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A Modified Vogel Approximation Method for Solving Balanced Transportation Problems

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Abstract

A modified Vogel Approximation Method is proposed and compared with those of the existing methods available for solving balanced transportation problems (in linear programming) for Basic Feasible Solutions (IBFS). The method is shown to be better than existing ones (excluding Vogel Approximation Method) since it does not only considers each unit cost in its solution algorithm, but also minimises total cost (comparatively) just like Vogel Approximation Method.

Keywords: Balanced Transportation Problem; Modified Vogel Approximation Method; Vogel Approximation Method; Standard Deviation of Costs.

1. Introduction

The problem of physical distribution (transportation) of goods and services from several supply origins to several demand destinations is one important application of linear programming,[1]. To express a transportation problem in terms of a linear programming model, which can be solved by the method of Simplex, is quite easy [1].

But because a transportation problem usually involves a large number of variables and constraints, solving it with the Simplex method is often time-consuming.

However, instead of using the regular simplex tableau, advantage is taken of the special structure of the transportation model to present the algorithm in a more convenient form [2,1].

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Transportation algorithms such as the "Stepping Stone Method" and the MODI (Modified Distribution) Method have been developed for this purpose [1].

The structure of transportation problem involves a large number of shipping routes from several supply origins to several demand destinations [3]. The objective is to determine the number of units of an item (commodity or product) that should be shipped from an origin to a destination in order to satisfy the required quantity of goods or services at each destination centre. This should be done within the limited quantity of goods or services available at each supply origin, at the minimum transportation cost and/or time [4,1,5].

There are various types of transportation models and the simplest of them was first presented in 1941. It was further developed in 1949 and in 1951. Thereafter, several extensions of transportation models and methods have been subsequently developed. The general problem is represented by the network below.

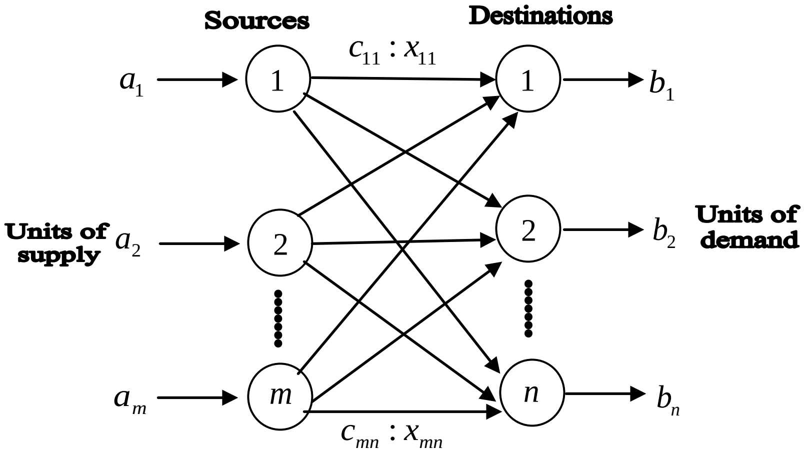

Figure 1: Network for a General Transportation Problem

There are m sources and n destinations, each represented by a "node". The "arcs" linking the sources and destinations represent the routes between the sources and the destinations. Arcs (i, j) joining source i to destination j carries two pieces of information: (1) the transportation cost per unit - c_{ij}, and (2) the amount shipped - x_{ij}. The amount of supply at source i is - a_i and the amount of demand at destination j is - b_j, [2,6,7]. The objective of the model is to determine the unknowns x_{ij} that will minimize the total transportation cost while satisfying all the supply and demand restrictions [8,2,1]. Mathematically, the problem, in general, may be stated as follows:

$$Min\,C = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} x_{ij}$$

$$s.t: \quad \sum_{j=1}^{n} x_{ij} = S_i \quad i = 1,2,3,\cdots,m$$

$$\sum_{i=1}^{m} x_{ij} = D_j \quad j = 1,2,3,\cdots,n$$

$$x_{ij} \geq 0, \forall\ i, j$$

Where;

$S_i = Supply\ at\ origin\ i$

$D_j = Demand\ at\ destination\ j$

$x_{ij} = Number\ of\ units\ shipped\ from\ origin\ i\ to\ destination\ j$ $\quad C = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} x_{ij}$ is the total shipping cost and

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j \quad (condition\ for\ the\ existence\ of\ feasible\ solution - rim\ conditions)$$

A balanced transportation problem having m - origins and n - destinations is usually represented in tabular form as shown below [1,9].

