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Common Versus Herschel-Bulkley Drilling Fluid Models: Effect of Their Rheological Parameters on Dynamic Particle Settling Velocity

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Abstract

Rheological modelling of drilling fluids in oil fields is usually described by Bingham plastic and Ostwald-deWaele models. These models gain popularity because their specific descriptive parameters are fairly easy to estimate. Standard methods use Fann VG meter dial reading at 600 and 300 rpm to determine these rheological parameters. Unfortunately, these points correspond to higher shear rates which seldom prevail during particle settling. Recently, many researchers pointed out that the non-Newtonian behavior of drilling fluids can be described well by the three parameters Herschle-Bulkley model. Again, the determination of its parameters using the standard API method make use of dial readings at 6 & 3 rpm to determine yield stress and 600 and 300 rpm to determine the other two parameters. Furthermore, the use of non-linear regression techniques to determine these parameters though deemed more accurate, sometimes give meaningless negative yield stress values. This work aims to investigate different techniques and shear rates to derive rheological parameters and show their influence on the magnitude of effective viscosity and hence settling velocity. It is demonstrated that very small differences among the values of the model parameters determined by different techniques/dial readings can lead to substantial differences in predicted settling velocities. Results of this work shows that the use of Herschle-Bulkley rheological parameters was by far the most accurate for representing the example muds rheograms as well as predicting the settling velocities, particularly when using non-linear regression values. Moreover, the simplified API method to determine Herschle-Bulkley rheological parameters may lead to considerable errors that may negate the usefulness of the model. It has been shown that Chien [10] correlation is sensitive to mud rheology and is suitable only for thin fluids. However, Chien [10] correlation was not affected by the annular fluid velocities.

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1. Introduction

In drilling operations, the term rheology refers to relationship of the shear-stress/shear-rate or dial-reading/sleeve-speed viscometer data of the drilling fluid. Shear stress (τ) is the force per unit area required to move a fluid at a given shear rate. Shear rate (γ) is the change in fluid velocity divided by the gap or width of the channel through which the fluid is moving. Generally, rheological properties are used to design and evaluate wellbore hydraulics and assess the performance of drilling fluids. Rheological models are useful tools to describe mathematically the relationship between shear-stress and shear-rate of a given fluid.

The most common rheological models to describe the rheological behavior of drilling fluids are the two parameters Bingham plastic (BP) model and the Ostwald-de Waele (power law) model [1,2]. Bingham plastic model or the power law (PL) model are used most often because of their simplicity and it is fairly easy to solve for their specific rheological parameters [1,3]. However, these models do not very well "simulate fluid behavior across the entire rheological spectrum, particularly in the law shear rate range" [1]. Moreover, the Bingham plastic model was found to overestimate the fluid yield stress (yield point) while the power law model lead to substantial errors if the fluid exhibit yield stress [1,3].

Three parameter model have been proposed by Herschle and Bulkley (HB) in 1962 [3]. This model is currently deemed more accurate in predicting the behavior of vast majority of drilling fluids in comparison to the two parameter models that are widely accepted in the oil industry. The Herschle-Bulkley model is sometimes referred to as modified power law or yield power law. This model is claimed to calculate the yield stress more accurately than that calculated by Bingham plastic and more accurately characterize mud behavior across the entire shear rate range [1]. However, there is not wide acceptance and wide spread application of this model because of difficulty in finding analytical solutions for differential equations and complexity of calculations [3]. The American petroleum institute (API) has suggested a simple way to determine the three rheological parameters of this model using only four Fann 35 VG dial readings [4]. The standard API method make use of dial readings at 6 & 3 revolution per minute (rpm) to determine yield stress (τ_0) and those at 600 and 300 rpm to determine the other two parameters. The HB model is currently recommended by API [2,4]. A more accurate method to determine the HB model parameters was suggested by several researchers [1,3]. These methods require trial-and-error solutions which are not a difficult task particularly with the advent of personal computers and their online use in the field. However, caution must be exercised when using such method as they may provide negative values for the yield stress which is meaningless. This study aims at investigating the effect of the simple procedure suggested by API on the predictability power of this model to the fluid rheology and settling velocities.

Hole cleaning has always been and will continue to be an integral part of the drilling process, but directional and horizontal drilling have elevated problems and concerns to levels not experienced during vertical drilling [2]. Slip velocity correlations have been developed in the past fifty years and recommendations of proper annular velocities have been suggested in order to ensure good hole cleaning. Several particle slip velocity correlations are available in the drilling literature, not the least are Chien [5], Zeidler [6], Moore [7], Walker and Mayes [8], Peden and Luo [9] and Chien [10]. Samble and Bourgoyne [11] have evaluated experimentally the correlations of Chien [5], Moore [7] and Walker & Mayes [8]. The experimental data used for this evaluation were obtained in Newtonian and non-Newtonian fluids for both static and flowing conditions. Among the evaluated correlations, the procedure proposed by Moore [7] gave the lowest average error for all fluids studied. Skalle et al. [12] pointed that Chien [10] and Walker Mayes [8] correlations still have a good sound in petroleum industry. Furthermore, Chien [10] correlation was adopted by the API as the recommended procedure for drilling fluids [13].

A major issue in the realm of particle settling velocity is the prediction of the drilling fluid's effective viscosity during the settling process. The apparent viscosity suggested by several researchers [5,7] represents the viscosity at the specific shear rate pertinent to that annular location in an annular flow situation, and does not necessarily represents the viscosity around the settling particles [9]. When fluid velocity approaches zero and the fluid becomes stagnant, apparent viscosity will approach infinity.

Moore [7] correlation employed the PL model parameters (K and n) that are basically determined at Fann viscometer readings of R_{600} and R_{300} to determine effective viscosity. Similarly Chien [5] correlation made use of BP model parameters (PV and YP) that are again determined from viscometer readings of R_{600} and R_{300}. However, API [14] methods for power law fluids recommend use of Fann viscometer readings of R_{100} and R_3 for calculating pressure loss inside the annulus. Becker and coworkers [15] stated that Fann rotary speed of 300 and 600 rpm induce shear rates higher than those that typically occur in actual drilling. Chien [10] emphasized that the effective viscosity working on the settling particles should be determined at the settling shear rate which is basically unknown. He recommended use of rheological parameters that are determined with a viscometer at low shear rates. However, Skalle et al. [12] stressed that the relative error is large at such low shear-stress readings. Muherei et al. [16] showed that different values of power law rheological parameters are resulted from using different rheometric data pairs for the same fluid. The effect was not small while re-calculating the fluid rheogram and predicting slip velocities, signifying thus the importance of making the best simulation of the rheological behavior of drilling fluids. Some uncommon data pairs represent very well the mud rheogram while other data pairs give good predictions to the settling velocities [16].

There is thus a debate as which of the Fann viscometer readings should be used to determine the more representing rheological parameters. The scope of this present work is to compare HB model with BP and PL models in terms of re-producing the rheology of example fluids as well as predicting particle dynamic settling velocities under similar conditions. Focus was also devoted to the effect of using different dial reading to determine the rheological parameters on predicting mud rheogram and particle settling velocities. The settling velocities according to Moore [7] as well as Chien [5,10] are computed using different previously estimated rheological parameters. Settling velocity values are plotted and compared with regard to observed settling velocity. The effect of rheological parameters on settling velocities are also investigated.

