (AIC) and Bayesian information criteria (BIC) smaller in relation to those of the other model, in addition, all parameters are statistically different from zero, with purely random residuals and the model managed to correctly remove the correlation of the squares of the residuals. Regarding the AR(1)-ARCH(1) model of the series of metical/rand exchange rate returns, it appears that it continues to have the best fit since its residuals are white noise and it managed to correctly remove the correlation of squares of the residues.

Table 18: Statistics for the selection of metical/dollar exchange rate return models (estimation period from January 2010 to December 2020)

Statistics	AR(1)-GARCH (1,1)	AR(1)-EGARCH (1,1)
AIC	-4,5957	-5,1340
BIC	-4,5079	-5,0243

Source: The authors, processed in R 4.0.3

6.1. Return and Volatility Forecasts

Volatility forecasts (table 19) point to slightly low values, which suggests that there will not be major fluctuations in the short term. Forecasts for the volatility of metical/dollar exchange rates for the year 2021 point to a decreasing trend, which means that fluctuations will tend to decrease slightly. Regarding the metical/rand exchange rate, forecasts point to an increasing trend in volatility for 2021, which means that fluctuations will tend to increase slightly.

Table 19: Forecasts of returns and exchange rate volatility from January to December 2021

Month	AR(1)-EGARCH (1,1)		AR(1)-ARCH (1)	
	Metical/dollar Exchange Rate		Metical/rand Exchange Rate	
	Return	Volatility	Return	Volatility
January	3,92E-03	0,02345	2,62E-02	0,03501
February	1,28E-03	0,02235	1,28E-02	0,04029
March	4,17E-04	0,02133	6,28E-03	0,04263
April	1,36E-04	0,02036	3,08E-03	0,04373
May	4,44E-05	0,01946	1,51E-03	0,04425
June	1,45E-05	0,01861	7,39E-04	0,04450
July	4,72E-06	0,01781	3,62E-04	0,04462
August	1,54E-06	0,01706	7,75E-02	0,04468
September	5,03E-07	0,01635	8,70E-05	0,04471
October	1,64E-07	0,01569	4,26E-05	0,04473
November	5,35E-08	0,01506	2,09E-05	0,04473
December	1,75E-08	0,01447	1,02E-05	0,04474

Source: The authors, processed in R 4.0.3

6.2. Discussion of the Results

In the present study, an analysis was made of the volatility of the metical/dollar and metical/rand exchange rates in Mozambique for the period from January 2010 to December 2020, having found that the conditional heteroscedasticity models are generally adequate to model the volatility of financial time series. This conclusion corroborates the results of the study carried out in Mozambique by [12], which analysed the volatility of the inflation rate for the period from 1996 to 2006, having reached the same conclusion about the applicability of ARCH class models in financial time series. The return series for the metical/rand exchange rate in Mozambique did not show asymmetric behaviour for the period under analysis, meaning that positive and negative shocks have the same impact on the conditional variance. These results corroborate the work of [3] who studied the behaviour of exchange rate volatility using several different exchange rate series for the period between 1986 and 1997 and found that the return series did not show signs of an asymmetric response to any shocks to rate volatility. Two models were estimated to describe the volatility of the returns of the metical exchange rate in relation to the dollar in Mozambique, namely the AR(1)-GARCH(1,1) and AR(1)-EGARCH(1,1) models, both adjusted with normal error distribution, with the AR(1)-EGARCH(1,1) model presenting better results in predictions. These results are similar to those obtained by [4], in their studies they carried out a forecast using a rolling window with 21 observations of the dollar/euro exchange rate pair, they concluded that the model that presented the best results was the EGARCH estimated based on a distribution normal of errors. In the present study, the leverage effect was verified, showing that the increase in volatility is greater after negative shocks than after positive shocks, of the same intensity in the returns on the metical/dollar exchange rate. These results corroborate in the same direction with the results of the study by [4]. The results of this study reveal that the selected AR(1)-EGARCH(1,1) model

performed better in predictions both within and outside the sample. These results differ with the work of [24] who studied the use of ARMA-EGARCH models in the euro/dollar exchange rate pair, and used data within the sample referring to the period between August 1, 2007 and July 31, 2010 and out-of-sample data between August 1, 2010 and July 31, 2013, using daily quotes, having concluded that the predictive power of the selected model using out-of-sample data has no correlation with the fit of the same model within the sample.

7. Conclusions

- i. The exchange rates under study, from January 2010 to December 2020, presented periods of high variation, followed by periods of lower variation, thus suggesting an agglomeration of volatility;
- ii. Among the models considered suitable for modelling the conditional variance (risk) of the metical/dollar exchange rate return series equation in Mozambique, the AR(1) - EGARCH(1,1) model stands out, which presented the best forecast performance;
- iii. In relation to the returns on the metical/rand exchange rate, it was possible to find the AR(1)-ARCH(1) model, which was the only one that proved to be adequate to describe the variance equation;
- iv. The metical/dollar exchange rate showed evidence of asymmetry, that is, volatility showed different responses to positive and negative shocks. Furthermore, the leverage effect was verified, indicating that the increase in volatility is greater after negative shocks than after positive shocks, of the same intensity in the returns on the metical/dollar exchange rate. These results enable the market to recognize that bad news, originating from the domestic or international economy, tends to have more significant effects than good news;
- v. Still in relation to the metical/dollar exchange rate, in addition to the asymmetry recorded, there is a strong persistence of shocks in volatility, indicating that such shocks may take several periods to dissipate;
- vi. Forecasts of exchange rate volatility analysed in this study point to slightly low values for the year 2021, which suggests that there will not be major fluctuations in the short term.