Table 1: Transportation Problem

	Destination							
Origin	1	2	3	.	.	.	N	S
1	C_{11}	C_{11}	C_{11}	.	.	.	C_{11}	S_1
2	C_{21}	C_{11}	C_{11}	.	.	.	C_{11}	S_2
3	C_{31}	C_{32}	C_{11}	.	.	.	C_{11}	S_3
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
M	C_{m1}	C_{11}	C_{11}	.	.	.	C_{nm}	S_m
D_j	D_1	D_2	D_3	.	.	.	D_n	

The balanced transportation problem is indeed a linear programming problem. But due to its special structure and intricacies, simple special algorithms have been developed for solving it. These existing methods are basically for finding the initial basic feasible solution and the optimal basic feasible solution to any given balanced transportation problem [9,1]. They are: Northwest-corner, Least-cost and Vogel Approximation methods.

2. The Algorithms for Existing Methods

The following are the algorithms for the existing methods in their order of efficiency in terms of minimization of total cost, usage of the given unit costs in the solution steps, and approximation of the optimal solution for any given tractable balanced transportation problem. However we remark that this paper would only be restricted to making comparison of the techniques using their abilities of minimizing total costs.

Northwest-corner Method [2,1]:

Step 1: Assign the maximum possible number of units to the cell at the Northwest corner. That is, cell - $(1, 1)$. Hence, $x_{11} = \min(S_1, D_1)$

Step 2: If there is any remaining unit in row one, set - $x_{11} = D_1$, $S_1 = D_1$ and move to cell - $(1, 2)$. But if $x_{11} = S_1$ then set $D_1 = D_1 - S_1$ and move to the next cell.

Step 3: Repeat step 2 until the bottom right corner cell is reached at which point, all the quantity of units available have been exhausted while all the demands have been satisfied.

Least-cost Method [2,1]:

Step 1: Identify the least cost cell/route and assign the maximum possible number of units to it.

Step 2: Delete the row or column having exhausted supply at that origin or having satisfied demand at that destination respectively.

Step 3: If the supply at every origin is exhausted and the demand at every destination satisfied, then stop. Otherwise return to step one.

Vogel Approximation Method [5]:

Step 1: Identify two lowest costs in each row and column. Find the row difference and column difference.

Step 2: Identify the row or column with the greatest cost difference and assign the maximum possible number of units to the least cost route in that row or column.

Step 3: If the assignment in step two satisfies the demand at that destination then delete the corresponding column. Otherwise delete the corresponding row when it exhausts the supply at the origin.

Step 4: If every supply is exhausted and every demand satisfied stop. Otherwise, return to step one.

3. Application of the Existing Method

Case Study:

Table one is a display of a balanced transportation problem. Find the initial basic feasible solution and optimal solution using: Northwest-corner Method, Least-cost Method and Vogel Approximation Method.

Table 2: Case Study

	Destination				
Origin	**1**	**2**	**3**	**4**	**S_i**
1	8	6	10	9	**35**
2	9	12	13	7	**50**
3	14	9	16	5	**40**
D_j	**45**	**20**	**30**	**30**	

Using the Northwest-corner Method to solve the above problem we have the following initial basic feasible solution below.

$x_{11} = 35$,

$x_{21} = 10$

$x_{22} = 20$

$x_{23} = 20$

$x_{33} = 10$

$x_{34} = 30$

With this initial basic feasible solution we calculate the total cost - C. That is,

$$C = x_{11}c_{11} + x_{21}c_{21} + x_{22}c_{22} + x_{23}c_{23} + x_{33}c_{33} + x_{34}c_{34}$$

$$\Rightarrow C = (35 \times 8) + (10 \times 9) + (20 \times 12) + (20 \times 13) + (10 \times 16) + (30 \times 5)$$

$$\Rightarrow C = 1180$$

Using the Least-cost Method to solve the above problem we have the following initial basic feasible solution below.