2. Settling velocity correlations

2.1. Stokes and Rittingers correlations

Traditionally, the settling velocity of a solid particle has been studied in stagnant fluids based on the force balance principle. The case of particle settling in a stagnant fluid is simplified by assuming that the particles are separated and do not interact with each other and that they are under the influence of gravity alone. Ideally there are two forces acting on such a particle; the force developed as a result of friction between the particle and the liquid (F_D) and the force due to the effective weight (F_{eff}) of the particle which is the difference between gravity and buoyancy forces, F_g *and* F_B, respectively. The effective force is described as follows:

$$F_{eff} = Vg(\rho_p - \rho_l) \tag{1}$$

Where:

F_{eff} is the effective force on particle, dyne

V is the particle volume, cm^3

g is the gravitational force = 981 cm/sec^2

ρ_p & ρ_l are the particle and liquid densities, respectively in g/cm^3

The fall of the particle in the fluid results in a resistant force called the fluid drag force. This frictional or drag force is difficult to quantify. Generally, it was found to be a function of the properties of both the liquid and the solid and can be expressed mathematically as:

$$F_D = C_D A_P \rho_l \frac{U_P{}^2}{2} \tag{2}$$

Where:

F_D is the drag force on particle, dyne

C_D is the drag coefficient, dimensionless

A_p is the particle projected area, cm^2

ρ_l is the liquid density in g/cm^3

U_p is the particle settling velocity, cm/sec

In accord to Newton's second Law, the forces should set equal to each other. For a spherical particles, the volume of the particle (V) and its projected area (A_P) can be expressed in terms of particle diameter and the following equation result:

$$U_P = \sqrt{\frac{4 \times g(\rho_p - \rho_l) D_p}{3 \times C_D . \rho_l}} \tag{3}$$

Equation (3) is the Newton's law for terminal settling velocity of a spherical particle. However, the drag coefficient in equation (3) has been found to be a function of the particle Reynolds number (N_{RP}). For very slow particle fall and laminar slip regime ($N_{RP} < 0.5$: $C_D = 24/N_{RP}$), Stokes arrived at the following expression [9]:

$$U_P = 100 \frac{g(\rho_p - \rho_l) {D_p}^2}{18\mu} \tag{4}$$

Where:

μ is the fluid viscosity in millipascal.sec (mPa.sec) or centipoise (cP)

D_p is the particle diameter, cm

Equation (4) is the Stokes law for the terminal settling velocity of a spherical particle under laminar flow conditions. In turbulent flow ($N_{RP} > 1000$: $C_D = 0.5$) the drag coefficient becomes constant and Rittinger's equation may be used:

$$U_P = 51.15 \sqrt{\frac{(\rho_p - \rho_l) D_p}{\rho_l}} \tag{5}$$

2.2. Moore correlation

Moore [7] suggested use of an apparent Newtonian viscosity obtained by equating the laminar flow frictional pressure losses in the Newtonian model with frictional pressure losses in the power law fluid model; the following expression was resulted:

$$\mu_a = \frac{K}{11.975} \left(\frac{(d_H - d_P)}{U_a} \right)^{(1-n)} \left(\frac{2 + \frac{1}{n}}{0.25} \right)^n \tag{6}$$

Where:

μ_a is the fluid apparent viscosity, mPa.sec

K is the PL consistency index, $Pa.sec^n$

n is the PL flow behavior index, dimensionless

d_H is the wellbore diameter, cm

d_p is the drillpipe diameter, cm

U_a is the fluid annular velocity, cm/sec

This correlation is for spherical particles and use average laminar flow velocity but does not account for the effect of particle slippage upon fluid shear stress. Moore [7] pointed that for fully turbulent flow ($N_{RP} = 2000$), the friction factor remains constant at a value of about 1.5. Substituting this value in Newton's equation (eq. 3), the following relation was obtained:

$$U_p = 29.53\sqrt{D_p\left(\frac{\rho_p - \rho_l}{\rho_l}\right)} \tag{7}$$

For laminar flow ($N_{RP} \leq 1$) the drag coefficient has been set equal to $40/N_{RP}$ [7], substitution in equation (3) gives:

$$U_p = 3270\frac{D_p^{\,2}}{\mu_a}\left(\rho_p - \rho_l\right) \tag{8}$$

For particle Reynolds number in the range of 10 to 100, the drag coefficient should be set equal to $= 22/(N_{RP})^{0.5}$ [7]. Similarly by substituting this value in equation (3), the following equation was attained:

$$U_p = 15.233\frac{D_p}{\mu_a^{0.333}}\frac{\left(\rho_p - \rho_l\right)^{0.667}}{\rho_l^{0.333}} \tag{9}$$

2.3. Chien correlations

Chien [5,10] presented two empirical correlations for the settling velocity of drill cuttings for rotary drilling operations. Both for determination of the settling velocity of cuttings in all slip regimes. In 1972, Chien [5] suggested use of a drag coefficient which equals to $24/N_{RP} + 1.714$. Substituting these values in equation (3), the following equation resulted:

$$U_p = 0.07\times\left(\frac{\mu_a}{\rho_l D_p}\right)\times\left[\sqrt{\frac{152437\times D_p}{\left(\frac{\mu_a}{\rho_l D_p}\right)^2}\left(\frac{\rho_p - \rho_l}{\rho_l}\right) + 1} - 1\right] \tag{10}$$

For mixtures of bentonite and water, Chien [5] suggested that the plastic viscosity can be used as the effective viscosity, while for polymer-based drilling fluids; the effective viscosity is calculated as shown below [5]:

$$\mu_a = PV + 117\frac{YP \times D_p}{U_a} \tag{11}$$

Where:
PV is the BP plastic viscosity, mPa.sec
YP is the BP yield point, Pa

For turbulent flow regime, Chien [5] suggested that the drag coefficient is 1.72 ($C_D = 1.72$). Chein [5] substituted this value in equation (3) and arrived at the following equation:

$$U_p = 27.58\sqrt{D_p\left(\frac{\rho_p - \rho_l}{\rho_l}\right)} \tag{12}$$

However, in a later publication Chien [10] emphasized on particle shape factor or sphericity (ψ). Particle sphericity is the ratio between the surface area of a sphere of volume equal to that of the particle and the surface area of the particle. He [10] developed a correlation to predict settling velocity of irregularly shaped particles in Newtonian and non-Newtonian fluids for all types of slip regimes. In this correlation, the drag coefficient is approximated by: $C_D = 30/N_{RP} + 67.289/\ e^{5.03\psi}$. Substituting this values in equation (3), He [8] obtained the following equation:

$$U_p = 0.00223 \times e^{5.03\psi}\ \frac{\mu_e}{\rho_l D_p} \times \left[\sqrt{\frac{3913210 \times e^{5.03\psi} D_p}{\left(\frac{\mu_e}{\rho_l D_p}\right)^2}\left(\frac{\rho_p - \rho_l}{\rho_l}\right) + 1} - 1\right] \tag{13}$$

Where: μ_e is the fluid effective viscosity, mPa.sec

3. Rheological models

In conventional drilling, drilling fluids are modelled with classical rheological models like Bingham plastic (BP) or power law (PL) model and fluid behavior is defined with only two points of the rheological relation (R_{600} and R_{300}). Nowadays, API recommend use of Herschle-Bulkley (HB) model [1]. The BP model is widely used in the drilling fluid industry to describe characteristics of many types of drilling fluids. Fluids obeying this model exhibit a linear shear-stress/shear-rate behavior after an initial shear stress (YP) threshold has been exceeded. Generally, the descriptive formula can be written as follows:

$$\tau = YP + PV(\gamma) \tag{14}$$

The term "YP" is the yield point which is the threshold stress (intercept) and "PV" is the plastic viscosity demonstrated by the slope of the line. However, the PL rheological model receives great attentions because it describes the behavior of polymeric fluids better than the BP model [1]. The PL model can be expressed by:

$$\tau = K\gamma^n \tag{15}$$

The term "K" is defined as the fluid consistency index and describes the thickness of the drilling fluid. The exponent "n" is called flow behavior index. There is no term for yield point and fluids that follow this model have no shear stress when shear rate is zero. The flow behavior index "n" indicate the degree of non-Newtonian. Unlike, the former rheological models which have two rheological parameters, the HB have three controlling parameters. The HB model can be mathematically described by:

$$\tau = \tau_0 + K_{HB}\gamma^{\beta} \tag{16}$$

The terms "K_{HB}" and "β" are similar to those of power law model "K" and "n", respectively. However, the calculated values will be different because of the presence of yield stress. The parameter "τ_0" is the fluid yield stress at zero shear rate. In theory this yield stress is identical to the Bingham plastic yield point (YP) though it is calculated value is always smaller. The Herschle-Bulkley includes the power law and Bingham plastic model as special cases. It reduces to Bingham plastic model if "n = 1"; and turns to power law model if "τ_0 = 0".

3.1. Example drilling fluids

The example drilling fluids was selected from pioneered study of Sifferman et al. [17] on dynamic settling velocity. In their [17] study drill cutting transport was studied in a full scale vertical annuli. Two of Sifferman et al [17] drilling fluids are selected, i.e. the intermediate and thin. Both fluids have a similar density (1.44 g/cm^3; 12 ppg (pound per gallon)).

