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Numerical Study of Kermack-Mckendrik SIR Model to Predict the Outbreak of Ebola Virus Diseases Using Euler and Fourth Order Runge-Kutta Methods

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Abstract

Mathematical Modeling has emerged as a vital tool for understanding the dynamics of the spread of many infectious diseases, one amongst is Ebola virus. The main focus of this paper is to model mathematically the transmission dynamics of Ebola virus. For this purpose we tend to use basic SIR model of Ebola Virus to predict the outbreak of the diseases. As we cannot fully solve the 3 basic equations of SIR model with a certain formula solution, we introduce Euler and fourth-order Runge-Kutta methods (RK4). These two proposed strategies are quite efficient and practically well suited for solving initial value problem (IVP) for ordinary differential equations (ODE).We discuss the numerical comparisons between Euler method and Runge-Kutta methods and also discuss regarding their performances with the actual data. The population that we used for this model had roughly a similar number of individuals as the number was living in Republic of Liberia during 2014.

Keywords: Ebola; Outbreak; Evolution; Mathematical Modeling.

1. Introduction

Mathematical models are a powerful tool for investigating human infectious diseases, such as Ebola virus, contributing to the understanding of the dynamics of disease and providing useful predictions about the potential transmission of a disease and the effectiveness of possible control measures, which can provide valuable information for public health policy makers.

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The Ebola virus disease was first discovered in 1976 in the present Democratic Republic of Congo [1]. Since then, there have been many outbreaks; with the greatest was 2014 outbreak [2] which has spread through many countries. According to Bekoe (2015), from the first confirmed case recorded on 23 March 2014 which more than 18 months, at least 11,312 people have been reported died from the disease in six countries; Liberia, Sierra Leone, Guinea, Nigeria, Mali and US. Since the virus keeps spreading through contacts and the mortality rate of 0.7[3], it is needed to understand the patterns and epidemiology of the disease. By these conditions, mathematical model of the outbreaks of Ebola virus can be helpful as it is a platform for understanding the behavior of a dynamical system. The objectives are to understand better the mathematical dynamics of an infected population when an outbreak occurs. Another goal is to use mathematical modeling to examine and to analyze the viral dynamics of the Ebola virus. To model this outbreak, the systems of differential equations are used. To study the known data, several distinct models will be used and each model is different depends on the parameters acquired. I decided to model the Ebola Epidemics in Liberia in 2014 and compare their spread using an SIR model.

1.1 Objectives

The purposes of this research are

1. To apply SIR model to predict the outbreaks of Ebola virus.
2. To determine the effect of the initial number of infectives of the population.
3. Compare with real data and fit with model.

1.2 Scope

In the proposal study, we will only investigate the outbreaks of Ebola virus by applying the mathematical model. The data has been collected and it covers the area in the continent of Africa, on Liberia that was recorded by CDC [4]. The calculation will be done by using tools of MATLAB software.

2. Methodology

2.1 Formulation of SIR model

The SIR model is used to illustrate the transfer of the epidemic through the interaction of the following three different variables:

$S =$ Number of people

that are susceptible to Ebola

$I =$ Number of people infected with Ebola

$R =$ Number of people recovered

from Ebola with total immunity

It makes sense to assume that a fixed population of N people, whereby there are no births and deaths by natural cause i.e.

$N = S + I + R$ [5]

This is because the population is fixed and therefore, there are only three compartments in which the population may fit into. Thus, the total of the number of people susceptible infected and recovered in equivalent to the total population. The assumption that N is fixed, with no births or deaths, makes sense given 60 days, although it is a simplification.

These variables change over time, so we will define the variable $t =$ time in days. We will set $t = 0$ at the start of August 2014.

The model uses two parameters β and γ with $\beta, \gamma > 0$.

Given these parameters, the model uses 3 differential equations.

The rate of change of the number of people susceptible to the disease over time

$$\frac{dS}{dt} = -\beta IS \qquad (1)$$

The rate of change of the number of people recovered over time

$$\frac{dR}{dt} = \gamma I \qquad (2)$$

The rate of change of the number of people infected.

$$\frac{dI}{dt} = \beta IS - \gamma I \qquad (3)$$

Parameterization of the model

In order to calculate β (the rate of infection) and γ (the rate of recovery), it helps to define two more parameters.

$D =$ Duration of disease for those recovered

$M =$ Mortality rate for those who die per day

(0.7 for Ebola)

This leads to two further equations.

The rate at which the disease is spread

$$\gamma = \frac{1}{D} \quad [6] \tag{4}$$

The infection rate of the disease

$$\beta = \frac{M}{S} \quad [7] \tag{5}$$

2.2 Transformation of Runge-Kutta Equations for SIR modeling

RK4 is one of the classic methods for numerical integration of ODE models.

Consider the following initial problem of ODE

$$\frac{dy}{dt} = f(t, y)$$

$y(t_o) = y_o$

Where $y(t)$ is the unknown function (scalar or vector) which I would like to approximate.