$x_{12} = 15$,

$x_{13} = 20$

$x_{21} = 30$

$x_{23} = 20$

Table 3: Northwest-corner Method

	Destination										
Origin	1	2	3	4	S_i	Stag1	Stag2	Stag3	Stag4	Stag5	Stag6
1	8	6	10	9	**35**	35/35 =0	0	0	0	0	0
2	9	12	13	7	**50**	50	50/10= 40	40/20= 20	20/20 =0	0	0
3	14	9	16	5	**40**	40	40	40	40	40/10= 30	30/30 =0
D_i	**45**	**20**	**30**	**30**	125						
Stag 1	45/35= 10	20	30	30		90					
Stag 2	10/10= 0	20	30	30			80				
Stag 3	0	20/20 =0	30	30				60			
Stag 4	0	0	30/20= 10	30					40		
Stag 5	0	0	10/10= 0	30						30	
Stag 6	0	0	0	30/30 =0							0

$x_{32} = 10$

$x_{34} = 30$

With this initial basic feasible solution we calculate the total cost - C . That is,

$$C = x_{12}c_{12} + x_{13}c_{13} + x_{23}c_{23} + x_{23}c_{23} + x_{32}c_{32} + x_{34}c_{34}$$

$$\Rightarrow C = (15 \times 8) + (20 \times 6) + (30 \times 9) + (20 \times 13) + (10 \times 16) + (30 \times 5)$$

$$\Rightarrow C = 1080$$

Table 4: Least-cost Method

	Destination										
Origin	1	2	3	4	S_i	Stag1	Stag2	Stag3	Stag4	Stag5	Stag6
1	8	6	10	9	**35**	35	35/20= 15	15/15 =0	0	0	0
2	9	12	13	7	**50**	50	50	50	50/30= 20	20/20 =0	0
3	14	9	16	5	**40**	40/30= 10	10	10	10	10	10/10 =0
D_i	**45**	**20**	**30**	**30**	125						
Stag 1	45	20	30	30/30 =0		95					
Stag 2	45	20/20 =0	30	0			75				
Stag 3	45/15= 30	0	30	0				60			
Stag 4	30/30= 0	0	30	0					30		
Stag 5	0	0	30/20= 10	0						10	
Stag 6	0/	0	10/10	0							0

Table 5: Key for the Least-cost Method

Order of Deletion	Name of Colour	Colour Description
First – Fourth column	Red	
Second – Second column	Yellow	
Third – Cells (1, 1) & (1, 3)	Green	

Fourth – Cells (2, 1) & (3, 1)	Deep Blue	
Fifth – Cell (2, 3)	Blue	
Sixth – Cell (3, 3)	White	

Using Vogel Approximation Method to solve the above problem we have the following initial basic feasible solution below.

$x_{12} = 10,$

$x_{13} = 25$

$x_{21} = 45$

$x_{23} = 5$

$x_{32} = 10$

$x_{34} = 30$

With this initial basic feasible solution we calculate the total cost - C. That is,

$$C = x_{12}c_{12} + x_{13}c_{13} + x_{21}c_{21} + x_{23}c_{23} + x_{32}c_{32} + x_{34}c_{34}$$

$$\Rightarrow C = (10 \times 6) + (25 \times 10) + (45 \times 9) + (5 \times 13) + (10 \times 9) + (30 \times 5)$$

$$\Rightarrow C = 1020$$

4. Modified Vogel Approximation Method

This method is developed as follows. Rather than using the idea of identifying two lowest costs as we have it in the algorithm of the existing Vogel Approximation Method, we make use of the idea of “standard deviation of row/column costs”.

Table 6: Vogel Approximation Method

	Destination					St1		St2		St3		St4		St5	
Ori gin	1	2	3	4	S_i	Rd 1	1	Rd2	2	Rd3	3	Rd4	4	R d5	5
1	8	6	10	9	**35**	8/6 =2	35	8/6 =2	35	8/6 =2	35/10 =25	10/ 8=2	25	10	25
2	9	12	13	7	**5**	9/7	50	12/	50	12/	50	13/	50/4	13	5/5

					0	=2		9=3		9=3		9=4	5=5		=0
3	14	9	16	5	**40**	9/5=4	40/30=10	14/9=5	10/10=0	-	0	-	0	-	0
D_i	**45**	**20**	**30**	**30**	125										
Cd1	9/8=1	9/6=3	13/10=3	7/5=2											
1	45	20	30	30/30=0			95								
Cd2	9/8=1	9/6=3	13/10=3	-											
2	45	20/10=10	30	0					85						
Cd3	9/8=1	12/6=6	13/10=3	-											
3	45	10/10=0	30	0							75				
Cd4	9/8=1	-	13/10=3	-											
4	45/45=0	0	30	0									30		
Cd5	-	-	13/10=3	-											
5	0	0	30/5=25	0											25
Cd6	-	-	10	-											
6	0	0	25/25=0	0											