Table 1: drilling fluid and particle properties [11,17,18]

Fluid Rheology	RPM	600	300	200	100	6	3
	Shear Rate, sec^{-1}	1022	511	341	170	10	5
Intermediate	Dial Reading	49	35	30	25	15	13
	Shear Stress, lb/100ft^2	52.3	37.4	32	26.7	16	13.9
	Shear Stress, Pa	25	17.9	15.3	12.8	7.7	6.6
Thin	Dial Reading	24	16	13	10	3	3
	Shear Stress, lb/100ft2	25.6	17.1	13.9	10.7	3.2	3.2
	Shear Stress, Pa	12.3	8.2	6.6	5.11	1.5	1.5
Other Properties	Fluid Density, g/cm^3	1.44	Particle density, g/cm^3			2.4	
	Particle equivalent diameter, cm	0.4963	Particle Sphericity			0.77	
	Drillpipe size, cm	8.89	Casing size			30.48	

Mud type	Intermediate			Thin		
Annular mud velocity, cm/sec	20.32	15.24	10.16	20.32	15.24	10.16
Particle slip velocity, cm/sec	5.28	5.03	4.67	9.75	9.60	10.16

The six Fann 35 VG meter dial readings and corresponding revolutions per minute as well as other fluid and particle (medium) properties are listed in Table-1 [11,17,18]. Observed particle slip velocities at different annular velocities for the two fluids are also shown in Table-1 [11].

3.2. Calculation of rheological parameters

To calculate power law "n" and "K" values as well as Bingham plastic "PV" and "YP", a mud's Fann 35 VG meter dial readings and corresponding revolutions per minute are required. Two data pairs are required for a solution. Generally, R_{600}/R_{300}, R_{100}/R_3, R_6/R_3 and are in common use. However, for HB rheological parameters, API [4] recommended use of R_6/R_3 for calculating yield shear stress (τ_0). Estimated yield stress and R_{600}/R_{300} are involved in calculating "β" and "K_{HB}" for HB model. This study employs common and recommended data pairs and other different data pairs for the calculation of these rheological parameters. Equations 17 and 18 are general equations for determining the PL flow behavior index and consistency index, respectively. Similarly, equation 19 and 20 are general equations for determining the BP plastic viscosity and yield point, respectively. Equation 21-23 are API equations recommended for calculating HB rheological parameters.

$$n = \frac{\log(\tau_2/\tau_1)}{\log(\gamma_2/\gamma_1)} \tag{17}$$

Where:
τ_1 & τ_2 *are the shear stress at lower and higher shear rates, respectively, Pa*
γ_1 & γ_2 *are the lower and higher shear rates, sec*$^{-1}$

$$K = \frac{\tau_2}{\gamma_2^{\,n}} \tag{18}$$

$$PV = \frac{(\tau_2 - \tau_1)}{(\gamma_2 - \gamma_1)} \tag{19}$$

$$YP = \left[\tau_1 - \frac{\gamma_1 \times (\tau_2 - \tau_1)}{(\gamma_2 - \gamma_1)}\right] \tag{20}$$

$$\tau_0 = 2\tau_3 - \tau_6 \tag{21}$$

Where:
τ_3 & τ_6 *are the shear stress at 3 and 6 rpm, respectively, Pa*

$$\beta = \frac{\log[(\tau_{600} - \tau_0)/(\tau_{300} - \tau_0)]}{\log(600/300)} \tag{22}$$

Where:
τ_{300} & τ_{600} *are the shear stress at 300 and 600 rpm, respectively, Pa*

$$K_{HB} = \frac{(\tau_{600} - \tau_0)}{1022^n} \tag{23}$$

A more accurate method to calculate HB rheological parameters involves use of all rheometric data (Fann 35 VG 6 dial readings and their corresponding shear rates). Herschle-Bulkley is a nonlinear model, hence a computer algorithm that employs a least squares method is required to calculate these parameters. Generally, three partial derivatives that minimize the absolute error are solved to produce the following equations [1]:

$$\tau_0 = \frac{\sum \tau_i \sum \gamma_i^{2\beta} - \sum \tau_i \gamma_i^{\beta} \sum \gamma_i^{\beta}}{N \sum \gamma_i^{2\beta} - \left(\sum \gamma_i^{\beta}\right)^2} \tag{24}$$

$$K_{HB} = \frac{N \sum \tau_i \gamma_i^{\beta} - \sum \tau_i \sum \gamma_i^{\beta}}{N \sum \gamma_i^{2\beta} - \left(\sum \gamma_i^{\beta}\right)^2} \tag{25}$$

$$Error = \tau_0 \sum \gamma_i^{\beta} \ln \gamma_i^{\beta} + K_{HB} \sum \gamma_i^{2\beta} \ln \gamma_i - \sum \tau_i \gamma_i^{\beta} \ln \gamma_i \tag{26}$$

As has been mentioned earlier, determination of these parameters using standard techniques can sometimes yield negative values of yield stress. To avoid this, an iterative procedure was written in Microsoft excel that could reject the situation of a negative yield stress.

4. Result and discussion

4.1. Predicting mud rheogram

Tables 2-3 are a statistical comparison of the accuracy of each model to predict shear stress for the example fluids. Tables 2 (A, B and C) contain the values of the Sifferman et al [17] intermediate mud rheological parameters (n, K, YP, PV, τ_0, K_{HB}, β) calculated using different rheometric data. The shear stress at six Fann 35 VG rpm are calculated again using these rheological parameters. While, Tables 3 (A, B and C) contain the rheological parameters for Sifferman et al [17] thin mud.

Referring to Table 2 and 3, it is obvious that the HB model represents the full rheological spectrum of the example muds with the lowest average error and standard deviation, particularly when all rheometric data were used to derive its rheological parameters. Remarkably, the common data pairs (R_{600}/R_{300}, R_{100}/R_3 and R_6/R_3) when used to derive the rheological parameters of the Bingham plastic and power law models resulted in the highest average error and standard deviation for both fluids. Nonetheless, the uncommon data pairs give similar average error and standard deviation to those of the rheological parameters derived from linear (BP) and nonlinear regression (PL) of all rheometric data particularly for the intermediated mud (Tables 2 (A, B & C)). Unacceptable average error and standard deviations are shown when using low shear rate data (R_6/R_3) for both muds when modeled as Bingham plastics. For the intermediate mud, the average error is about 308% (SD = 295) while for thin mud an average error of 53% (SD = 41) is observed.

Table 2A: Intermediate mud as a power law

Data Pair		R_{600}/R_{300}		R_{100}/R_3		R_6/R_3		R_{300}/R_6		R_{300}/R_3		NLR	
n		0.4851		0.1867		0.208		0.2165		0.2152		0.2282	
K, $Pa.sec^n$		0.8685		4.8974		4.7298		4.6372		4.6749		4.41	
Measured Shear Stress, Pa		Calculated Shear Stress, Pa											
rpm	τ	τ	%E	τ	%E	τ	%E	τ	%E	τ	%E	τ	%E
600	25.04	25.04	0	17.86	-28.67	19.99	-20.17	20.79	-16.97	20.77	-17.05	21.44	-14.38
300	17.89	17.89	0	15.69	-12.3	17.31	-3.24	17.89	0	17.89	0	18.3	2.29
200	15.33	14.69	-4.17	14.55	-5.09	15.91	3.78	16.39	6.91	16.39	6.91	16.68	8.81
100	12.78	10.5	-17.84	12.78	0	13.77	7.75	14.1	10.33	14.12	10.49	14.24	11.42
6	7.67	2.68	-65.06	7.56	-1.43	7.67	0	7.67	0	7.71	0.52	7.5	-2.22
3	6.64	1.92	-71.08	6.64	0	6.64	0	6.6	-0.6	6.64	0	6.4	-3.61
Average Error, %			26.36		7.92		1.98		0.05		0.15		0.39
Standard Deviation			33.02		11.18		9.68		9.41		9.47		9.35

- Generally, use of linear regression (LR) rheological parameters of BP and non-linear regression (NLR) rheological parameter of PL described the mud rheograms with reasonable accuracy compared to other data pairs. Furthermore, HB model surpasses other models for describing the rheological behavior of both fluids. However, BP R_{300}/R_6 and BP R_{300}/R_3 as well as PL NLR, PL R_{300}/R_6 and PL R_{300}/R_3 provided lower average error than HB API standard method. Moreover, when re-calculating the intermediate mud rheogram HB R_{600}/R_{200}, HB R_{600}/R_{100} and HB R_{300}/R_{100} obtained lower average error and standard deviation than the API R_{600}/R_{300}. This indicated that the simplified API method to determine HB rheological parameters may lead to considerable errors that may render the model performance lower than the other models.