The Iterative formula of RK4 method for solving ODE is as follows

$k_1 = hf(t_n, y_n)$

$k_2 = hf\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1\right)$

$k_3 = hf\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2\right)$

$k_4 = hf(t_n + h, y_n + k_3)$

$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

For simplicity, here we use the simplest SIR model to examine whether the RK4 method has been implemented correctly. The SIR model is defined as follows

$$\frac{dS}{dt} = -\beta IS$$

$$\frac{dI}{dt} = \beta IS - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

where $S(t)$ is the number of susceptible people in the population at time t, $I(t)$ is the number of infectious people at time t, $R(t)$ is the number of recovered people at time t, β is the transmission rate, γ represents the recovery rate, and

$N = S(t) + I(t) + R(t)$ is the fixed population.

According to the general iterative formula, the iterative formulas for $S(t), I(t)$ and $R(t)$ of SIR model can be written out

$$S_{n+1} = S_n + \frac{\Delta t}{6}(k_1^S + 2k_2^S + 2k_3^S + k_4^S)$$

$$k_1^S = f(t_n, S_n, I_n) = -\beta S_n I_n$$

$$k_2^S = f\left(t_n + \frac{\Delta t}{2}, S_n + \frac{k_1^S \Delta t}{2}, I_n + \frac{k_1^I \Delta t}{2}\right)$$

$$= -\beta\left(S_n + \frac{k_1^S \Delta t}{2}\right)(I_n + \frac{k_1^I \Delta t}{2})$$

$$k_3^S = f\left(t_n + \frac{\Delta t}{2}, S_n + \frac{k_2^S \Delta t}{2}, I_n + \frac{k_2^I \Delta t}{2}\right)$$

$$= -\beta\left(S_n + \frac{k_2^S \Delta t}{2}\right)(I_n + \frac{k_2^I \Delta t}{2})$$

$$k_4^S = f(t_n + \Delta t, S_n + k_3^S \Delta t, I_n + k_3^I \Delta t)$$

$$= -\beta(S_n + k_3^S \Delta t)(I_n + k_3^I \Delta t)$$

$$I_{n+1} = I_n + \frac{\Delta t}{6}(k_1^I + 2k_2^I + 2k_3^I + k_4^I)$$

$$k_1^I = f(t_n, S_n, I_n) = \beta S_n I_n - \gamma I_n$$

$$k_2^I = f\left(t_n + \frac{\Delta t}{2}, S_n + \frac{k_1^S \Delta t}{2}, I_n + \frac{k_1^I \Delta t}{2}\right) = \beta\left(S_n + \frac{k_1^S \Delta t}{2}\right)\left(I_n + \frac{k_1^I \Delta t}{2}\right) - \left(I_n + \frac{k_1^I \Delta t}{2}\right)$$

$$k_3^I = f\left(t_n + \frac{\Delta t}{2}, S_n + \frac{k_2^S \Delta t}{2}, I_n + \frac{k_2^I \Delta t}{2}\right)$$

$$= \beta\left(S_n + \frac{k_2^S \Delta t}{2}\right)\left(I_n + \frac{k_2^I \Delta t}{2}\right) - \left(I_n + \frac{k_2^I \Delta t}{2}\right)$$

$$k_4^I = f(t_n + \Delta t, S_n + k_3^S \Delta t, I_n + k_3^I \Delta t)$$

$$= \beta(S_n + k_3^S \Delta t)(I_n + k_3^I \Delta t) - (I_n + k_3^I \Delta t)$$

$$R_{n+1} = R_n + \frac{\Delta t}{6}(k_1^R + 2k_2^R + 2k_3^R + k_4^R)$$

$$k_1^R = f(t_n, I_n) = \gamma I_n$$

$$k_2^R = f\left(t_n + \frac{\Delta t}{2}, I_n + \frac{k_1^I \Delta t}{2}\right) = \gamma\left(I_n + \frac{k_1^I \Delta t}{2}\right)$$

$$k_3^R = f\left(t_n + \frac{\Delta t}{2}, I_n + \frac{k_2^I \Delta t}{2}\right) = \gamma\left(I_n + \frac{k_2^I \Delta t}{2}\right)$$

$$k_4^R = f(t_n + \Delta t, I_n + k_3^I \Delta t) = \gamma(I_n + k_3^I \Delta t)$$

2.3 Transformation of Euler Equations for SIR modeling

If we have a "slope formula," i.e., a way to calculate dy/dt at any point (t, y), then we can generate a sequence of y-values,

$$y_0, y_1, y_2, y_3 \dots\dots\dots\dots$$

By starting from a given y_0 and computing each *rise* as *slope* x *run*. That is,

$$y_{n+1} = y_n + slope_n \, \Delta t$$

where Δt is a suitably small step size in the time domain.