St6	
Rd6	6
10	25/25=0
-	0
-	0

	0

Table 7: Key for the Vogel Approximation Method

Order of Deletion	Name of Colour	Colour Description
First – Fourth column	Red	
Second – Cells (3, 1), (3,2) & (3, 3)	Yellow	
Third – Cells (1, 2) & (2, 2)	Green	
Fourth – Cells (1, 1) & (2, 1)	Deep Blue	
Fifth – Cell (2, 3)	Sky Blue	
Sixth – Cell (1, 3)	White	

Step 1: Find the row and column costs-standard deviations.

Step 2: Identify the row or column with the greatest costs-standard deviation and assign the maximum possible number of units to the least cost route in that row or column.

Step 3: If the assignment in step two satisfies the demand at that destination then delete the corresponding column. Otherwise delete the corresponding row when it exhausts the supply at the origin.

Step 4: If every supply is exhausted and every demand satisfied, stop. Otherwise return to step one.

Using this algorithm to solve the same selected balanced transportation problem of section three gives:

$x_{12} = 10$,

$x_{13} = 25$

$x_{21} = 45$

$x_{23} = 5$

$x_{32} = 10$

$x_{34} = 30$

With this initial basic feasible solution we calculate the total cost - C. That is,

$$C = x_{12}c_{12} + x_{13}c_{13} + x_{21}c_{21} + x_{23}c_{23} + x_{32}c_{32} + x_{34}c_{34}$$

$$\Rightarrow C = (10\times 6) + (25\times 10) + (45\times 9) + (5\times 13) + (10\times 9) + (30\times 5)$$

$$\Rightarrow C = 1020$$

Table 8: Modified Vogel Approximation Method

	Destination					St1		St2		St3		St4		St5	
Origin	1	2	3	4	S_i	RS D1	1	RS D2	2	RS D3	3	RS D4	4	RS D5	5
1	8	6	10	9	**35**	1.5	35	1.6	35	1.6	35	2.0	35/10 =25	0.0	25/25=0
2	9	12	13	7	**50**	2.3	50	1.7	50	1.7	50/45=5	0.5	5	0.0	5
3	14	9	16	5	**40**	4.3	40/30 =10	2.9	10/10=0	0.0	-	0.0	-	0.0	-
D_i	**45**	**20**	**30**	**30**	125										
CS D1	2.6	2.4	2.4	1.0											
1	45	20	30	30/30=0			95								
CS D2	2.6	2.4	2.4	0.0											
2	45	20/10 =10	30	-					85						
CS D3	0.5	3.0	1.5	0.0											
3	45/45=0	10	30	-							40				
CS D4	0.0	3.0	1.5	0.0											
4	-	10/10	30	-									30		

		=0													
CS D5	0.0	0.0	1.5	0.0											
5	-	-	30/25=5	-											5
CS D6	0.0	0.0	0.0	0.0											
6	-	-	5/5=0	-											

RSD6	6
0.0	-
0.0	5/5=0
0.0	-

Table 9: Key for Modified Vogel Approximation Method

Order of Deletion	Name of Colour	Colour Description
First – Fourth column	Red	
Second – Cells (3, 1), (3,2) & (3, 3)	Yellow	
Third – Cells (1, 1) & (2, 1)	Green	
Fourth – Cells (1, 2) & (2, 2)	Deep Blue	
Fifth – Cell (1, 3)	Blue	
Sixth – Cell (1,4)	White	

The table below gives a comparison of the existing methods and the proposed Modified Vogel Approximation Method in terms of minimization of total cost.

Table 10: Comparison of Methods

	Method	IBF Solution
1	Northwest-corner	1180
2	Least-cost	1080
3	Vogel Approximation	1020
4	Modified Vogel Approximation	1020

5. Conclusion

The concept of balanced transportation problems and the existing methods available for solving this type of linear programming problem in their order of efficiency has been examined. Our paper proposes a Modified Vogel Approximation Method - which is so far the best available method for solving balanced transportation problems. We have used our modification as well as other existing methods in solving a selected balanced transportation problem; and have compared the efficiency of all the methods at reducing the total cost using the same selected problem. Following the above, it is evident that the modified method is of better status than its predecessors (except Vogel Approximation Method which has the same IBF solution) in terms of reducing total cost.

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