Table 2B: Intermediate mud as a Bingham plastic

Data Pair		R_{600}/R_{300}		R_{100}/R_3		R_6/R_3		R_{300}/R_6		R_{300}/R_3		LR	
YP, Pa		10.74		6.4501		5.61		7.4614		6.5264		8.2507	
PV, Pa.sec		0.014		0.0372		0.2016		0.0204		0.0222		0.0174	
Measured Shear Stress, Pa		Calculated Shear Stress, Pa											
rpm	τ	τ	%E	τ	%E	τ	%E	τ	%E	τ	%E	τ	%E
600	25.04	25.05	0.04	44.46	77.56	211.6	745.05	28.31	13.06	29.21	16.65	26.03	3.95
300	17.89	17.89	0	25.46	42.31	108.61	507.1	17.88	-0.06	17.87	-0.11	17.14	-4.19
200	15.33	15.51	1.17	19.12	24.72	74.27	384.47	14.41	-6	14.09	-8.09	14.18	-7.5
100	12.78	13.12	2.66	12.79	0.08	39.94	212.52	10.94	-14.4	10.31	-19.33	11.21	-12.28
6	7.67	10.88	41.85	6.83	-10.95	7.67	0	7.67	0	6.75	-11.99	8.43	9.91
3	6.64	10.81	62.8	6.64	0	6.64	0	7.57	14.01	6.64	0	8.34	25.6
Average Error, %			18.09		22.29		308.2		1.1		3.81		2.58
Standard Deviation			27.31		33.34		295.1		10.98		12.43		13.81

4.2. Settling velocity

The settling problem of drilled cuttings in drilling fluids is complicated by the non-Newtonian behavior of these fluids, i.e. their shear-dependent viscosities. Theoretically the viscosity affecting the particle settling velocity in a non-Newtonian fluid should be that of the fluid envelop surrounding the particle and this viscosity depends on the shear rate distribution around the particle. Particle settling velocities are computed according to the Moore [7] and Chien [5,10] correlations and compared to the observed settling velocities. It should be pointed out that Moore [7] and Chien [5] suggested an apparent viscosity which depend on annular fluid flow. Accordingly, these viscosities are functions of the fluid velocity and are independent of the particle settling velocity. If the fluid velocity approaches zero, i.e. the fluid is stagnant, the apparent viscosity for annular pipe flow will approach infinity and the settling velocity would be zero. This hinders application of these correlations for predicting settling velocities of particles in quiescent fluids. Moreover, Moore [7] apparent viscosity employs PL rheological parameters while Chien [5] apparent viscosity involves BP rheological parameter. Therefore, these two correlations are not suitable to compare different models but are good for comparing different rheological parameters of either Bingham plastic or power law which are derived by different set of rheometric data.

Table 2C: Intermediate mud as a Herschle-Bulkley

Data Pair		API-R_{600}/R_{300}		API-R_{600}/R_{200}		API-R_{600}/R_{100}		API-R_{300}/R_{100}		API-R_{200}/R_{100}		All 6R	
β		0.662		0.6305		0.5564		0.4898		0.439		0.638	
K_{HB}, Pa.sec^n		0.1978		0.2461		0.4112		0.579		0.7516		0.2203	
τ_0, Pa		5.61		5.61		5.61		5.61		5.61		6.4034	
Measured Shear Stress, Pa		Calculated Shear Stress, Pa											
rpm	τ	τ	%E	τ	%E	τ	%E	τ	%E	τ	%E	τ	%E
600	25.04	25.04	0	25.04	0	25.04	0	22.86	-8.71	21.35	-14.74	24.73	-1.24
300	17.89	17.89	0	18.16	1.51	18.82	5.2	17.89	0	17.22	-3.75	18.18	1.62
200	15.33	15	-2.15	15.33	0	16.15	5.35	15.68	2.28	15.33	0	15.49	1.04
100	12.78	11.54	-9.7	11.89	-6.96	12.78	0	12.78	0	12.78	0	12.24	-4.23
6	7.67	6.53	-14.86	6.68	-12.91	7.11	-7.3	7.42	-3.26	7.69	0.26	7.37	-3.91
3	6.64	6.19	-6.78	6.3	-5.12	6.63	-0.15	6.9	3.92	7.15	7.68	7.03	5.87
Average Error, %			5.58		3.91		0.52		0.96		1.76		0.14
Standard Deviation			5.98		5.51		4.63		4.51		7.37		3.81

4.2.1. Moore correlation

Figure 1 (A & B) are dynamic settling velocities of the particle in both fluids while modelling the fluids as power law fluids using Moore correlation. As seen in Figure 1 (A & B), the dynamic particle settling velocities at three different annular velocities are determined using different rheometric data including non-linear regression (NLR) rheological data.

It is evident that all types of data used in this figure significantly underestimate the observed settling velocity. It is also clear that the rheological parameter derived from R_{600}/R_{300} giving better results. The observed particle settling velocity in the intermediated mud were 5.28, 5.03 and 4.67 cm/sec when the annular velocities were 20, 15 and 10 cm/sec, respectively. Similarly, the observed particle settling velocity in the thin mud were 9.75, 9.6 and 10.16 cm/sec when the annular velocities were 20, 15 and 10 cm/sec, respectively. The absolute error while using R_{600}/R_{300} data was 37% for the intermediate mud and 19% for the thin mud. For the intermediate mud all other data were performing similarly with an absolute error approaching 78%. Similarly, the absolute error for all data excluding R_{600}/R_{300} and R_6/R_3 in the thin mud was revolving between 41% - 54%. It was not possible to estimate the settling velocity using R_6/R_3 data of the thin mud because both readings have identical value at 1.53 Pa (PV = 0). Finally, it could be noticed that both the observed and the estimated Moore [7] settling velocities are not very sensitive to annular velocities.

Table 3A: Thin mud as a power law

Data Pair		R_{600}/R_{300}		R_{100}/R_3		R_6/R_3		R_{300}/R_6		R_{300}/R_3		NLR	
n		0.5838		0.3439		0		0.4285		0.364		0.399	
K, Pa.sec^n		0.2146		0.8732		1.53		0.5652		0.8451		0.6901	
Measured Shear Stress, Pa		Calculated Shear Stress, Pa											
rpm	τ	τ	%E	τ	%E	τ	%E	τ	%E	τ	%E	τ	%E
600	12.26	12.26	0	9.46	-22.84	1.53	-87.52	11.01	-10.2	10.53	-14.11	10.96	-10.6
300	8.18	8.18	0	7.46	-8.8	1.53	-81.3	8.18	0	8.18	0	8.31	1.59
200	6.64	6.46	-2.71	6.49	-2.26	1.53	-76.96	6.87	3.46	7.06	6.33	7.07	6.48
100	5.11	4.31	-15.66	5.11	0	1.53	-70.06	5.11	0	5.48	7.24	5.36	4.89
6	1.53	0.83	-45.75	1.94	26.8	1.53	0	1.53	0	1.97	28.76	1.74	13.73
3	1.53	0.56	-63.4	1.53	0	1.53	0	1.14	-25.49	1.53	0	1.32	-13.73
Average Error, %			21.25		1.18		52.64		5.37		4.7		0.39
Standard Deviation			27.04		16.21		41.15		10.89		14.04		10.55

Table 3B: Thin mud as a Bingham plastic

Data Pair		R_{600}/R_{300}		R_{100}/R_3		R_6/R_3		R_{300}/R_6		R_{300}/R_3		LR	
YP, Pa		4.1		1.4193		1.53		1.3943		1.4628		2.2919	
PV, Pa.sec		0.008		0.0217		0		0.0133		0.0131		0.0104	
Measured Shear Stress, Pa		Calculated Shear Stress, Pa											
rpm	τ	τ	%E	τ	%E	τ	%E	τ	%E	τ	%E	τ	%E
600	12.26	12.27	0.08	23.59	92.41	1.53	-87.52	14.98	22.19	14.85	21.13	12.92	5.38
300	8.18	8.19	0.12	12.51	52.93	1.53	-81.3	8.19	0.12	8.16	-0.24	7.61	-6.97
200	6.64	6.82	2.71	8.81	32.68	1.53	-76.96	5.92	-10.84	5.92	-10.84	5.83	-12.2
100	5.11	5.46	6.85	5.11	0	1.53	-70.06	3.66	-28.38	3.69	-27.79	4.06	-20.55
6	1.53	4.18	173.2	1.64	7.19	1.53	0	1.53	0	1.6	4.58	2.4	56.86
3	1.53	4.14	170.59	1.53	0	1.53	0	1.46	-4.58	1.53	0	2.35	53.59
Average Error, %			58.93		30.87		52.64		3.58		2.19		12.69
Standard Deviation			87.54		36.73		41.17		16.49		16.3		34.02