It really doesn't matter in this calculation if the slope formula happens to depend not just on t and y but on other variables, say x and z -- as long as we know how x and z are related to t and y. If x and z happen to be other dependent variables in a system of differential equations, I can generate values of x and z in the same way.

Of course, for the SIR model, we want the dependent variable names to be S, I and R. Thus we have three Euler formulas of the form

$$S_{n+1} = S_n + slope_n \, \Delta t$$

$I_{n+1} = I_n + slope_n \Delta t$

$R_{n+1} = R_n + slope_n \Delta t$

More specifically, given the SIR equations,

$\frac{dS}{dt} = -\beta IS$; $\frac{dI}{dt} = \beta IS - \gamma I$; $\frac{dR}{dt} = \gamma I$

The Euler formulas become

$S_{n+1} = S_n - \beta I_n S_n \Delta t$

$I_{n+1} = I_n + (\beta I_n S_n - \gamma I_n)\Delta t$

$R_{n+1} = R_n + \gamma I_n \Delta t$

To calculate something from these formulas, we must have explicit values *for* $\beta,\ \gamma,\ S(0),\ I(0),\ R(0)\ and\ \Delta t$.

3. Application, Comparison and Result Discussion

If we now take the example of the Ebola outbreak in Liberia 2014, we can assign the parameters with the following values. The total population of Liberia, N = 4294000 [8], and according to data from WHO, the number of people infected, I =391[9] and the number of people dead is 227 [9]. Seeing as R includes the number of people who have received permanent immunity, this includes those who have died as they have permanent immunity, in addition to those who have recovered with permanent immunity.

Therefore, number of people recovered $R = 227 + (0.3 \times 391) = 344$

We will now use this data to provide the parameters with the following values.

$N = 4294000$; $I = 391$; $R = 344$

Therefore, $S = N - I + R = 4294000 - (344 + 391) = 4293265$

The duration of the disease ranges from 2 to 21 days [10], therefore we could roughly estimate the duration of the disease at the midpoint, i.e.11 (approx.) days.

$D = 11$; $\gamma = \frac{1}{11} = 0.09$

According to WHO, the mortality rate of Ebola is 0.7 [3] and the number of people susceptible is 4293265.

Therefore, β (the rate of infection) $= \frac{0.7}{4293265} = 1.63 \times 10^{-7}$

In order to use the SIR model to predict the evolution of the disease, it would be helpful if we could solve the system of differential equations.

Unfortunately, we cannot completely solve these equations with an explicit formula solution.

Therefore, we will use numerical approaches. We will use Euler method and fourth order Runge-Kutta method to extract the solution.

3.1 Euler Method

For each day, we will calculate the values of S, I and R using

1. $S_{n+1} = S_n - \beta I_n S_n$
2. $I_{n+1} = I_n + (\beta I_n S_n - \gamma I_n)$
3. $R_{n+1} = R_n + \gamma I_n$

We take the initial values as

$S_0 = 4293265$; $I_0 = 391$; $R_0 = 344$; $\gamma = 0.09$; $\beta = 1.63 \times 10^{-7}$;

We will do this explicitly for the transition from t = 0 to t = 1. Using equations 1, 2 and 3, the following values for S, I and R can be calculated.

$S_1 = S_0 - \beta I_0 S_0 \times \Delta t = 4293265 - (1.63 \times 10^{-7} \times 391 \times 4293265) \times 1$

$= 4292991.377 \approx 4292991$

$I_1 = I_0 + (\beta I_0 S_0 - \gamma I_0) \times \Delta t = 391 + (1.63 \times 10^{-7} \times 391 \times 4293265 - 0.09 \times 391) \times 1$

$= 629.4326582 \approx 630$

$R_1 = R_0 + \gamma I_0$

$= 344 + 0.09 \times 391$

$= 379.19 \approx 379$

Here we use MATLAB to evaluate S, I, R over a two month period. **Table-3.1** shows the result.

Table 3.1: Euler Method

Day	**S**	**I**	**R**	**S+I+R**
01	4293265	391	344	4294000
02	4292991	630	379	4294000
03	4292550	1014	436	4294000
04	4291842	1630	528	4294000
05	4290700	2625	675	4294000
06	4288865	4225	910	4294000
07	4285912	6798	1290	4294000
08	4281162	10936	1902	4294000
09	4273530	17584	2886	4294000
10	4261283	28248	4469	4294000
11	4241662	45327	7011	4294000
12	4210323	72586	11091	4294000
13	4160509	115868	17623	4294000
14	4081931	184017	28052	4294000
15	3959494	289892	44614	4294000
16	3772399	450898	70703	4294000
17	3495141	687575	111284	4294000
18	3103424	1017410	173166	4294000
19	2588759	1440508	264733	4294000
20	1980912	1918710	394378	4294000
21	1361382	2365556	567062	4294000
22	836453	2677585	779962	4294000
23	471386	2801669	1020945	4294000
24	256117	2764788	1273095	4294000
25	140695	2631379	1521926	4294000
26	80349	2454901	1758750	4294000
27	48198	2266111	1979691	4294000
28	30394	2079964	2183642	4294000
29	20090	1903072	2370838	4294000
30	13858	1738028	2542114	4294000
31	9932	1585531	2698537	4294000
32	7365	1445400	2841235	4294000
33	5630	1317049	2971321	4294000
34	4422	1199723	3089855	4294000
35	3557	1092613	3197830	4294000