8. Recommendations

For future studies related to exchange rate volatility, it is recommended to use multivariate GARCH models to ensure greater accuracy in forecasts, considering that multivariate models in time series tend to achieve better results.

9. Study Limitations

The main limitations encountered during the conduct of this study were:

- i. **Lack of specific bibliographic material:** The literature on exchange rate volatility in emerging economies, such as Mozambique, is limited. The scarcity of studies on the behavior of the metical/dollar and metical/rand exchange rates made it difficult to contextualize the results obtained, as well as to compare them with other economies with similar characteristics. This gap in specialized literature sources prevented a more in-depth analysis of the underlying variables and their long-term effects.

- ii. **Insufficient similar studies in Mozambique:** The absence of comparable studies focused on exchange rate volatility in Mozambique posed a significant challenge for validating the results. There was no robust database or comparative analysis on the metical/rand exchange rate, making it difficult to directly compare with other economies in the region. This factor limited the ability to extrapolate the results to other similar economic realities and restricted the practical application of the results in local exchange rate policies.

Bibliography

- [1] Banco de Moçambique (2015).Relatório anual. Maputo, Moçambique.
- [2] Banco de Moçambique (2016).Relatório anual. Maputo, Moçambique.
- [3] Brooks, C. (2008). Introductory Econometrics for Finance. 2nd Edition. Cambridge.
- [4] Bunjaku, B., & Nāsholm, A. (2010). Forecasting Volatility - A comparison study of model based forecasts and implied volatility. Master Thesis.
- [5] Bollerslev, T. P. (1986). Generalized Autoregressive Conditional Heteroskedasticity. Journal of Econometrics, 31 (3), 307 – 327.
- [6] Costa, P., Couto, G. & Martins, G. (2003). Análise da volatilidade do premio de risco do mercado de capitais. Português. 36pp. 2ª versão. ISEG/UTL
- [7] Engle, R.F. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of the United Kingdom Inflation; Econometric, 50. pp 987-1006.
- [8] Francq, C & Zakoian, J.M (2010). Inconsistency of the MLE and inference based on weighted LS for LARCH. models." Journal of Econometrics 159.1 (2010): 151-165
- [9] Franses, F.H & van Dijk D.R. (2000). Non-linear Time Series Models in Empirical Finances.
- [10] Gokcan, S. (2000). Forecasting Volatility of Emerging Stock Markets: Linear versus Non-Linear.
- [11] Gujarati, D.N (2006). Econometria básica. 4ª Edição (tradução). São Paulo. Editora Campus.
- [12] Loquiha, O. (2009). Modelos Auto Regressivos de Heterocedasticidade Condicional - Uma aplicação a análise de volatilidade da taxa de inflação de Moçambique no período de 1996-2006.Trabalho de Licenciatura em Estatística na UEM.
- [13] Mankiw, N.G, (2001). Principles of Macroeconomics, 6a Edition, Amazon.
- [14] Mishkin, S. (2004). Economics of Money, Banking and Financial Markets 7th edition, Pearson Addison

Wesley, Boston.

- [15] Morettin, P. (2008). *Econometria Financeira*, 2008.
- [16] Morettin, P. A. & Toloi, C. M. C. (2004). *Análise de Séries Temporais*. São Paulo: Edgard Blucher LTDA, 2004, 535 p.
- [17] Morettin, P. A. & Toloi, C. M. C. (2006). *Previsão de séries temporais*. 2 Edição. São Paulo.
- [18] Nelson, D.B. (1991). *Conditional Heteroskedasticity in Asset Returns: A New Approach*. *Econometric*, 59. pp 347-370.
- [19] Pessanha, J.F.M, Leal, L.H.C & Silva, M.A.M (2004). *Uma Aplicação de Modelos de séries temporais na previsão da demanda por Transporte Aéreo de Passageiro*. Rio de Janeiro. Pp5-11.
- [20] Poon, S. & Granger, C. W. J. (2003). *Forecasting Volatility in Financial Markets: A Review*. *Journal of Economic Literature*, Vol. 41 (2), 478-539.
- [21] Pungel, T. A. (2007). *International Economics*. 13a Edition. New York: McGraw- Hill Irwin.
- [22] Tíbulo, C. (2017). *Modelos de séries temporais aplicados a dados de humidade relativa do ar*. Dissertação (Mestrado em Engenharia de Produção) - Universidade Federal de Santa Maria - UFSM/RS.
- [23] Xiao, L. & Abdurrahman, A. (2007). "Volatility modelling and forecasting in finance." *Forecasting volatility in the financial markets*.
- [24] Zhong, Y. (2013). *Forecast volatility in value of the EUR/USD*. Unpublished Master Thesis. University of Nottingham