Table 3C: Thin mud as a Herschle-Bulkley

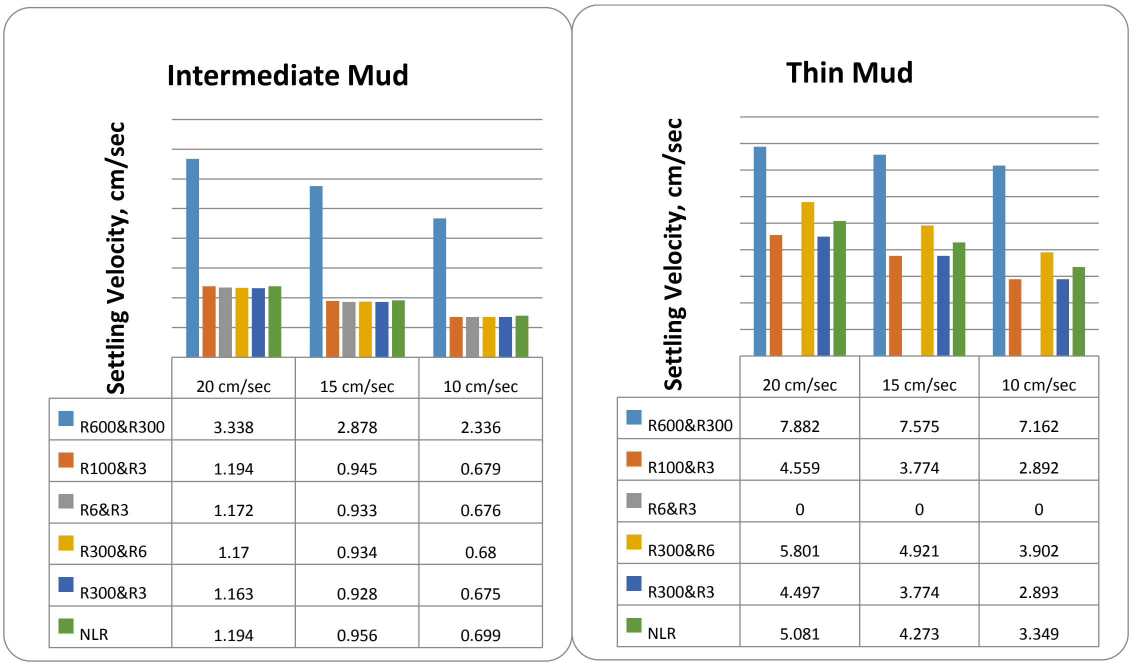
Data Pair		API-R_{600}/R_{300}		API-R_{600}/R_{200}		API-R_{600}/R_{100}		API-R_{300}/R_{100}		API-R_{200}/R_{100}		All 6R	
β		0.6902		0.6753		0.6126		0.5637		0.5134		0.601	
K_{HB}, Pa.sec^n		0.0899		0.0996		0.1538		0.1978		0.2561		0.1737	
τ_0, Pa		1.53		1.53		1.53		1.53		1.53		0.993	
Measured Shear Stress, Pa		Calculated Shear Stress, Pa											
rpm	τ	τ	%E	τ	%E	τ	%E	τ	%E	τ	%E	τ	%E
600	12.26	12.27	0.08	12.26	0	12.26	0	11.36	-7.34	10.51	-14.27	12.17	-0.73
300	8.18	8.18	0	8.25	0.86	8.55	4.52	8.18	0	7.82	-4.4	8.36	2.2
200	6.64	6.56	-1.2	6.64	0	7	5.42	6.82	2.71	6.64	0	6.77	1.96
100	5.11	4.65	-9	4.73	-7.44	5.11	0	5.11	0	5.11	0	4.8	-6.07
6	1.53	1.98	29.41	2.01	31.37	2.17	41.83	2.26	47.71	2.37	54.9	1.7	11.11
3	1.53	1.81	18.3	1.83	19.61	1.95	27.45	2.03	32.68	2.12	38.56	1.46	-4.58
Average Error, %			6.27		7.4		13.2		12.63		12.47		0.65
Standard Deviation			14.47		14.81		17.36		22.13		27.54		6.13

4.2.2. Chien (1972) correlation

Dynamic particle settling velocities are calculated by Chien [5] correlation for both fluids and are depicted in Figure 3 (A & B). It is clear that this correlation significantly overestimate the observed settling velocities. All data excluding R_6/R_3 are performing similarly with an absolute error spanning between 148% - 167% for the intermediate mud and 57% - 65% for the thin mud. Data of R_6/R_3 are not stable since it shows the lowest error (22%) for the intermediate mud and the highest error (82%) for the thin mud. Likewise those of Moore [7], the settling velocities of Chien [5] correlation are also not widely affected by annular fluid velocities.

4.2.3. Chien (1994) correlation

The effective viscosity suggested by Chien [10] is determined at the settling shear rate. The settling shear rate is the ratio of particle settling velocity to its diameter. The effective viscosity of fluid is equal to shear stress at that shear rate (BP: $\mu_e = YP/\gamma + PV$; PL: $\mu_e = K\gamma^{n-1}$; HB: $\mu_e = \tau/\gamma + K_{HB}\,\gamma^{\beta-1}$).



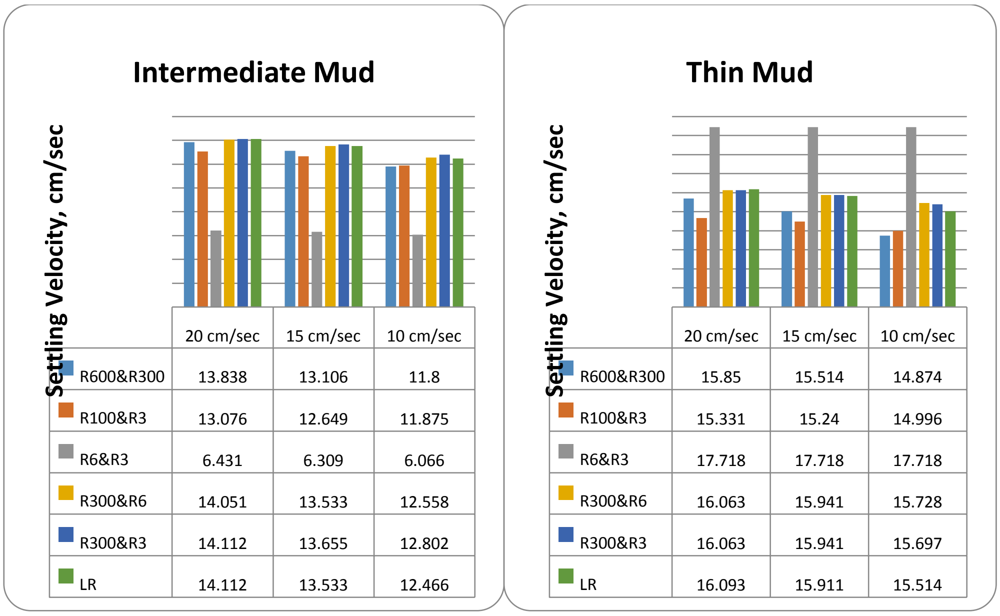
Intermediate Mud	20 cm/sec	15 cm/sec	10 cm/sec
R600&R300	3.338	2.878	2.336
R100&R3	1.194	0.945	0.679
R6&R3	1.172	0.933	0.676
R300&R6	1.17	0.934	0.68
R300&R3	1.163	0.928	0.675
NLR	1.194	0.956	0.699

Thin Mud	20 cm/sec	15 cm/sec	10 cm/sec
R600&R300	7.882	7.575	7.162
R100&R3	4.559	3.774	2.892
R6&R3	0	0	0
R300&R6	5.801	4.921	3.902
R300&R3	4.497	3.774	2.893
NLR	5.081	4.273	3.349