36	2923	994911	3296166	4294000
37	2449	905843	3385708	4294000
38	2088	824679	3467233	4294000
39	1807	750738	3541455	4294000
40	1586	683393	3609021	4294000
41	1409	622065	3670526	4294000
42	1266	566222	3726512	4294000
43	1150	515378	3777472	4294000
44	1053	469091	3823856	4294000
45	972	426953	3866075	4294000
46	905	388595	3904500	4294000
47	847	353679	3939474	4294000
48	799	321896	3971305	4294000
49	757	292968	4000275	4294000
50	720	266637	4026643	4294000
51	689	242671	4050640	4294000
52	662	220858	4072480	4294000
53	638	201004	4092358	4294000
54	617	182935	4110448	4294000
55	599	166489	4126912	4294000
56	583	151521	4141896	4294000
57	568	137899	4155533	4294000
58	555	125501	4167944	4294000
59	544	114217	4179239	4294000
60	533	103948	4189519	4294000

3.2 Runge-Kutta (RK4) Method

For each day, we will calculate the values of $S,\ I$ and R using

$$k_1^S = -\beta S_n I_n$$

$$k_1^I = \beta S_n I_n - \gamma I_n$$

$$k_1^R = \gamma I_n$$

$$k_2^S = -\beta\left(S_n + \frac{k_1^S \Delta t}{2}\right)(I_n + \frac{k_1^I \Delta t}{2})$$

$$k_2^I = \beta\left(S_n + \frac{k_1^S \Delta t}{2}\right)\left(I_n + \frac{k_1^I \Delta t}{2}\right) - \left(I_n + \frac{k_1^I \Delta t}{2}\right)$$

$$k_2^R = \gamma\left(I_n + \frac{k_1^I \Delta t}{2}\right)$$

$$k_3^S = -\beta\left(S_n + \frac{k_2^S \Delta t}{2}\right)(I_n + \frac{k_2^I \Delta t}{2})$$

$$k_3^I = \beta\left(S_n + \frac{k_2^S \Delta t}{2}\right)\left(I_n + \frac{k_2^I \Delta t}{2}\right) - \left(I_n + \frac{k_2^I \Delta t}{2}\right)$$

$$k_3^R = \gamma\left(I_n + \frac{k_2^I \Delta t}{2}\right)$$

$$k_4^S = -\beta(S_n + k_3^S \Delta t)(I_n + k_3^I \Delta t)$$

$$k_4^I = \beta(S_n + k_3^S \Delta t)(I_n + k_3^I \Delta t) - (I_n + k_3^I \Delta t)$$

$$k_4^R = \gamma(I_n + k_3^I \Delta t)$$

$$S_{n+1} = S_n + \frac{\Delta t}{6}(k_1^S + 2k_2^S + 2k_3^S + k_4^S)$$

$$I_{n+1} = I_n + \frac{\Delta t}{6}(k_1^I + 2k_2^I + 2k_3^I + k_4^I)$$

$$R_{n+1} = R_n + \frac{\Delta t}{6}(k_1^R + 2k_2^R + 2k_3^R + k_4^R)$$

We take the initial values as

$S_0 = 4293265$; $I_0 = 391$; $R_0 = 344$; $\gamma = 0.09$; $\beta = 1.63 \times 10^{-7}$

We will do this explicitly for the transition from t = 0 to t = 1. Using those equations the following values for S, I and R can be calculated.

$$k_1^S = -\beta S_0 I_0$$

$$= -1.63 \times 10^{-7} \times 4293265 \times 391 = -273.622658245000$$

$$k_1^I = \beta S_0 I_0 - \gamma I_0 = 1.63 \times 10^{-7} \times 4293265 \times 391 - 0.09 \times 391$$

$$= 238.432658245000$$

$$k_1^R = \gamma I_0 \quad = 0.09 \times 391 \quad = 35.1900000000000$$

$$k_2^S = -\beta \left(S_0 + \frac{k_1^S \Delta t}{2} \right) (I_0 + \frac{k_1^I \Delta t}{2})$$

$$= -1.63 \times 10^{-7} \times \left(4293265 - \frac{273.622658245000}{2} \right) \times \left(391 + \frac{238.432658245000}{2} \right)$$

$$= -357.039129094785$$

$$k_2^I = \beta \left(S_0 + \frac{k_1^S \Delta t}{2} \right) \left(I_0 + \frac{k_1^I \Delta t}{2} \right) - \left(I_0 + \frac{k_1^I \Delta t}{2} \right)$$