(A) (B)

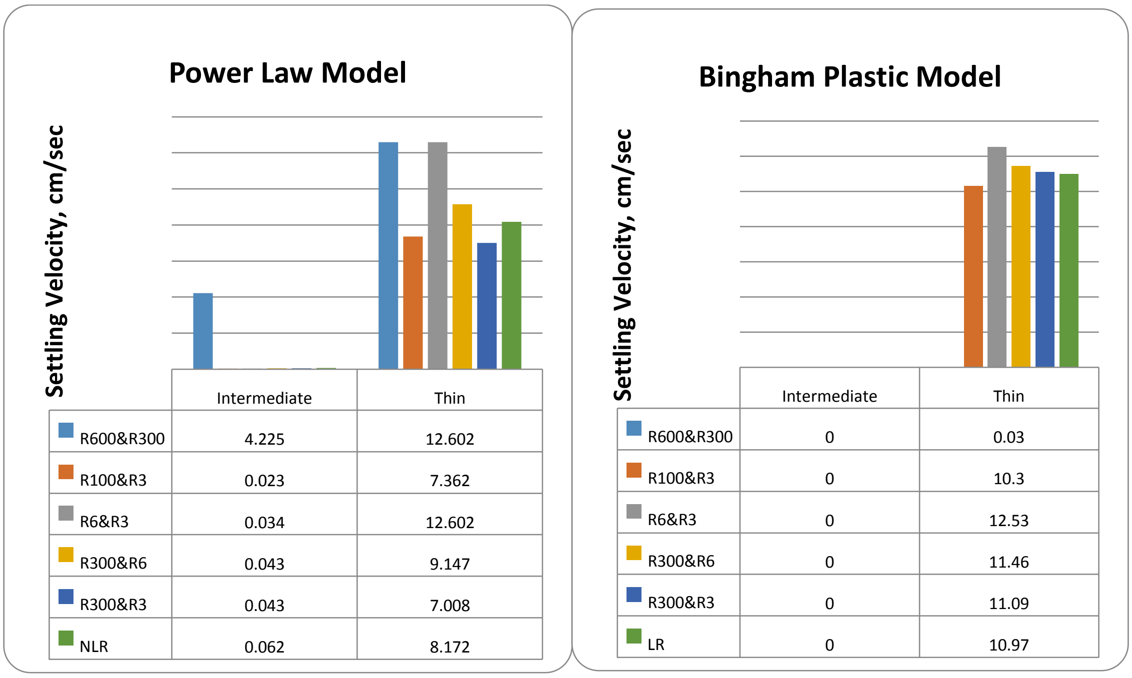
Figure 1: Moore [7] dynamic particle settling velocities – PL rheological parameters

Since the shear rate prevailing while the particle is settling out of the fluid is not known, a trial and error or numerical iteration method is required to predict the particle settling velocity in non-Newtonian fluid. Simple iteration methods were written in Microsoft Excel to facilitate calculation. For non-Newtonian fluids, viscosity depends on the shear rate and knowledge of the settling shear rate is important for evaluating the viscous forces experienced by the particle. In the turbulent regime the viscosity has a minor effect on drag force; therefore the settling shear rate has no important role in turbulent slip. However, in contrast to the previous suggestions [5,7], Chien [10] suggestion depends only upon the particle settling velocity and is independent of the fluid velocity.



(A) (B)

Figure 2: Chien [5] dynamic particle settling velocities – BP rheological parameters



(A) (B)

Figure 3: Chien [10] static particle settling velocities BP & PL models

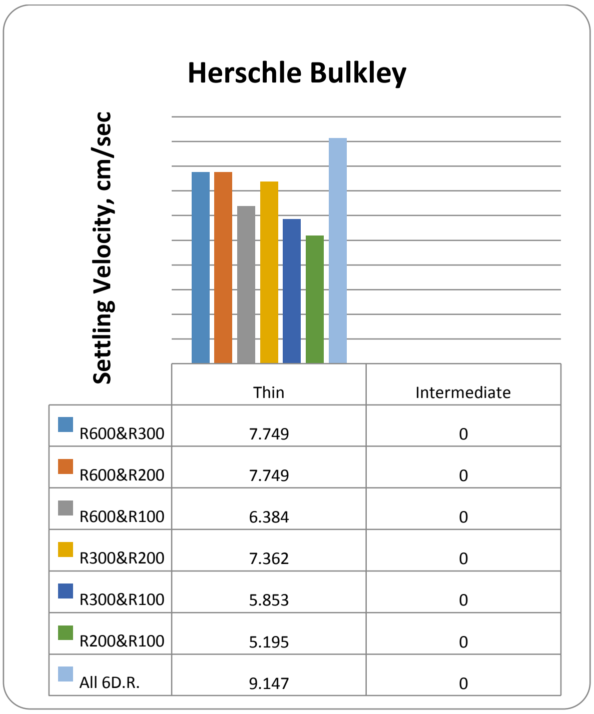
It has been demonstrated previously that the particle settling velocities as determined by Moore [7] and Chien [5] were not very sensitive to the annular velocities prevailed during the settling process. Sifferman et al [17] as well as Sample and Bourgoyne [11] observed through experimental work that the particle settling velocities are not significantly affected by the rate at which the fluid is moving in the annulus. Accordingly, Chien [10] settling velocity correlation which do not take into account the annular velocity may work better also for dynamic particle settling velocities. Figure 3 (A & B) shows the particle settling velocities estimated by Chien [10] correlation for both fluids simulated as to follow either the power law model (Figure 3-A) or the Bingham plastic model (Figure 3-B).

Clearly, the Chien [10] correlation is sensitive to mud rheology. The correlation was not able to predict the settling velocities of the intermediate mud as the resulted effective viscosities was significantly large to get any settling through this mud. Doubtless, simulating the thin mud using both fluid models obtained widely different settling velocity estimations particularly for R_{600}/R_{300} data. While the power law model using this pair of data significantly overestimated (+29% Error) the observed settling velocity, the Bingham plastic model for the same pair of data could not estimate the observed settling velocity. Amazingly, R_{600}/R_{300} and R_6/R_3 data yielded the same settling velocity for power law model and thin fluid. Moreover, R_6/R_3 data obtained nearly the same settling velocity (12.6 vs. 12.5) for the thin mud using both models (PL and BP). Obviously, R_{300}/R_6 data exceled other data (-6% Error) in predicting particle settling velocity for the power law model followed by NLR data (-16% Error) and R_{100}/R_3 data (-25% Error). Excluding R_{600}/R_{300} and R_6/R_3 data all remained data underestimate the observed settling velocity using power law fluid and overestimate the observed settling velocity using Bingham plastic model. For Bingham plastic model, the lowest error was shown with R_{100}/R_3 data (+6% Error) followed by LR data (+13% Error) and R_{300}/R_3 data (+14% Error). For the same model, the highest error was registered by R_6/R_3 data (+29% Error) followed by R_{300}/R_6 (+ 18% Error). However, it is important to point out that these estimations using Chien [10] correlation for both power law and Bingham plastic obtained settling velocities with significantly lower errors than settling velocity estimations obtained by Moore [7] and Chien [5] correlations.

Settling velocities for HB model were plotted in Figure 4 for both fluids. Again, Chien [10] correlation could not estimate the settling velocities of the intermediate mud. It appears that this is a significant drawback adhered to this correlation. As it is the case with power law model all data of Figure 4 underestimate the observed settling velocity.

As seen in Figure 4, the rheological parameters determined by trial and error calculations showed the lowest error (-6% Error). This is exactly the same value obtained by R_{300}/R_6 using power law model. The simplified method of adopted by API to determine HB rheological parameters significantly underestimate the observed settling velocity (-21% Error). This error is greater even with some data of PL and BP models. The investigation of other rheometric data besides API standard method do not show improvements (lower errors than API suggested standard).

It was mentioned earlier that Chien [10] settling velocity is dependent on the particle settling shear rate and independent of annular fluid velocities. Some experimental results [11] showed that the particle settling velocity is basically independent of the fluid velocity. But some investigators [19] thought that the particle should settle faster in dynamic fluids than in stagnant fluids.



	Thin	Intermediate
R600&R300	7.749	0
R600&R200	7.749	0
R600&R100	6.384	0
R300&R200	7.362	0
R300&R100	5.853	0
R200&R100	5.195	0
All 6D.R.	9.147	0

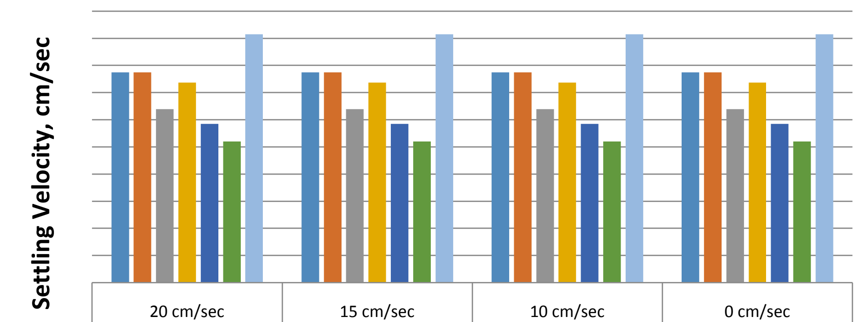
Figure 4: Chien [10] static particle settling velocities HB model

It is known that the viscosity of non-Newtonian fluids depend on the shear rates. In case, if settling happens in a quiescent non-Newtonian fluids, the shear rate on a particle equals to the particle slip velocity divided by its diameter (U_p/D_p) [10,19,20]. For flowing fluids, experimental results of Novotny [20] on proppant particles using Newtonian fluids showed that the shear rate imposed on the particle did not affect the slip velocity. Guliyev [19] suggested that the case of non-Newtonian fluids may be different and employed a shear rate which is a vector sum of the shear rates due to particle sinking (U_p/D_p) and the shear rate imposed by the fluid (γ_f). The total shear rate (γ_t) is defined by Guliyev [19] as: $\gamma_t = ((U_p/D_p)^2 + \gamma_f^{\ 2})^{0.5}$; $\gamma_f = U_a/D_e$). It is hypothesized in this work that by substituting this total shear rate in place of slip velocity shear rate employed by Chien [10] correlation, it is possible to get good estimations for dynamic particle settling velocities. The current estimations are lower than observed settling velocities, thus by incorporating the effect of annular fluid shear rates, effective viscosities become lower and particles will settle faster. The effective viscosity of the HB model can be expressed as: $\mu_e = (\tau/\gamma_t + K_{HB}\, \gamma_t^{\ \beta-1})$. The results of this modification are shown in Figure 5.

Unfortunately, the estimated settling velocities are exactly the same as previously obtained (Figure 5). The shear rate of the annular fluid does not add to the settling behaviour of particles. One reason may be the fact that the shear rate employed in Chien [10] correlation was determined by trial and error and does not differentiate between the shear rate due to particle settling and that due to annular fluid velocity.

4.2.4. Effect of rheological parameters on settling velocity

The effect of flow behaviour index and consistency index for power law and Herschle-Bulkley models are shown in Figure 6 and 7, respectively. It is evident from Figure 6 that the relationship between the flow behaviour index and estimated settling velocities is consistent with settling velocity decreased as the flow behaviour index decreased. The upper trend for PL "n" and the lower trend for HB "β". As the flow behaviour index decreased the fluid behaves more like non-Newtonian fluids.



	20 cm/sec	15 cm/sec	10 cm/sec	0 cm/sec
R600&R300	7.749	7.749	7.749	7.749
R600&R200	7.749	7.749	7.749	7.749
R600&R100	6.384	6.384	6.384	6.384
R300&R200	7.362	7.362	7.362	7.362
R300&R100	5.853	5.853	5.853	5.853
R200&R100	5.195	5.195	5.195	5.195
All 6D.R.	9.147	9.147	9.147	9.147

Figure 5: Chien [10] dynamic particle settling velocities in thin mud - HB model

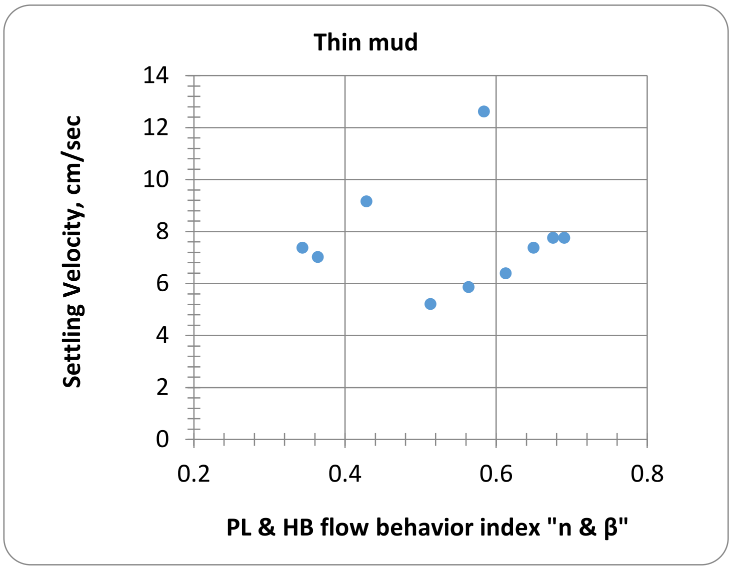


Figure 6: Effect of flow behaviour index on settling velocity

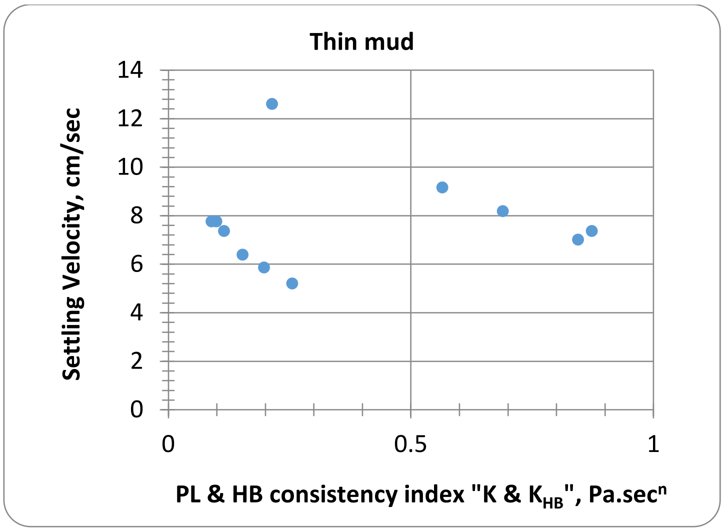


Figure 7: Effect of consistency index on settling velocity

Obviously, the trend reverses for the consistency index (Figure 7) with the estimated settling velocities decrease with an increase in consistency index. Again, the upper trend for PL consistency index "K" and the lower trend for HB consistency index "K_{HB}". Similar results are obtained by Muherei and Basaleh [16] for power law fluids. It is to be noted that the HB yield stress is constant in previous cases (1.53 Pa), hence it was not possible to investigate its effect on estimated settling velocities. However, Kenny et al. [21] stressed on the importance of considering the three rheological parameters when hole cleaning is modeled. To include the effect of both PL rheological parameters and the effect of all HB rheological parameters, we use the effective viscosity pertaining to the shear rate of particle sinking. Accordingly, the effective viscosities were plotted against particle settling velocities as illustrated in Figure 8. No discrimination was made between various models. The trend was consistent and linear with particle settling velocities decrease with increasing effective viscosities.