$$= -1.63 \times 10^{-7} \times \left(4293265 + \frac{-273.622658245000}{2} \right) \times \left(391 + \frac{238.432658245000}{2} \right) - \left(391 + \frac{238.432658245000}{2} \right) = 311.119659473760$$

$$k_2^R = \gamma \left(I_0 + \frac{k_1^I \Delta t}{2} \right) = 0.09 \times \left(391 + \frac{238.432658245000}{2} \right) = 45.9194696210250$$

$$k_3^S = -\beta \left(S_0 + \frac{k_2^S \Delta t}{2} \right) (I_0 + \frac{k_2^I \Delta t}{2})$$

$$= -1.63 \times 10^{-7} \times \left(4293265 + \frac{-357.039129094785}{2} \right) \left(391 + \frac{311.119659473760}{2} \right)$$

$$= -382.467864374178$$

$$k_3^I = \beta \left(S_0 + \frac{k_2^S \Delta t}{2} \right) \left(I_0 + \frac{k_2^I \Delta t}{2} \right) - \left(I_0 + \frac{k_2^I \Delta t}{2} \right)$$

$$= 1.63 \times 10^{-7} \times \left(4293265 + \frac{-357.039129094785}{2} \right) \times \left(391 + \frac{311.119659473760}{2} \right) - \left(391 + \frac{311.119659473760}{2} \right)$$

$$= 333.277479697859$$

$$k_3^R = \gamma \left(I_0 + \frac{k_2^I \Delta t}{2} \right) = 0.09 \times \left(391 + \frac{311.119659473760}{2} \right) = 49.1903846763192$$

$$k_4^S = -\beta (S_0 + k_3^S \Delta t)(I_0 + k_3^I \Delta t)$$

$$= -1.63 \times 10^{-7} \times (4293265 - 382.467864374178) \times (391 + 333.277479697859)$$

$$= -506.805816985307$$

$$k_4^I = \beta(S_0 + k_3^S \Delta t)(I_0 + k_3^I \Delta t) - (I_0 + k_3^I \Delta t)$$

$$= 1.63 \times 10^{-7} \times (4293265 - 382.467864374178) \times (391 + 333.277479697859) - (391 + 333.277479697859)$$

$$= 441.62084381250$$

$$k_4^R = \gamma(I_0 + k_3^I \Delta t)$$

$$= 0.09 \times (391 + 333.277479697859) = 65.1849731728073$$

$$\therefore S_1 = S_0 + \frac{\Delta t}{6}(k_1^S + 2k_2^S + 2k_3^S + k_4^S)$$

$$= 4293265 + \frac{1}{6}(-273.622658245000 - 2 \times 357.039129094785 - 2 \times 382.467864374178 - 506.805816985307)$$

$$= 4292888.42625631 \approx 4292888$$

$$\therefore I_1 = I_0 + \frac{\Delta t}{6}(k_1^I + 2k_2^I + 2k_3^I + k_4^I)$$

$$= 391 + \frac{1}{6}(238.432658245000 + 311.119659473760 + 333.277479697859 + 441.62084381250)$$

$$= 719.141296733456 \approx 719$$

$$\therefore R_1 = R_0 + \frac{\Delta t}{6}(k_1^R + 2k_2^R + 2k_3^R + k_4^R)$$

$$= 344 + \frac{1}{6}(35.1900000000000 + 45.9194696210250 + 49.1903846763192 + 65.1849731728073)$$

$$= 392.432446961249 \approx 393$$

Here we use MATLAB to evaluate S, I, R over a two month period.**Table-3.2** shows the result.

Table 3.2

RK4 Method				
Day	S	I	R	S+I+R
01	4293265	391	344	4294000
02	4292888	719	393	4294000
03	4292196	1322	482	4294000
04	4290923	2432	645	4294000
05	4288583	4471	946	4294000
06	4284286	8214	1500	4294000
07	4276407	15077	2516	4294000
08	4261994	27626	4380	4294000
09	4235753	50457	7790	4294000
10	4188376	91623	14001	4294000
11	4104130	164649	25221	4294000
12	3958283	290515	45202	4294000
13	3717118	496967	79915	4294000
14	3346977	809182	137841	4294000
15	2838838	1226389	228773	4294000
16	2237427	1696343	360230	4294000
17	1636735	2124419	532846	4294000
18	1127150	2428029	738821	4294000
19	747957	2580728	965315	4294000
20	489487	2605010	1199503	4294000
21	321556	2540850	1431594	4294000
22	214459	2424226	1655315	4294000
23	146134	2280695	1867171	4294000
24	102037	2126431	2065532	4294000
25	73073	1971024	2249903	4294000
26	53657	1819893	2420450	4294000
27	40360	1675938	2577702	4294000
28	31056	1540566	2722378	4294000
29	24413	1414309	2855278	4294000
30	19575	1297197	2977228	4294000
31	15987	1188973	3089040	4294000
32	13279	1089223	3191496	4294000
33	11204	997456	3285340	4294000
34	9589	913148	3371263	4294000