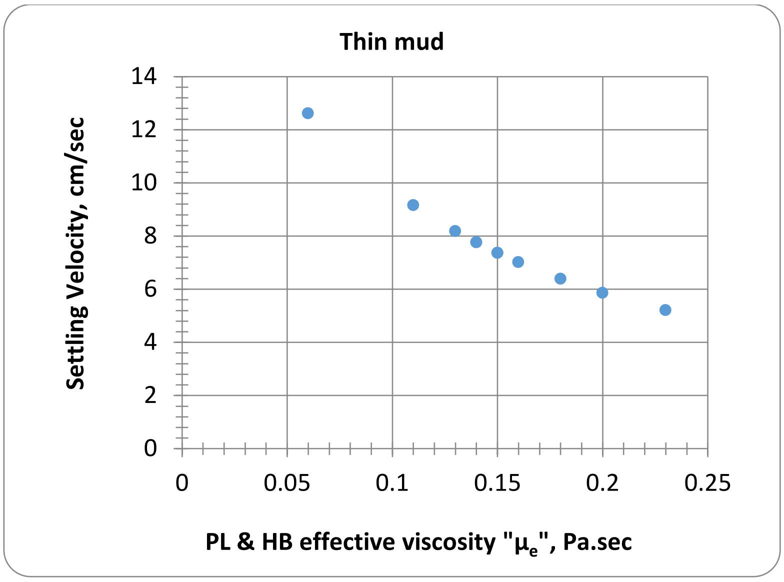


Figure 8: Effect of effective viscosity on settling velocity

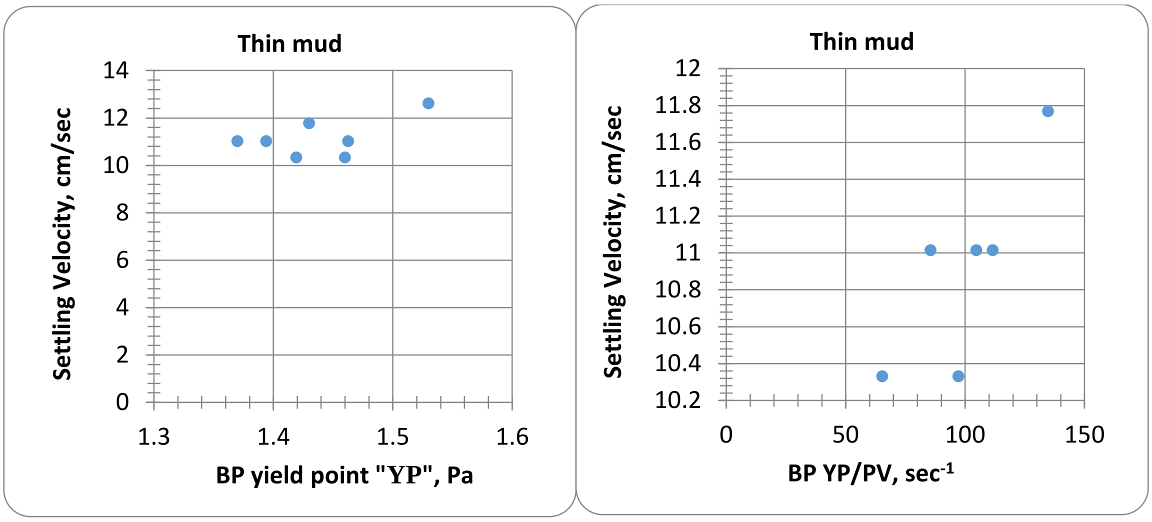


Figure 9: Effect of BP yield point and YP/PV on settling velocity

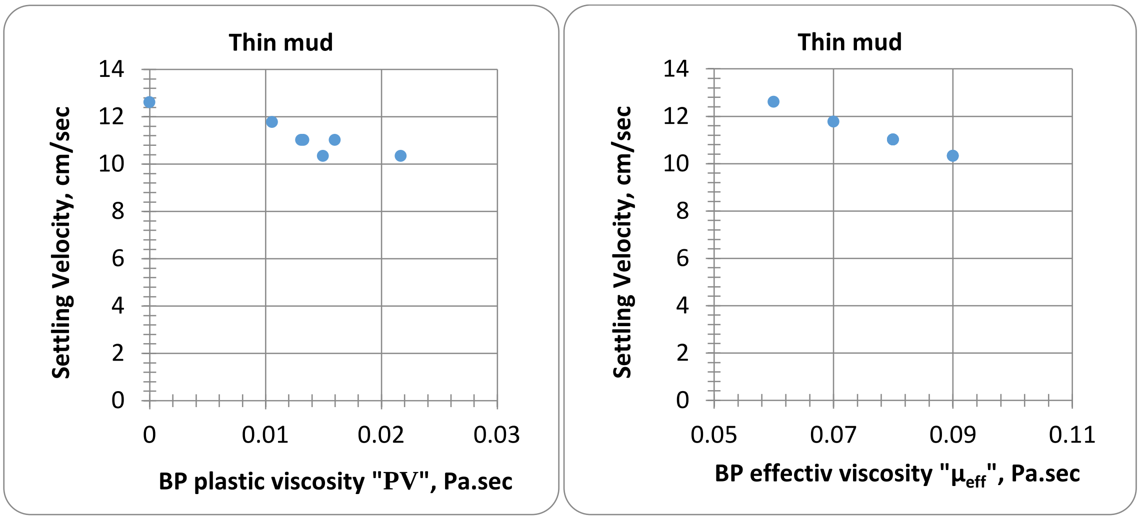


Figure 10: Effect of BP plastic and effective viscosities on settling velocity

The effect of Bingham plastic rheological parameters on particle settling velocities for the thin mud were considered in detail for all possible rheometric data of the Fann 35 VG meter. Unfortunately, it was not possible to estimate particle settling velocities for R_{600}/R_{300}, R_{600}/R_{200}, R_{600}/R_{100}, R_{600}/R_{3}, R_{300}/R_{200}, R_{300}/R_{100} and R_{200}/R_{100}. The yield points obtained from such data spans between 3.6 and 4.1 pascal which are more than double those obtained from other data pairs (R_{600}/R_{6}, R_{300}/R_{6}, R_{300}/R_{3}, R_{200}/R_{6}, R_{200}/R_{3}, R_{100}/R_{3} and R_{6}/R_{3}). This may be the reason why those data yielded a particle settling velocity approaching zero. It has been found earlier that Chien [10] correlation is very sensitive to fluid rheology and found not suitable to predict settling velocities in thick and intermediate muds. To illustrate the effect of BP rheological parameters on estimated settling velocities by Chien [10] correlation, rheological parameters determined from data pairs of R_{600}/R_{6}, R_{300}/R_{6},

R_{300}/R_3, R_{200}/R_6, R_{200}/R_3, R_{100}/R_3 and R_6/R_3 are plotted against the Chien [10] settling velocities.

As depicted in Figure 9, it was not possible to obtain a consistent relationship between BP yield point and settling velocities as well as between the BP yield-point/plastic-viscosity ratios with settling velocities. Fairly good and consistent trend was found between BP plastic viscosity and settling velocities. Excellent and perfect trend is shown between effective viscosity and settling velocities. Both trends are similar with settling velocities decreasing as the BP plastic viscosity and effective viscosity increases (Figure 10). It should be noted that the BP effective viscosities are determined from both rheological parameters (YP and PV) at the shear rate prevailing during particle settling.

5. Conclusions

(1) Herschle-Bulkley rheological parameters obtained from all rheometric data give paramount performance in re-producing the fluids rheogram. When re-calculating the thin mud rheogram, the lowest average error and standard deviation was seen with HB rheological parameters that are obtained out of non-linear regression of all data.

(2) Moore [7] correlation considerably underestimate the observed settling velocities while Chien [5] correlation significantly overestimate the observed settling velocities of both muds under three different fluid annular velocities. Both correlations yielded settling velocities that are less sensitive to annular velocities.

(3) Chien [10] correlation is very sensitive to mud rheology. The correlation was found not suitable for thick and intermediate muds with higher yield points in excess of 3 Pa. Generally, for the thin mud this correlation slightly underestimate the observed settling velocities for power law model and slightly overestimate observed settling velocities for BP model. For PL model, the rheological parameters obtained from R_{300}/R_6 showed the lowest error (-6 %E) followed by NLR rheology (-16 %E) while for BP model, the lowest error was observed with R_{100}/R_3 rheology (+6 %E) followed by LR and R_{300}/R_6 rheological data (+14 %E).

(4) Likewise the case with PL model, Chien [10] correlation for HB model slightly underestimate observed settling velocity with the lowest error been registered with rheological parameters obtained by solving non-linear regression equations (-6 %E).

(5) The simplified API method to determine Herschle-Bulkley rheological parameters may lead to considerable errors with regard to both describing the fluid rheology as well as predicting settling velocities that may by one way or another negate the usefulness of the model.

(6) Settling velocity estimated by Chien [10] correlation where found to decrease with decreasing fluid behaviour index and increasing fluid consistency index for both PL and HB models.

(7) Effective viscosities incorporate effect of all rheological parameters and represent excellent and consistent relationship with estimated settling velocities. The settling velocities decrease as the effective viscosity increases.

6. Recommendations

Based on the results shown in this study, it is strongly recommended to revise the API standards for determining Herschle-Bulkley rheological parameters and to weigh simplification against benefits.

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