35	8316	835770	3449914	4294000
36	7300	764806	3521894	4294000
37	6479	699764	3587757	4294000
38	5809	640176	3648015	4294000
39	5257	585604	3703139	4294000
40	4798	535640	3753562	4294000
41	4414	489905	3799681	4294000
42	4089	448050	3841861	4294000
43	3813	409750	3880437	4294000
44	3577	374709	3915714	4294000
45	3374	342652	3947974	4294000
46	3199	313328	3977473	4294000
47	3046	286506	4004448	4294000
48	2913	261974	4029113	4294000
49	2796	239538	4051666	4294000
50	2694	219019	4072287	4294000
51	2603	200255	4091142	4294000
52	2523	183095	4108382	4294000
53	2452	167404	4124144	4294000
54	2389	153057	4138554	4294000
55	2333	139937	4151730	4294000
56	2283	127941	4163776	4294000
57	2238	116972	4174790	4294000
58	2197	106943	4184860	4294000
59	2161	97773	4194066	4294000
60	2128	89389	4202483	4294000

Figure 3.1-3.3 shows the comparison between Euler and RK-4 method in the cases of infectives, recovered and susceptives respectively.

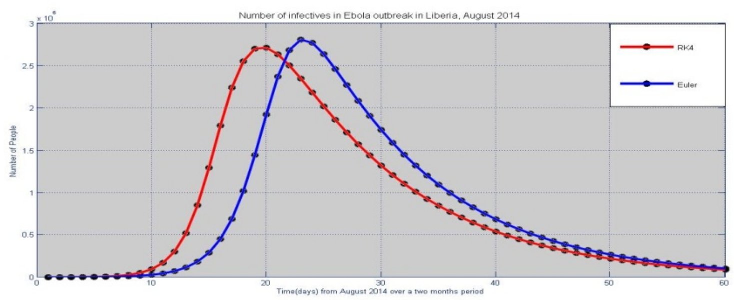

Figure 3.1

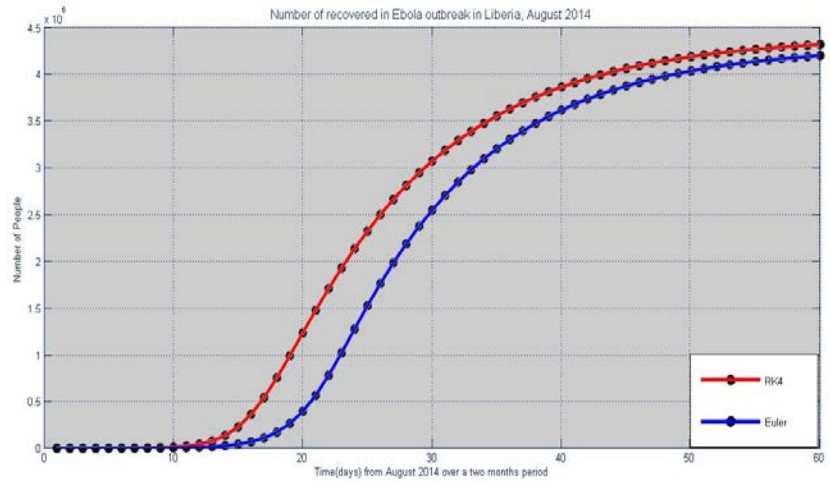

Figure 3.2

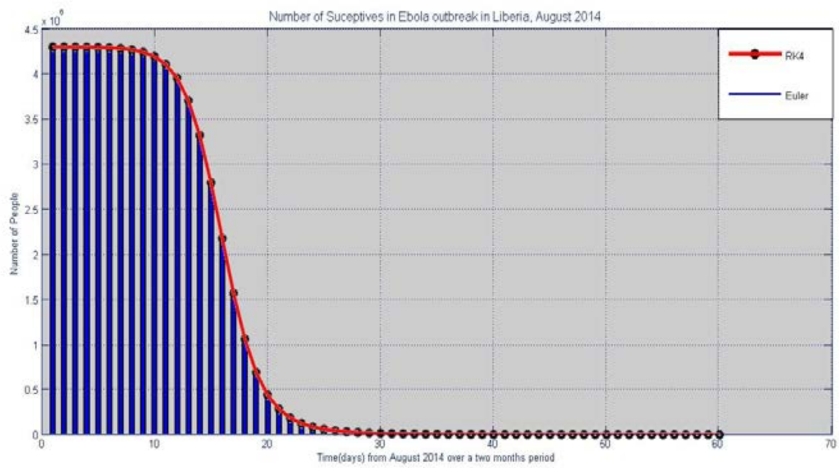

Figure 3.3

For predict the evolution, we only discuss about the infectives.

Each strategies show that initially, the number of individuals infected will increase steeply, however, over an extended period of time, the numbers eventually decrease. This happens simultaneously because the range of individuals recovered will increase because as those infected decreases, they're being transferred into the recovered class. The main reason is owing to the raised awareness of the disease leasing to additional medical support being given in order to assist combat the transmission of the disease. Furthermore, an increased awareness ends up in additional folks being conscious of strategies of protection. The steep increase within the beginning of the primary fifteen days is possibly to flow from to the good uncertainty that lied with Ebola allowing a greater rate of transmission. The peak of every graph illustrates the utmost number of individuals ever to be infected and once this point; there's a transition whereby the numbers decrease.

3.3 Comparing the model to actual data

However, in order for the model to be valid and allow informing government policy, it obviously needs to correspond fairly close to reality.

The **Table-3.3.1** below compares the data collected from the SIR model (using Euler and RK4) for the number of people infected and the real life data of the number of people infected.

Table 3.3.1

Time(Days)	I -Actual	I- Euler	I -RK4	Time(Days)	I -Actual	I- Euler	I -RK4
1	391	391	391	35	1871	1092613	835770
2	486	630	719	37	2046	905843	699764
6	554	4225	8214	41	2081	622065	489905
10	599	28248	91623	45	2407	426953	342652
11	670	45327	164649	47	2710	353679	286506
13	786	115868	496967	51	3022	242671	200255
17	834	687575	2124419	53	3280	201004	167404
19	972	1440508	2580728	55	3458	166489	139937
20	1082	1918710	2605010	60	3696	103948	89389
26	1378	2454901	1819893				

Using the value of **Table-3.3.1** we can plot a graph using MATLAB which compares the model data to the actual data for the number of people infected (**Figure-3.4**).

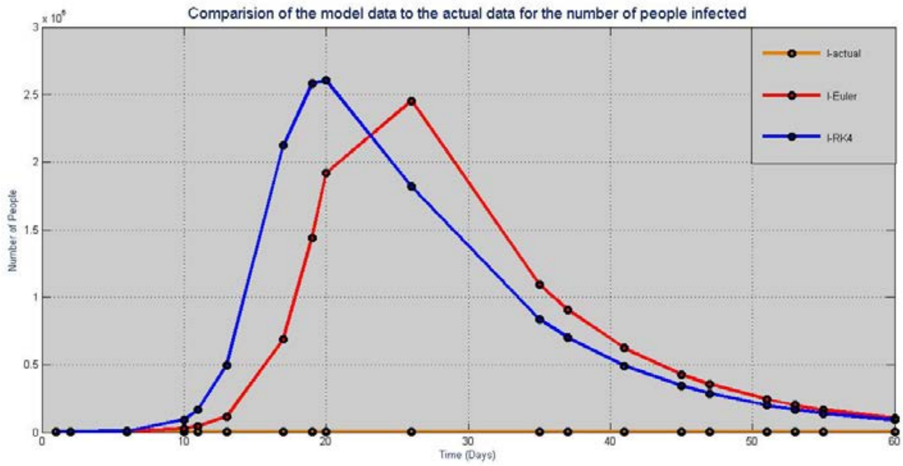

Figure 3.4

As the graph demonstrates, the real data does not correspond very well to the data received from the model. Although the actual data may seem to follow a straight like graph, this is untrue as it is only depicted in this manner due to the limitations on the axis of the graph. The difference between the real life data and the data from the model is so vast that the straight line looks like a graph of $y = 0$. Therefore, we decided to plot it separately (**Figure-3.5**).

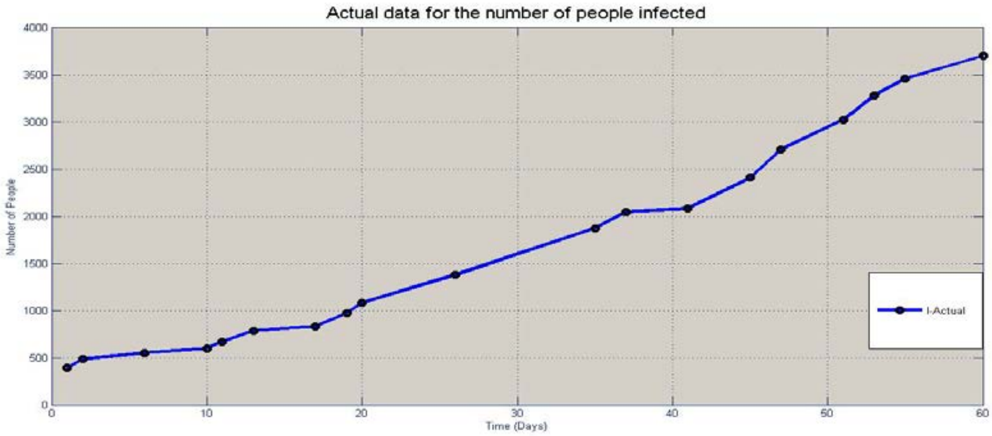

Figure 3.5

3.4 Curve Fitting with Actual Data

The model has significantly overestimated the number of individuals who can become infected with Ebola fever. This is often because of the many limitations that the model presents. One of the main limitations includes the incorrect beta and gamma values that were calculated. Once fixing the beta and gamma values, we were able to find another gamma value that resulted in similar values to the real data. Here is that the graph to indicate this, with the suitable gamma value of 0.66. The **Table-3.4.1** below compares the data collected from the fitted model for the number of people infected and the real life data of the number of people infected.

Table-3.4.1

Time(Days)	I -Actual	I- Euler	I -RK4	Time(Days)	I -Actual	I- Euler	I -RK4
1	391	391	391	35	1871	1555	1419
2	486	408	407	37	2046	1679	1525
6	554	482	475	41	2081	1952	1758
10	599	569	555	45	2407	2261	2020
11	670	593	576	47	2710	2430	2163
13	786	644	623	51	3022	2796	2472
17	834	759	726	53	3280	2994	2639
19	972	824	783	55	3458	3201	2814
20	1082	858	813	60	3696	3757	3287
26	1378	1092	1019				

Using the value of **Table-3.4.1** we can plot a graph using MATLAB which compares curve fitted data with real data for the number of people infected (**Figure-3.6**).

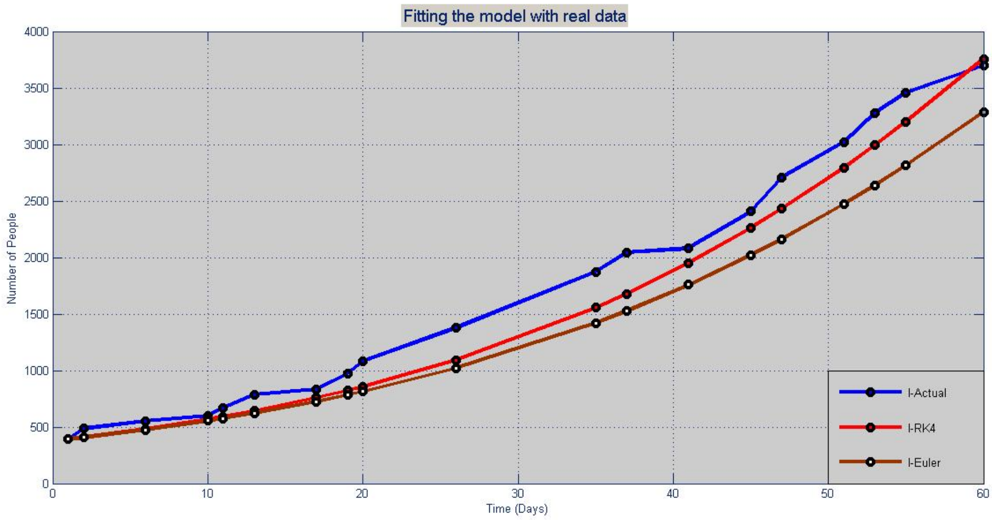

Figure 3.6

3.5 Result Discussion

Although, this doesn't match the graph exactly, it shows a much better correlation of the number of individuals infected. Between Euler and RK4 technique, Rk4 provides better correlation as because Rk4 methods divide its step size more than Euler methods and the local truncation error is less than Euler methods. Therefore, in order to improve the model, many changes should be done, together with altering the gamma value. The value that we eventually used to alter the model, led to being in 2 decimal places. This goes to illustrate the necessary precision required as very little deviance will cause massive changes. This can be because; the gamma is calculated through extreme simplification, leaving great possibilities for more room for errors. Furthermore, it is tough to differentiate between the numbers of people who have died and also the numbers of people who have survived with permanent immunity as they both fall under the same class of being 'recovered'. It is also a fact that rate of recovery is faster that the time scale of birth and death. The infection rate and recovery rate plays a vital role in the model and in this method we find more accurate value of infection rate and recovery rate to fit the model with the actual data collected from CDC.

4. Conclusion

The results obtained from modeling data will lead to completely different views and interpretations. This is due to the unequal distribution of knowledge across the globe whereby in countries like Liberia, there is little access to the statistics which makes it troublesome to form constructive predictions regarding the outbreak. Through our research, we have gained further insight into the uses of mathematical modeling so as to work out the outbreak of diseases similarly as evaluating its flaws. Having chosen Ebola as the diseases of concentration, as it is incredibly relevant to this situation in continent, it has enabled a practical understanding of its rate of transmission.

5. Recommendations

The Numerical methods give a better prediction if the data collected from an authentic source. This result can help to estimate future predictions of the disease and consequently, it will facilitate to determine practical components like the quantity of beds required in the hospitable, number of vaccination and reallocation costs etc.

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